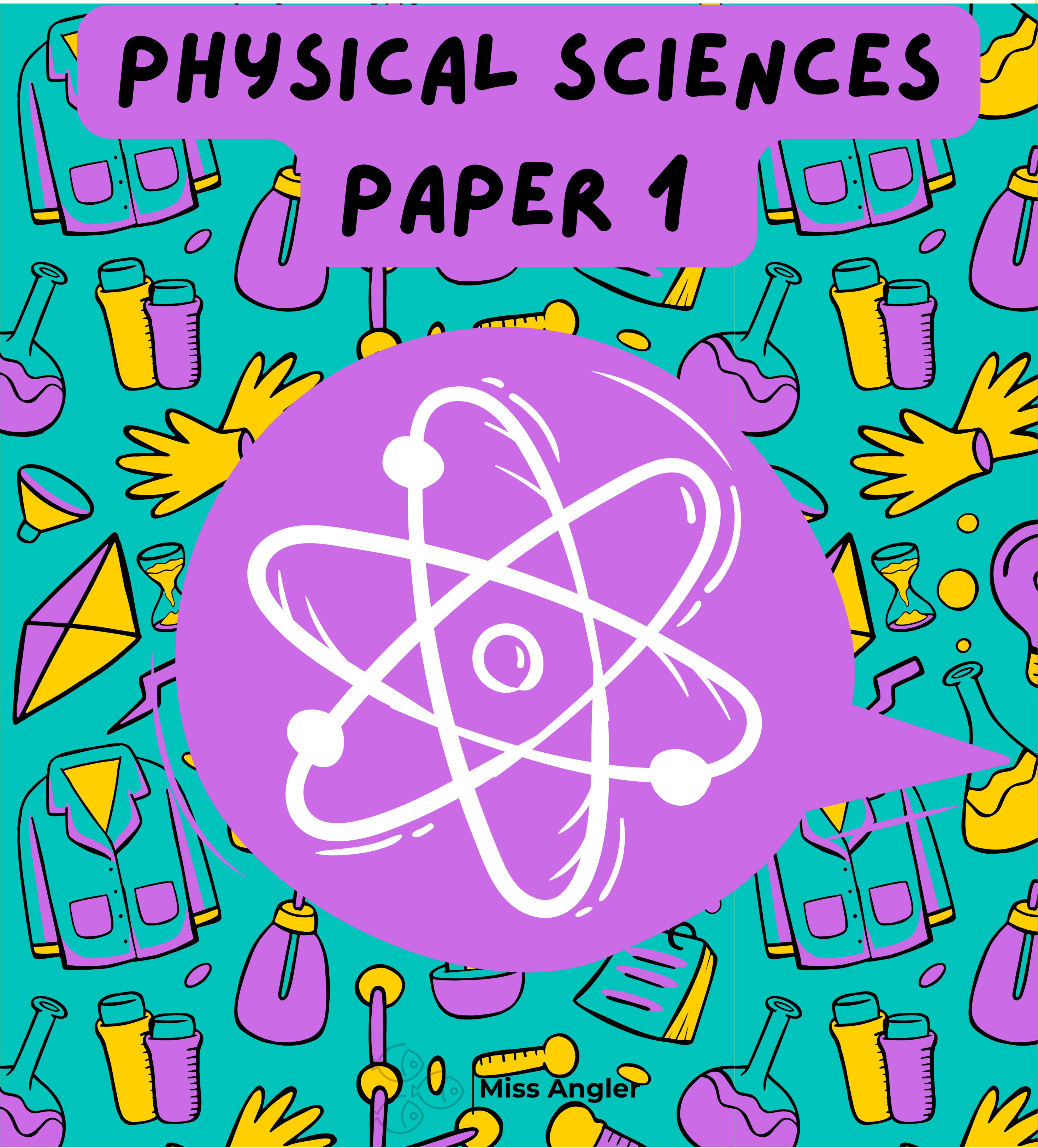
THE CHEAT SHEET

Grade

12

CAPS

PHYSICAL SCIENCES PAPER 1



Miss Angler

THE CHEAT SHEET: GR. 12 PHYSICAL SCIENCES: PAPER 1

Miss Angler

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Department of Basic Education pg. 10 & 11
Quizlet pg.87 & 221 (www.quizlet.com)
Libretext Chemistry pg. 240 (chem.libretexts.org)
Pngwing pg. 209 (www.pngwing.com)
Socratic pg. 88 & 89 (www.socratic.org)

CONTENTS PAGE

HOW TO USE THIS STUDY GUIDE

PLEASE REMEMBER: This study guide does not replace your textbook or the instruction of your teacher. Its purpose is to make learning and revision EASY!

Difficulty levels of questions

Your exam will include questions covering three difficulty levels.
Each level of question will require a different level of understanding and way of answering the question.

The difficulty levels are summarized in this study guide using the acronym **C.A.P:**

C — COMPREHENSION AND RECALL QUESTIONS

These are common questions which include definitions and calculation questions that look similar to the questions covered in class.
Approximately **50%** of Paper 1 (Physics) will include questions on this level.

A — ANALYSIS AND APPLICATION QUESTIONS

These are more complex questions which involves applying the knowledge and skills learned in this chapter.
Approximately **40%** of Paper 1 (Physics) will include questions on this level.

P — PROBLEM- SOLVING QUESTIONS

These are questions that require critical thinking and being able to make connections between different representations of information and integrating different topics. They are not familiar questions, but are able to be solved through critical analysis. Approximately **10%** of Paper 1 (Physics) will include questions on this level.

WORKED EXAMPLE KEY

This study guide will highlight which questions are classified under each difficulty level. Look out for **C, A or P** next to each question.

KEY NOTES

In this study guide, the worked examples have explanations added.

Note

Scenario

Answer

Worked examples

Multiple choice questions C + A

Question

PAPER 1 TIPS FOR SUCCESS

100%

Mathematical formulae

The following Mathematical formulae can be integrated into topics in paper 1 (Physics):

- **Quadratic formula** for quadratic equations in the form $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- **Theorem of Pythagoras:** $a^2 = b^2 + c^2$ **OR** $hyp^2 = side_1^2 + side_2^2$
- **Trigonometric ratios:**

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

Applied to right angled triangles only

- **How to solve an equation with TWO unknowns?**
  Set up a second equation and solve it using **simultaneous equations.**

Scientific method

Variables in a scientific investigation:

- **Independent variable:** Variable that is **deliberately** changed or manipulated in an investigation. The independent variable does not depend on the dependent variable in the investigation.
- **Dependent variable:** Variable that is being **measured** in an investigation.
  The dependent variable depends on the independent variable in the investigation.
- **Control variable(s) or fixed variable(s):** Variable(s) that are kept constant in an investigation,to ensure that the investigation is a fair test.

Scientific terms used in a scientific investigation:

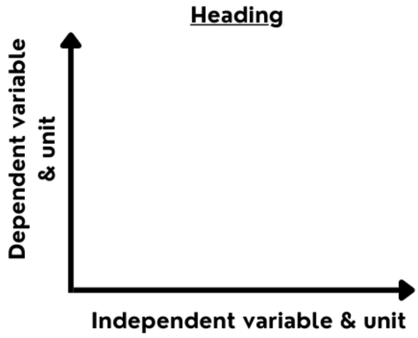
- **Investigative question**: **Question** about what is investigated. The investigative question must include:
  - The independent variable
  - The dependent variable
  - A question mark at the end.

A mark will be lost for the investigative question if there is no question mark.

- **Hypothesis:** An educated guess; a **statement** of what the relationship between the independent and dependent variable is.
  The hypothesis is an answer to the investigative question.
- **Fair test:** An investigation is a fair test if it only has **ONE** independent i.e., one variable being deliberately changed or manipulated.

Graphs

Graph sketching

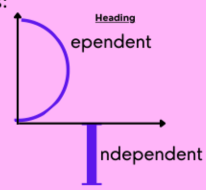


Tips for graph sketching:

- Sketch the graph and axes in pencil. The heading can be done in pen/pencil.
- The graph must be a reasonable size. A half a page can be used as a guideline.
- The graph must have a heading, which includes the **independent and dependent variable.** For example, velocity - time graph.
- Underline the heading.
- The axes must be labelled, with the **variables and units**. The independent variable is on the x - axis and the dependent variable on the y - axis.
- When sketching graphs, plotted values of x and y must be indicated on the **x - axis** and **y - axis**.

PRO-TIPS

Method to remember that the independent variable is on the x - axis and the dependent variable on the y- axis:



Graph interpretation

Straight - line graphs:

Gradient of the graph:

Gradient = $\dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Equation of the standard form of a straight - line graph: **y = mx + c**

Where:

m = gradient of the graph and/ or physical quantity represented by the gradient
c = y - intercept of the graph and/ or physical quantity represented by the y - intercept
x = input values and/ or physical quantity on the x - axis
y = output values and/ or physical quantity on the y - axis

PRO-TIPS

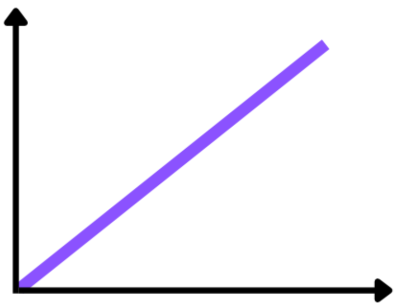
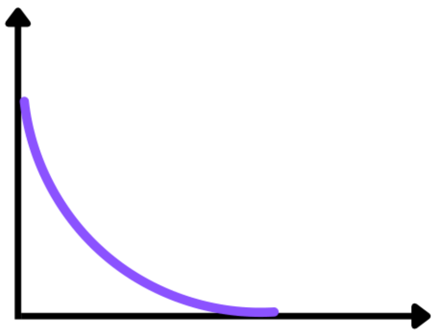
The variables in the straight - line graph equation can be replaced with physical quantities.

2D shape	Formula to calculate the area
Rectangle	A = l x b
Triangle	A = $\frac{1}{2}$b x $\perp$h
Square	A = l x b or A = s^2 (where s = side)

Area under the graph:

As with the gradient of the graph, the **area under the graph** represents a different physical quantity, which can be calculated.

Relationship between variables: Graphical representation

Directly proportional relationship	Inversely proportional relationship
$a \propto b$ • As variable 'a' increases, variable 'b' increases by the same factor. • In a directly proportional relationship, the **ratio** between the two variables is constant.	$a \propto \frac{1}{b}$ • As one variable 'b' increases, the other variable 'a' **decreases** by the same factor. • e.g., if variable 'b' is doubled, variable 'a' is halved. • In a inversely proportional relationship, the the **product** of two variables (e.g., a x b) is constant
 **NOTE:** A directly proportional relationship is represented by a straight - line graph which starts at the origin.	 **NOTE:** An inversely proportional relationship is represented by a smooth curve in quadrant I, since most real - life inversely proportionally relationships are positive.

General tips

- Format for **calculation questions**:

✓ Formula
✓ Substitution

To earn a formula mark, the substitution must be shown.

✓ Answer + units. **NOTE:** Round off only the final answer to two decimal places.
✓ If the physical quantity is a **vector quantity**, the direction must also be indicated, unless the question asks for magnitude only, then **only** the magnitude (size) must be stated.

Remember: For vector calculations, state your sign convention i.e., positive direction at the **START** of the calculation.

- Use bullet points as far as possible when explaining answers.
- Learn the definitions as prescribed from the **Grade 12 guidelines.**
- Use the formulae given on the **data sheet.**

PAPER 1: PHYSICS
SUMMARY OF THE GRADE 12 GUIDELINE DEFINITIONS

Mechanics

- **Normal force, N:** The force or the component of a force which a surface exerts on an object with which it is in contact, and which is perpendicular to the surface.

- **Frictional force, f:** The force that opposes the motion of an object and which acts parallel to the surface.

- **Static frictional force, f_s:** The force that opposes the tendency of motion of a stationary object relative to a surface.

- **Kinetic frictional force, f_k:** The force that opposes the motion of a moving object relative to a surface.

- **Newton's First Law of Motion:** A body will remain in its state of rest or motion at constant velocity unless a non-zero resultant/net force acts on it.

- **Newton's Second Law of Motion:** When a net force acts on an object, the object will accelerate in the direction of the force at an acceleration directly proportional to the force and inversely proportional to the mass of the object.

- **Newton's Third Law of Motion:** When object A exerts a force on object B, object B SIMULTANEOUSLY exerts an oppositely directed force of equal magnitude on object A.

Newton's law of Universal Gravitation

- **Newton's Law of Universal Gravitation:** Each body in the universe attracts every other body with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.

- **Weight:** The gravitational force the Earth exerts on any object on or near its surface.

Vertical projectile motion

- **Acceleration:** Rate of change of velocity.

- **Projectile:** An object which has been given an initial velocity and then it moves under the influence of the gravitational force only.

- **Free fall:** Motion during which the only force acting on an object is the gravitational force.

Momentum and impulse

- **Momentum:** The product of an object's mass and its velocity.
- **Newton's Second Law of Motion in terms of momentum:** The net (or resultant) force acting on an object is equal to the rate of change of momentum of the object in the direction of the net force.
- **Impulse:** The product of the resultant/net force acting on an object and the time the resultant/net force acts on the object.
- **Isolated system:** A system on which the resultant/net external force is zero.
- **The Principle of Conservation of Linear Momentum:** The total linear momentum of an isolated system remains constant (is conserved).

Work, energy and power

- **Work done:** Work done on an object by a constant force F as F Δx cos θ, where F is the magnitude of the force, Δx the magnitude of the displacement and θ the angle between the force and the displacement.
- **Work-energy theorem:** The work done on an object by a net force is equal to the change in the object's kinetic energy.
- **Conservative force**: A force for which the work done in moving an object between two points is independent of the path taken.
- **Non-conservative force:** A force for which the work done in moving an object between two points depends on the path taken.
- **The Principle of Conservation of Mechanical Energy:** The total mechanical energy in an isolated system remains constant. (A system is isolated when the net external force acting on the system is zero).
- **Power:** The rate at which work is done or energy is expended.

Doppler effect

- **Doppler Effect:** The change in frequency (or pitch) of the sound detected by a listener, because the sound source and the listener have different velocities relative to the medium of sound propagation.

Electrostatics

- **Coulomb's Law:** The magnitude of the electrostatic force exerted by one point charge (Q_1) on another point charge (Q_2) is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance (r) between them.

- **Electric field:** A region of space in which an electric charge experiences a force. The direction of the electric field at a point is the direction that a positive test charge would move if placed at that point.

- **Electric field at a point:** The electric field at a point is the electrostatic force experienced per unit positive charge placed at that point.

Electric circuits

- **Ohm's Law in words:** The potential difference across a conductor is directly proportional to the current in the conductor at constant temperature.

- **Power:** The rate at which work is done.

- **emf:** Maximum energy provided by a battery per unit charge passing through it.

Electrodynamics

- **Rms potential difference:** is the AC potential difference which dissipates/produces the same amount of energy as an equivalent DC potential difference.

- **Rms current:** is the alternating current which dissipates/produces the same amount of energy as an equivalent direct current (DC).

Optical phenomena (Photoelectric effect)

- **Photoelectric effect:** The process whereby electrons are ejected from a metal surface when light of suitable frequency is incident on that surface.

- **Threshold frequency, f_0:** The minimum frequency of light needed to emit electrons from a certain metal surface.

- **Work function, W_0:** The minimum energy that an electron in the metal needs to be emitted from the metal surface.

- **Atomic absorption spectrum:** is formed when certain frequencies of electromagnetic radiation passing through a substance is absorbed.

- **Atomic emission spectrum:** is formed when certain frequencies of electromagnetic radiation are emitted due to an atom making a transition from a higher energy state to a lower energy state.

NOTE:
Definitions are usually awarded **2 marks.**
A definition is asked per question or per section of work.
All definitions must be stated as per the grade 12 guideline definitions.

PAPER 1 DATA SHEET

DATA FOR PHYSICAL SCIENCES GRADE 12
PAPER 1 (PHYSICS)

TABLE 1: PHYSICAL CONSTANTS

NAME	SYMBOL	VALUE
Acceleration due to gravity	g	$9{,}8\ m\cdot s^{-2}$
Universal gravitational constant	G	$6{,}67 \times 10^{-11}\ N\cdot m^2\cdot kg^{-2}$
Speed of light in a vacuum	c	$3{,}0 \times 10^{8}\ m\cdot s^{-1}$
Planck's constant	h	$6{,}63 \times 10^{-34}\ J\cdot s$
Coulomb's constant	k	$9{,}0 \times 10^{9}\ N\cdot m^2\cdot C^{-2}$
Charge on electron	e	$-1{,}6 \times 10^{-19}\ C$
Electron mass	m_e	$9{,}11 \times 10^{-31}\ kg$
Mass of the Earth	M	$5{,}98 \times 10^{24}\ kg$
Radius of the Earth	R_E	$6{,}38 \times 10^{6}\ m$

TABLE 2: FORMULAE

MOTION

$v_f = v_i + a\Delta t$	$\Delta x = v_i\Delta t + \frac{1}{2}a\Delta t^2$ OR $\Delta y = v_i\Delta t + \frac{1}{2}a\Delta t^2$
$v_f^2 = v_i^2 + 2a\Delta x$ OR $v_f^2 = v_i^2 + 2a\Delta y$	$\Delta x = \left(\frac{v_i + v_f}{2}\right)\Delta t$ OR $\Delta y = \left(\frac{v_i + v_f}{2}\right)\Delta t$

FORCE

$F_{net} = ma$	$p = mv$
$f_s^{max} = \mu_s N$	$f_k = \mu_k N$
$F_{net}\Delta t = \Delta p$ $\Delta p = mv_f - mv_i$	$w = mg$
$F = \frac{Gm_1m_2}{d^2}$ OR $F = \frac{Gm_1m_2}{r^2}$	$g = \frac{GM}{d^2}$ OR $g = \frac{GM}{r^2}$

WORK, ENERGY AND POWER

$W = F\Delta x\cos\theta$	$U = mgh$ OR $E_P = mgh$
$K = \frac{1}{2}mv^2$ OR $E_k = \frac{1}{2}mv^2$	$W_{net} = \Delta K$ OR $W_{net} = \Delta E_k$ $\Delta K = K_f - K_i$ OR $\Delta E_k = E_{kf} - E_{ki}$
$W_{nc} = \Delta K + \Delta U$ OR $W_{nc} = \Delta E_k + \Delta E_p$	$P = \frac{W}{\Delta t}$
$P_{ave} = Fv_{ave}$	

WAVES, SOUND AND LIGHT

$v = f\lambda$	$T = \frac{1}{f}$
$f_L = \frac{v \pm v_L}{v \pm v_s} f_s$	$E = hf$ OR $E = h\frac{c}{\lambda}$
$E = W_o + E_{k(max)}$ OR $E = W_o + K_{max}$ where $E = hf$ and $W_0 = hf_0$ and $E_{k(max)} = \frac{1}{2}mv_{max}^2$ OR $K_{max} = \frac{1}{2}mv_{max}^2$	

ELECTROSTATICS

$F = \frac{kQ_1Q_2}{r^2}$	$E = \frac{kQ}{r^2}$
$V = \frac{W}{q}$	$E = \frac{F}{q}$
$n = \frac{Q}{e}$ OR $n = \frac{Q}{q_e}$	

ELECTRIC CIRCUITS

$R = \frac{V}{I}$	emf (ε) = $I(R + r)$
$R_s = R_1 + R_2 + \ldots$ $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$	$q = I\Delta t$
$W = Vq$ $W = VI\Delta t$ $W = I^2R\Delta t$ $W = \frac{V^2\Delta t}{R}$	$P = \frac{W}{\Delta t}$ $P = VI$ $P = I^2R$ $P = \frac{V^2}{R}$

ALTERNATING CURRENT

$I_{rms} = \frac{I_{max}}{\sqrt{2}}$ $V_{rms} = \frac{V_{max}}{\sqrt{2}}$	$P_{ave} = V_{rms}I_{rms}$ $P_{ave} = I_{rms}^2R$ $P_{ave} = \frac{V_{rms}^2}{R}$

RECAP FROM GRADE 11: MECHANICS

Physical quantities: Scalar and vector quantities

A physical quantity is a quantity that can be measured. Physical quantities can be classified as scalar quantities or vector quantities, depending on whether they are associated with direction or not.

- **Scalar quantity:** Physical quantity with magnitude (size) only.
  Examples of scalar quantities: distance, time, speed, mass, energy, frequency, charge.
- **Vector quantity:** Physical quantity with magnitude (size) and direction.
  Examples of vector quantities: displacement, velocity, force, acceleration, momentum, impulse, electric field.

Net/ resultant vector

When there are two or more vectors acting on an object, e.g., two or more forces acting on an object, these vectors can be replaced with ONE single vector, which will produce the same effect.

Remember to state your sign convention i.e., choose a positive direction when working with a vector quantity.

- **Resultant vector:** Single vector having the same effect as two or more vectors together; the vector sum of two or more vectors.

PRO-TIPS

- Draw **all** diagrams in pencil.
- **Label** the forces on a free - body diagram or force diagram, using symbols.
- Have a **key** on the side, naming the forces on the diagram.

Free- body diagrams & force diagrams

Free - body diagrams and force diagrams are diagrams that are used to represent the forces acting on an object.

Free - body diagram	Force diagram
In a free – body diagram: • The object is a single, isolated object which is represented as a **dot** (or a square block). • Each force acting on the object is represented as an **arrow.** The arrows representing the force must be **drawn in proportion to each other.** A larger force must be represented using longer arrow, and a smaller force using a shorter arrow. • The arrow representing the force **ALWAYS** points **AWAY** from the dot and shows the direction in which the force acts.	In a force diagram: • The actual object is drawn. • Each force acting on the object is represented as an **arrow**. The arrows representing the force must be drawn in proportion to each other. A larger force must be represented using longer arrow, and a smaller force using a shorter arrow • The arrow representing the force is **drawn from the surface of the object.**

Types of forces

Weight or gravitational force

Accepted symbols: w or F_g

Definition: weight: The gravitational force the Earth exerts on any object on or near its surface.

- Weight or the gravitational force is a **non – contact force** that acts over a distance.
- **SI units:** Newtons (N)

The weight/ gravitational force acting on an object can be calculated using the formula:

$$w = mg \textbf{ or } F_g = mg$$

What do these variables mean and what are the SI units?

w or F_g= weight or gravitational force in Newtons (N)

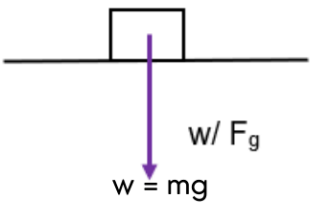
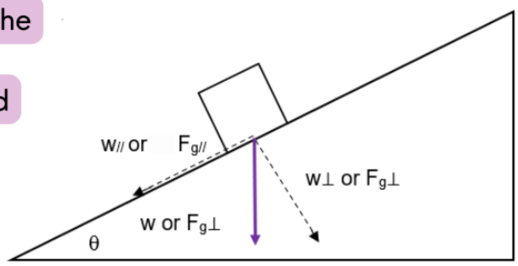
m = mass (of object) in kg.

g = gravitational acceleration (9,8 $m.s^{-2}$ downwards **on Earth**)

Definition: Mass: A measure of the amount of matter an object has.

NOTE:
- The weight of an object depends on the gravitational acceleration. Therefore, an object's weight can **CHANGE** depending on what celestial body (planet or moon) the object is on.
- The weight of the object also depends on the mass of the object, the greater the mass of the object, the greater the weight of the object (provided the gravitational acceleration remains constant).
- **ANY** object has mass, and therefore it experiences the gravitational force (weight) acting on it.
- The **mass** of the object does **NOT** change regardless of what celestial body (planet/ moon) the object is on.
- Weight or gravitational force always acts **vertically downwards** (towards the centre of the celestial body).

Calculating the weight or gravitational force of an object on a horizontal surface vs on an inclined plane:

Horizontal surface	Inclined plane
 w = mg **OR** F_g =mg	On an inclined plane, the gravitational force or weight can be resolved into its parallel and perpendicular components:  **Parallel component** $F_{g//}$=F_g sinθ **or** $w_{//}$ = wsinθ $F_{g//}$=mgsinθ **or** $w_{//}$= mgsinθ **Perpendicular component** $F_g\perp$ = $F_g$cosθ **or** $w_\perp$ = wcosθ $F_g\perp$ = mgcosθ **or** $w_\perp$= mgcosθ

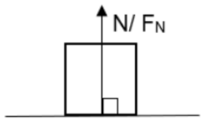
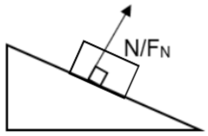
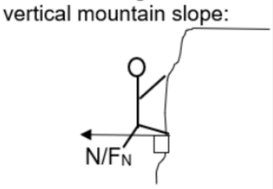
Normal force

Accepted symbols: N or F_N

Did you know? The word 'normal' in mathematics means 'perpendicular' or 'at right angles'.

Definition: Normal force, N or F_N: The force or the component of a force which a surface exerts on an object with which it is in contact, and which is perpendicular to the surface.

- An object experiences a normal force if it is in contact with a surface.
- The normal force always acts **PERPENDICULAR** to the surface:

Object of mass, **m**, at rest on a horizontal surface:	Object of mass, **m** kg, on an inclined plane:	A hiker, exerting a force with his foot against a vertical mountain slope:
		

PRO-TIPS

The frictional force is an **opposing** force. It always acts in the opposite direction of motion or in the opposite direction to the force trying to cause the object to move.

Frictional force

Definition: Frictional force, f or F_f : The force that opposes the motion of an object and which acts parallel to the surface.

The frictional force either prevents a stationary object from starting to move.

OR

It opposes the motion of an object that is already in motion.

There are **TWO** types of frictional forces:

Static frictional force	Kinetic frictional force
Static frictional force ,f_s: Force that opposes the tendency of motion of a stationary object relative to a surface.	**Kinetic frictional force, f_k:**Force that opposes the motion of a moving object relative to a surface.
• The static frictional force is a force that keeps the object at rest and prevents the object from moving. The static frictional force can change. • The object experiences a **MAXIMUM** static frictional force just before the object starts moving. Only the maximum static frictional force can be calculated using the formula below. • The static frictional force is directly proportional to the normal force (N).	• An object experiences a kinetic frictional force when the object is in motion. • The frictional force always acts in the opposite direction of motion. • The kinetic frictional force remains constant. • The kinetic frictional force is directly proportional to the normal force (N).
Formula to calculate the maximum static frictional force: $f_s^{max} = \mu_s N$	Formula to calculate the kinetic frictional force: $f_k = \mu_k N$

NOTE: The co -efficient of friction (μ) only depends on the nature of the surfaces in contact

Tension force (T or F_T)

Tension is a **pulling force** transmitted along a rope/ string/cable/cord.

The tension force is **constant** throughout the rope/string/cable/cord if it is assumed that:

- The rope/string is inextensible i.e., cannot be stretched.
- The rope/string is massless.

PRO-TIPS

The direction of the tension force is always **AWAY** from the point of contact of the rope. This is because it is a **PULLING FORCE**.

Newton's laws of motion

Newton's first law of motion

Newton's First Law of motion: A body will **remain** in its state of rest or motion at **constant velocity** unless a non-zero resultant/net force acts on it.

Newton's first law of motion deals with when the net or resultant force acting on the object is zero (F_{net}= 0 N), therefore, the **forces acting on the object are in equilibrium**.

Object remains at rest **OR** **Object is moving at a constant velocity**

Object remains at rest:

$v_i = v_f = 0 \text{ m.s}^{-1}$
$a = 0 \text{ m.s}^{-2}$
From $F_{net} = ma$
$F_{net} = 0$ N

Object is moving at a constant velocity:

$v_i = v_f$
$a = 0 \text{ m.s}^{-2}$
From $F_{net} = ma$
$F_{net} = 0$ N

PRO-TIPS

When the forces acting on an object are in equilibrium, the net/resultant force acting on the object is zero (0 N) and the object is either:

- At rest **OR**
- moving at a constant velocity.

Newton's second law of motion

Newton's second Law of motion: When a resultant/net force acts on an object, the object will **accelerate** in the direction of the force at an acceleration **directly** proportional to the force and **inversely** proportional to the **mass** of the object.

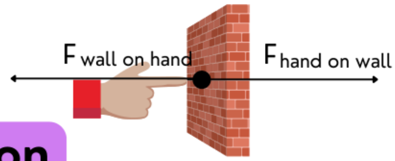
A net force acts on an object. Fnet ≠ 0N.
Forces acting on the object are **not** balanced/in equilibrium

A net force causes the object to accelerate in the direction of the net force.
Acceleration and net force act in the same direction.
There is a change in velocity (vi ≠ vf)
$a \neq 0 \text{ m·s}^{-2}$

Newton's third law of motion

Newton's Third Law of motion: When object A exerts a force on object B, object B **SIMULTANEOUSLY** exerts an oppositely directed force of equal magnitude on object A.

- When a force is exerted on an object, the object exerts a force equal in magnitude but opposite in direction back on the object exerting the force.
- This is called **action - reaction pairs**. The forces **act on different objects** and therefore are not in equilibrium.
- An example of an action - reaction pair:
  A hand exerts a force on the surface of the wall (applied force), the wall exerts a force equal in magnitude but opposite in direction on the hand (normal force).
- The action- reaction pair can be described as: Force of the hand on the wall and the force of the wall on the hand.



Worked examples: Mechanics revision

Multiple choice questions

C + A

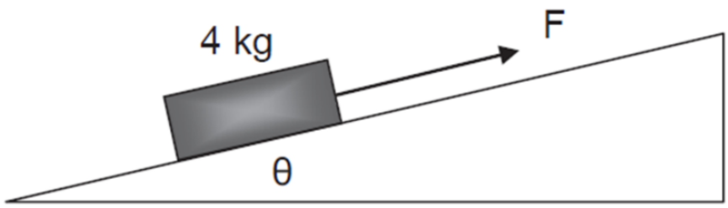
NOTE: All multiple choice questions are 2 marks

1.1 A man, with a weight of 980 N, stands on a horizontal floor. Which ONE of the following represents the force that the floor exerts on the man?

A The floor exerts a force greater than 980 N on the man.
B The floor exerts a force of 980 N upwards on the man.
C The floor exerts a force smaller than 980 N on the man.
D The floor exerts a force 980 N downwards on the man.

Answer: B
When the man, with a weight of 980 N, stands on a horizontal floor, he exerts a force of 980 N downwards on the floor. According to Newton's Third Law of motion, the floor will exert a force of equal magnitude but opposite in direction on the man, i.e., 980 N upwards.

1.2 A block with a mass of 4 kg is pulled upwards along a frictionless slope, inclined at an angle ,θ, with a force **F**, as shown in the sketch below.

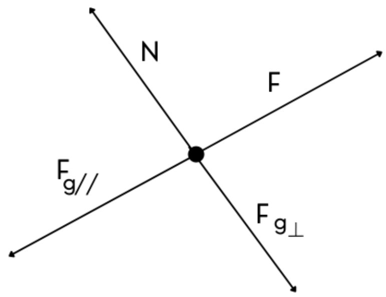


Which ONE of the following equations can be used to calculate the magnitude of the normal force (N)?

A $N = (4)(9,8)\sin\theta$
B $N = F - (4)(9,8)\cos\theta$
C $N = F + (4)(9,8)\cos\theta$
D $N = (4)(9,8)\cos\theta$

Answer: D

To visualise the forces acting on the object, draw a free - body diagram (or force diagram) showing the forces acting on the object, with the components:



From the free - body diagram, the following can be concluded: $N = Fg_{\perp}$, therefore

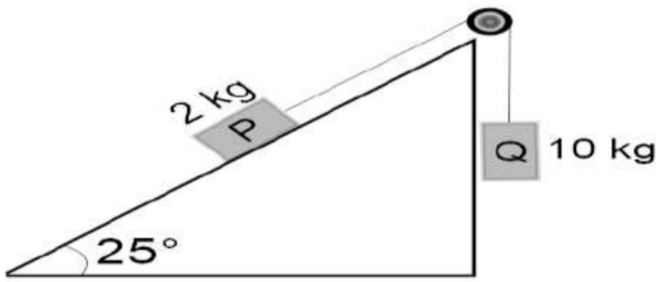
$N = mg\cos\theta$

$N = (4)(9,8)\cos\theta$

Worked example

C + A

Blocks **P** and **Q** are connected over a frictionless pulley system by a string of negligible mass. Block **P** accelerates up the rough slope. The co – efficient of kinetic friction between block **P** and the surface is 0,1.



1.1 State Newton's second law of motion in words. (2)

1.2 Calculate the tension in the string. (4)

1.3 Block **Q** is replaced by another block of mass 30 kg. The maximum force that the string can withstand is 25 N. Prove, using a calculation, that the string will break if block **Q** is replaced with another block of mass 30 kg. (5)

1.4 Sketch a velocity versus time graph showing the motion of block **P** from the moment it accelerates up the slope to when the string breaks. No values need to be indicated on the graph. (3)

1.1 When a resultant/net force acts on an object, the object will accelerate in the direction of the force at an acceleration directly proportional to the force and inversely proportional to the mass of the object.

1.2

- This is a **two - body system** that is connected by a string. Therefore, the magnitude of the acceleration of the system is the same, and therefore the magnitude of the acceleration of both blocks is the same.
- The magnitude of the tension in the string is the same for both blocks.
- The tension and acceleration of the system is unknown, therefore two equations for each block can be set up, and using simultaneous equations, the tension can be calculated:

Block **P** (2 kg)

Let up the slope be positive

$F_{net} = ma$
$T + (-F_{g//}) + (-f_k) = ma$
$T - mg\sin\theta - \mu_k.N = ma$
$T - mg\sin\theta - \mu_k.mg\cos\theta = ma$
$T - (2)(9,8)\sin 25^o - (0,1)(2)(9,8)\cos 25^o = 2a$
$T - 10,059... = 2a$
$T = 2a + 10,059...$ ------------------------------------ 1

PRO-TIPS
Only round off the final answer to a minimum of two decimal places.

Block **Q** (10 kg)

Let downwards the slope be positive

$F_{net} = ma$
$F_g + (-T) = ma$
$mg - T = ma$
$(10)(9,8) - T = 10a$
$98 - T = 10a$
$T = 98 - 10a$ -- 2

2 into 1: $2a + 10,059... = 98 - 10a$
$12a = 98 - 10,059....$
$a = 7,328...\ m.s^{-2}$ ------------------------ 3

3 into 1: $T = 2(7,328...) + 10,059...$
$T = 24,72\ N$

1.3 The mass of the hanging block has changed (increased). This will change the tension and the acceleration of the system. Therefore, the new magnitude of acceleration and tension must be calculated. The new tension and acceleration of the system is unknown, therefore two equations for each block can be set up, and using simultaneous equations, the new tension can be calculated:

Block **P** (2 kg)

Let up the slope be positive

$F_{net} = ma$
$T + (-F_{g//}) + (-f_k) = ma$
$T - mg\sin\theta - \mu_k.N = ma$
$T - mg\sin\theta - \mu_k.mg\cos\theta = ma$
$T - (2)(9,8)\sin 25^o - (0,1)(2)(9,8)\cos 25^o = 2a$
$T - 10,059... = 2a$
$T = 2a + 10,059...$ ------------------------------------ 1

Block **Q** (30 kg)

Let downwards the slope be positive

$F_{net} = ma$
$F_g + (-T) = ma$
$mg - T = ma$
$(30)(9,8) - T = 30a$
$294 - T = 30a$
$T = 294 - 30a$ --- 2

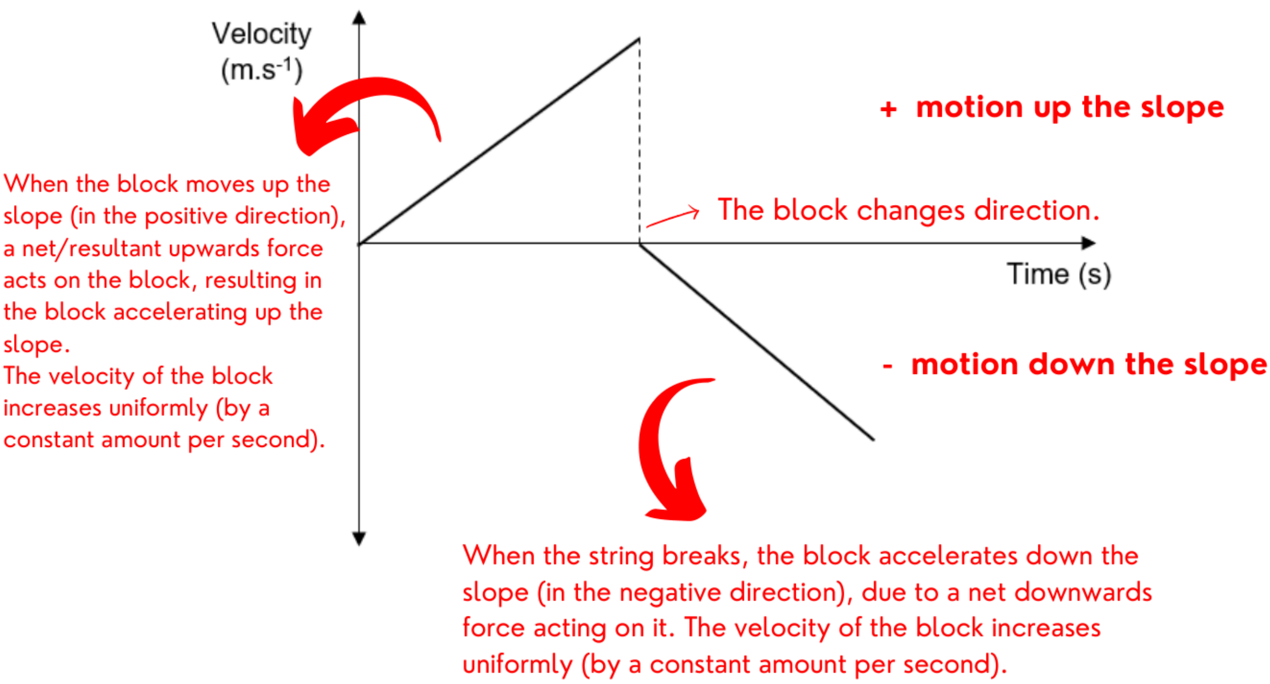
2 into 1: $2a + 10,059... = 294 - 30a$
$32a = 294 - 10,059....$
$a = 8,873...\ m.s^{-2}$ ------------------------ 3

3 into 1: $T = 2(8,873...) + 10,059...$
$T = 27,81\ N$

$27,81\ N > 25\ N$
$\therefore$ The string will break

1.4

Velocity – time graph representing the motion of block **P**:

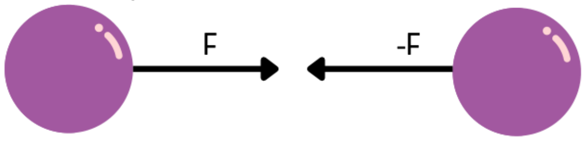


PRO-TIPS

Remember the following when drawing graphs:
- Heading
- Label the axes
- Draw the graph in pencil
- Use a ruler, if it is a straight - line graph.

Newton's law of Universal Gravitation

Newton's Law of Universal Gravitation: Each body in the universe attracts every other body with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.



PRO-TIPS

The gravitational force that one object of mass exerts on another object of mass is equal in magnitude but opposite in direction (regardless of the object's masses) due to **Newton's third law of motion.**

Formula to calculate the gravitational force between two objects of mass:

$$F = \frac{Gm_1m_2}{r^2}$$

What do these variables mean and what are the SI units?

F = magnitude of the gravitational force between two objects in Newtons (N)
G = gravitational constant = $6{,}67 \times 10^{-11}$ $N.m^2.kg^{-2}$
m_1 = mass of object 1 in kilograms (kg)
m_2 = mass of object 2 in kilograms (kg)
r = distance **between the centres** of the objects in metres (m)

Gravitational acceleration

Objects on the same celestial body (i.e., planet or moon) experience the same **gravitational acceleration**, even if they have different masses. The gravitational acceleration on a celestial body does not depend on the mass of the object, only on the mass of the celestial body and the radius of the celestial body.

Scenario:

- A person of mass, **m**, on the surface of the Earth, experiences a gravitational force due to the Earth.
- This is the weight of the person, which can be calculated using the formula:
- F_g = mg or w = mg.
- This can be integrated into Newton's law of Universal gravitation:

$$F = \frac{Gm_1m_2}{r^2}$$

$$F_g = \frac{GMm}{r^2}$$

$$mg = \frac{GMm}{r^2}$$

$\div$ m throughout:

$$g = \frac{GM}{r^2}$$

Formula to calculate the gravitational acceleration on any celestial body

What do these variables mean and what are the SI units?

g = magnitude of the gravitational acceleration in $m.s^{-2}$
G = gravitational constant = $6{,}67 \times 10^{-11}$ $N.m^2.kg^{-2}$
M = mass of the celestial body (i.e., planet or moon)
r = radius of the celestial body in metres (m)

Worked example

C + A

1.1 The Sun exerts an average force of 3,57 x 10^{22} N on the Earth.

1.1.1 Write down the magnitude of the force that the Earth exerts on the Sun? (2)

1.1.2 Calculate the mass of the Sun. Take the mass of the Earth as 6 x 10^{24} kg and the average distance between the centre of the Earth and the Sun as 1,5 x 10^{8} km. (5)

1.2 Determine the magnitude of the gravitational acceleration of the Earth at a distance 1,2 x 10^{7} m from the centre of the Earth.
Take the mass of the Earth as 6 x 10^{24} kg. (3)

1.1.1 3,57 x 10^{22} N

The gravitational force that the Sun and Earth exert on each other is equal in magnitude but opposite in direction, due to Newton's third law of motion

1.1.2 The magnitude of the gravitational force that the Sun and Earth exert on each other is known, the mass of the Earth and the distance between their centres. Using Newton's Law of Universal Gravitation, the mass of the sun can be determined.

$$F = \frac{Gm_1m_2}{r^2}$$

$$(3{,}57 \times 10^{22}) = \frac{(6{,}67 \times 10^{-11})(6 \times 10^{24})m_{sun}}{(1{,}5 \times 10^{8} \times 1000)^2}$$

SI units for distance: metres (m)
km --> m x 1000 (or x 10^3)

$$m_{sun} = \frac{(3{,}57 \times 10^{22})(1{,}5 \times 10^{8} \times 1000)^2}{(6{,}67 \times 10^{-11})(6 \times 10^{24})}$$

$$m_{sun} = 2{,}01 \times 10^{30} \text{ kg}$$

1.2 The gravitational acceleration at a distance above the Earth's surface can be determined using the gravitational acceleration formula as the mass of the Earth and the distance from the centre of the Earth is known.

$$g = \frac{GM}{r^2}$$

$$g = \frac{(6{,}67 \times 10^{-11})(6 \times 10^{24})}{(1{,}2 \times 10^{7})^2}$$

$$g = 2{,}78 \text{ m.s}^{-2}$$

Question asked magnitude only

PRO-TIPS

If the mass of the Earth is not given, use M = 5,98 x 10^{24} kg as given on the data sheet.

REMINDER :QUESTION DIFFICULTY

C — COMPREHENSION AND RECALL QUESTIONS
These are common questions which include definitions and calculation questions that look similar to the questions covered in class. Approximately **50%** of Paper 1 (Physics) will include questions on this level.

A — ANALYSIS AND APPLICATION QUESTIONS
These are more complex questions which involves applying the knowledge and skills learned in this chapter. Approximately **40%** of Paper 1 (Physics) will include questions on this level.

P — PROBLEM- SOLVING QUESTIONS
These are questions that require critical thinking and being able to make connections between different representations of information and integrating different topics. They are not familiar questions, but are able to be solved through critical analysis. Approximately **10%** of Paper 1 (Physics) will include questions on this level.

RECAP FROM GRADE 11: ELECTROSTATICS

Electrostatics is the study of the interaction of charges (electro -) that are mainly stationary or at rest. (Static = stationary or not moving).

Types of charge (Q)

There are **two** types of charge: Positive and negative charge

Electrons (e^-) carry negative charge

Charge on a electron ,q_e= - 1,6 x 10^{-19}C

Protons (p^+) carry positive charge

Charge on a proton , q_e = + 1,6 x 10^{-19}C

PRO-TIPS

- An electron is the smallest subatomic particle that makes up matter.
- A charge smaller than the charge on an electron does not exist in nature.

- A **neutral object** has the same number of protons and electrons.
- A **negatively charged object** has more electrons than protons (i.e., an excess of electrons)
- A **positively charged object** has more protons than electrons (i.e., an excess of protons or a deficit of electrons)

Principle of quantization of charge

Principle of quantisation of charge: All charges in the universe consist of an integer multiple of the charge on one electron, i.e.1,6 x 10^{-19} C.

$$Q = nq_e$$

OR

$$n = \frac{Q}{q_e}$$

Formula on the data sheet

What do these variables mean and what are the SI units?

Q = Amount of charge in coulombs (C).

q_e= Charge on an electron or charge on a proton, i.e., -1, 6 x 10^{-19} C or +1,6 x 10^{-19}C.

n = Number of electrons **OR** protons, expressed as an integer. For example, the number of EXCESS electrons or EXCESS protons that an object has.

NOTE: The number of protons and electrons can only exist as **WHOLE NUMBERS**.

Charge (Q) conversion chart

Symbol	Name	Conversion	
mC	**Milli**coulomb	mC → C	x 10^{-3}
μC	**Micro**coulomb	μC → C	x 10^{-6}
nC	**Nano**coulomb	nC → C	x 10^{-9}
pC	**Pico**coulomb	pC → C	x 10^{-12}

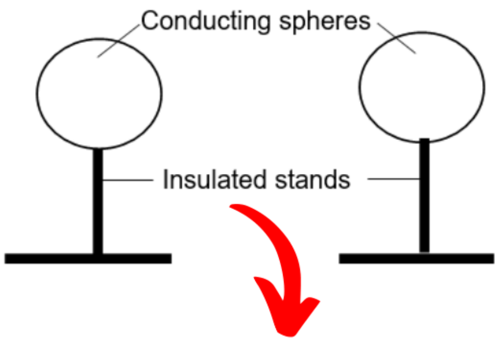
NOTE: The SI unit for charge is Coulombs (C)

Principle of conservation of charge

The principle of conservation of charge states that the net charge of an isolated system remains constant during any physical process (i.e. two charges making contact and then separating).

Applying the principle of conservation of charge

The principle of conservation of charge can only be applied to **identical conducting spheres** with the same shape, size and are made from the same material:



PRO-TIPS

- An isolated system in electrostatics is a system when no charge can be added or removed from the system.

The insulated stands (or an insulated surface) ensures that charge is not able to be discharged/leak back into the Earth.

Before the conducting spheres make contact

Before the conducting spheres make contact, the net charge of the system is the sum of the charges on each of the conducting spheres in the system:

$$Q_{net} = Q_1 + Q_2 + \dots$$

During and after contact

When the conducting spheres make contact, a conducting pathway is formed and charge is transferred from the sphere that has more electrons to the sphere that has fewer electrons, until charge is **equally shared** by both spheres. The new charge on each sphere can be calculated:

For <u>two</u> conducting spheres:

$$Q = \frac{Q_1 + Q_2}{2}$$

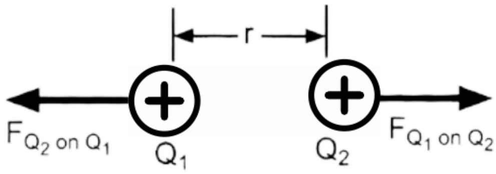
PRO-TIPS

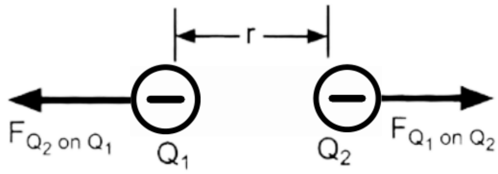
- Both spheres will have the **same magnitude** and **same** type of **charge** after contact.

Coulomb's law: Electrostatic force between charges

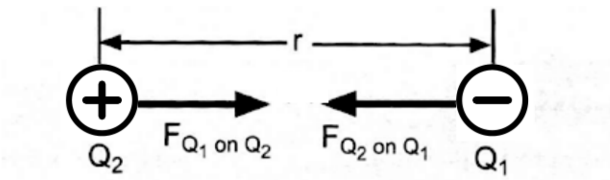
Charges exert an **electrostatic force** on each other. This is also called Coulomb forces.

- Like charges, e.g., positive and positive **OR** negative and negative exert an electrostatic force of repulsion on each other





- Unlike charges or opposite charges, e.g., positive and negative charges exert an electrostatic force of **attraction** on each other



Definition: Coulomb's Law: The magnitude of the electrostatic force exerted by one point charge (Q_1) on another point charge (Q_2) is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance (r) between them.

Coulomb's law: Calculating the electrostatic force between charges

$$F = \frac{kQ_1Q_2}{r^2}$$

What do these variables mean and what are the SI units?

F = **Magnitude** of the electrostatic force of attraction or repulsion between the two charges in Newtons (N)

Q_1 = **Magnitude** of point charge 1 in Coulombs (C)

Q_2 = **Magnitude** of point charge 2 in Coulombs (C)

r = Distance between the charges in meters (m)

k = Coulomb's constant = 9×10^{9} N.m^2.C^{-2}

Distance conversion chart

mm → m	÷ 1000
cm → m	÷ 100
km → m	x 1000

PRO-TIPS

- Only the **magnitudes** of the charges are substituted into the Coulomb's law equation. The type of charge is not included.
- Electrostatic force is a **VECTOR** quantity (it has magnitude and direction)

Electric field

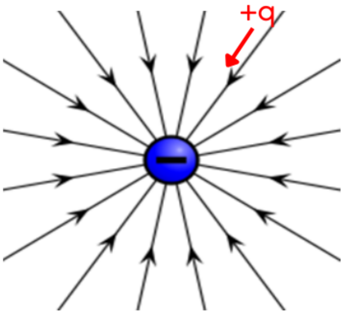
Every isolated charge produces its own electric field. Electric field lines are very similar to that of magnetic field lines.

Definition: Electric field: A region of space in which an electric charge experiences a force. The direction of the electric field at a point is the direction that a positive test charge would move if placed at that point.

NOTE:

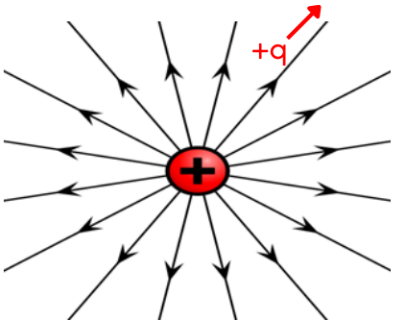
- A positive test charge is a small amount of positive charge, such that it that does not affect the electric field of the source charge emitting the electric field.
- When a small positive test charge (+q) is placed at a point in the electric field, the direction in which the positive test moves, determines the direction of the electric field.

Electric field around a negative isolated source charge:



The positive test charge (+q) moves towards the negative source charge, therefore the direction of electric field is **TOWARDS** a negative charge.

Electric field around a positive isolated source charge:



The positive test charge (+q) moves away from the positive source charge, therefore the direction of electric field is **AWAY FROM** a positive charge.

NOTE:

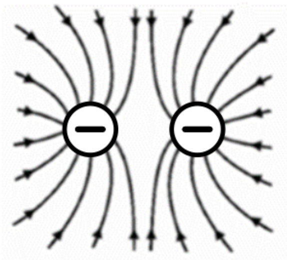
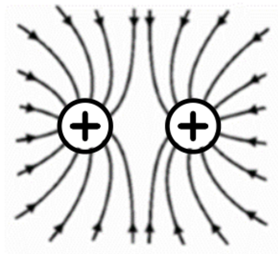
- The electric field is stronger where the electric field lines are closer together.
- The electric field weakens as the distance from the charge increases.

PRO-TIPS

When drawing the electric field around a charge:

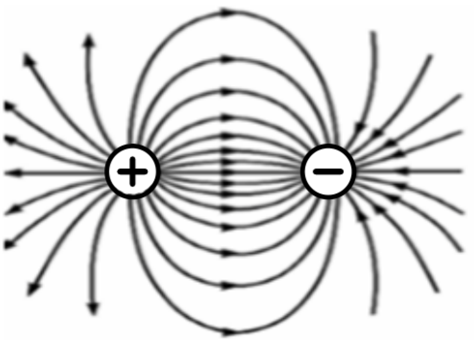
- The field lines must start from the charge.
- The field lines **MUST NOT CROSS** or touch.
- field lines must be drawn perpendicular to the charge.
- **ARROWS** indicating the direction of the electric field must be shown.

Electric field between two like charges:

ELECTRIC FIELD LINES BETWEEN TWO NEGATIVE CHARGES:	ELECTRIC FIELD LINES BETWEEN TWO POSITIVE CHARGES:
	

NOTE: There is an electrostatic force of REPULSION between like charges.

Electric field between two unlike charges:

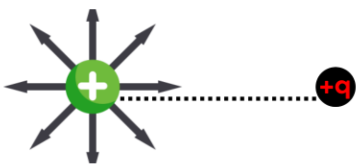


NOTE: There is an electrostatic force of ATTRACTION between unlike charges.

Calculating the electric field at a point (E)

Definition: Electric field at a point: is the electrostatic force experienced per unit positive charge placed at that point.

$$E = \frac{F}{q}$$



What do these variables mean and what are the SI units?

F = **Magnitude** of the electrostatic force experienced by the positive test charge(in newtons N)
q = **Magnitude** of the **charge on the positive test charge** in coulombs (C)
E = **Magnitude** of the electric field at a point in Newtons per Coulomb ($N.C^{-1}$).

NOTE:

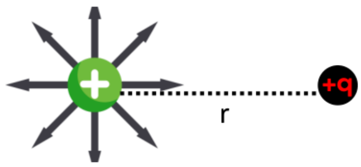
- Electric field is a **VECTOR QUANTITY**, therefore it has magnitude and direction.
- The direction of the electric field at a point is the direction that a positive test charge would move if placed at that point.

Another formula can be derived to calculate the electric field at a point, **E**, at a distance, **r** ,from the SOURCE point charge, **Q**:

$$E = \frac{F}{q}$$

$$E = \frac{\frac{kQq}{r^2}}{q}$$

$$E = \frac{kQ}{r^2}$$



What do these variables mean and what are the SI units?

k = Coulomb's constant: $9 \times 10^{9}\,N.m^{2}.C^{-2}$

r = Distance in meters (m) between the charge and any point (in space).

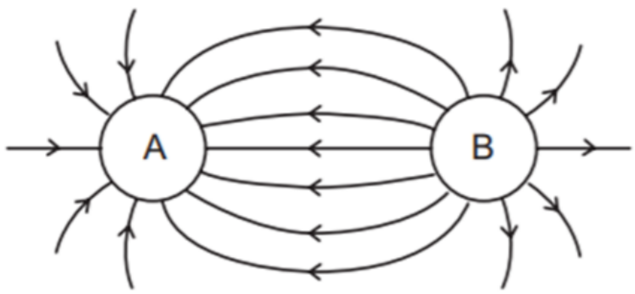
Q = **Magnitude** of charge on the **source charge** in Coulombs (C).

E = **Magnitude** of the electric field at a point in Newtons per Coulomb ($N.C^{-1}$).

Worked examples: Electrostatics revision

Multiple choice questions

1.1 The electric field pattern between two charged spheres, **A** and **B**, is shown below:



Which ONE of the following statements regarding the charge on spheres **A** and **B** is CORRECT?

A Sphere **A** and **B** are both positively charged.

B Sphere **A** and **B** are both negatively charged.

C Sphere **A** is positively charged and sphere **B** is negatively charged.

D Sphere **A** is negatively charged and sphere **B** is positively charged.

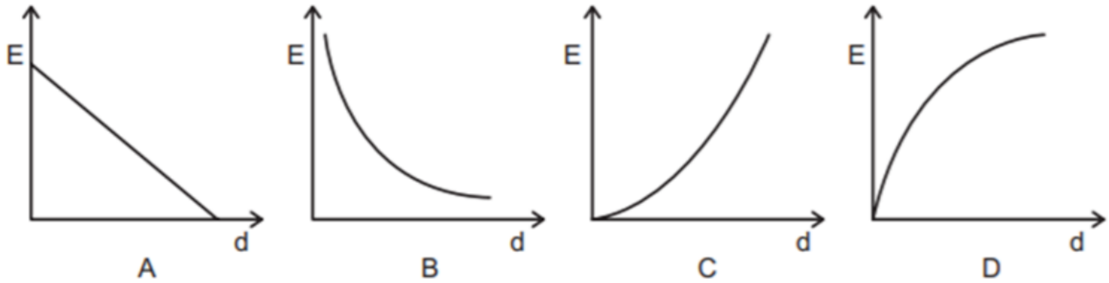
Answer: D

Remember: The direction of the electric field at a point is the direction that a test charge would move if placed at that point.

From the diagram, the electric field lines run **TOWARDS** charge **A**, this indicates that sphere **A** is negatively charged.

The electric field lines run **AWAY FROM** charge **B**, this indicates that charge **B** is positively charged.

1.2 Which ONE of the following sketch graphs best represents the relationship between the electric field strength **E** and the distance **d** from a given charge **Q**?



Answer: B

From the formula,

$$E = \frac{kQ}{r^2}$$

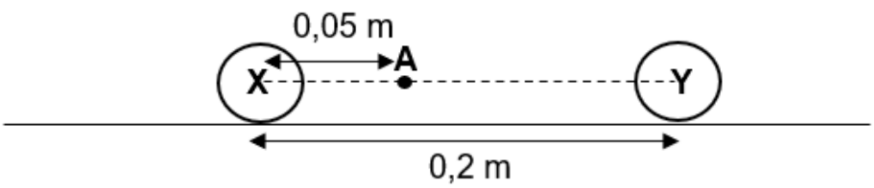
$$\therefore E \; \alpha \; \frac{1}{r^2}$$

The electric field at a point is **inversely proportional** to the square of the distance between the charges, this inversely proportional relationship is represented by the graph sketch in '**B**'.

Worked example

Two identical conducting spheres **X** and **Y** on an insulated surface are placed a distance of 0,2 m apart in a straight line, as shown below. The charge on sphere **X** is unknown and the charge on sphere **Y** is -3 pC.

Point **A** is located 0,05 m to the right of sphere **X**.



Sphere **X** and **Y** exert an electrostatic force of 2,5 x 10^{-7} N attraction on each other.

1.1 Calculate the magnitude of the charge on sphere **X**. (3)

1.2 Calculate the electric field at point **A** due to sphere **X**. (3)

1.3 Calculate the net electric field at point **A**. (4)

1.1 The charge on sphere **X** can be determined using Coulomb's law as the electrostatic force between sphere **X** and **Y** is known, the distance between the charges, and the magnitude of the charge on sphere **Y**. Remember that charge is measured in C, to convert pC --> C x 10^{-12}

$$F = \frac{kQ_1Q_2}{r^2}$$

$$(2{,}5 \times 10^{-7}) = \frac{(9 \times 10^9)Q_x(3 \times 10^{-12})}{(0{,}2)^2}$$

$$Q_x = \frac{(2{,}5 \times 10^{-7})(0{,}2)^2}{(9 \times 10^9)(3 \times 10^{-12})}$$

$$Q_x = 3{,}70 \times 10^{-7} \text{ C}$$

PRO-TIPS

Only the **magnitudes** of the charges are substituted into the Coulomb's law formula and the electric field formula

1.2 The type of charge (positive or negative) on sphere **X** needs to be determined to determine the direction of the electric field at point **A**, due to charge **X**.
Sphere **X** and **Y** exert an electrostatic force of attraction on each other, therefore they have unlike (or opposite) charges. Sphere **Y** is negatively charged, therefore sphere **X** is positively charged.
The direction of the electric field at point A due to sphere **X** is the direction that a positive test charge would move if placed at that point. Since sphere **X** is positively charged, the positive test charge will move AWAY from sphere **X**, from the diagram, this is to the right.

$$E = \frac{kQ}{r^2}$$

$$E = \frac{(9 \times 10^9)(3{,}70 \times 10^{-7})}{(0{,}05)^2}$$

$E = 1332000$ N.C^{-1} right (or away from sphere **X**)

OR

$E = 1{,}33 \times 10^6$ N.C^{-1} right (or away from sphere **X**)

1.3 The net electric field at point **A** is the vector sum of the electric fields at point **A**.
Sphere **X** is positively charged, therefore the direction of the electric field at point **A** due to sphere **X** is to the right.
Sphere **Y** is negatively charged, therefore if a positive test charge was placed at point **A**, it will move towards sphere **Y**, which is to the right.
The net electric field at point **A** can therefore be determined:

Let right be positive

$$E_{net} = E_X + E_Y$$

$$E_{net} = E_X + \frac{kQ}{r^2}$$

$$E_{net} = (+1332000) + \frac{(9 \times 10^9)(3 \times 10^{-12})}{(0{,}2 - 0{,}05)^2}$$

$E_{net} = 1{,}33 \times 10^6$ N.C^{-1} right

NOTE:

- Since point **A** lies between sphere **X** and **Y**
- The distance between sphere **Y** and point **A** is the total distance between the spheres (0,2 m) **minus** the distance that sphere **X** is from point **A** (0,05 m).

NOTE: The answer in question 1.2 and question 1.3 are similar because the magnitude of the charge on sphere **Y** is **very small**.

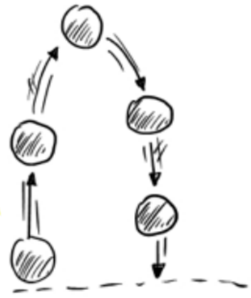
VERTICAL PROJECTILE MOTION:

Motion in one dimension

WHAT IS VERTICAL PROJECTILE MOTION?

Vertical projectile motion is motion vertically **upwards** or vertically **downwards** and the **ONLY** force acting on the object (projectile) is the **gravitational force**.

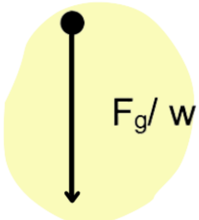
Any object that is thrown, kicked, or shot perpendicularly upwards or downwards in a gravitational field travelling vertically, in **one dimension**.



Definitions

- **Projectile:** An object which has been given an initial velocity and then it moves under the influence of the gravitational force only.
- A **projectile** (at all times during its motion) is in free–fall, if air resistance or air friction is ignored.
- **Free – fall:** Motion during which the only force acting on an object is the gravitational force.
- The acceleration of a projectile is **ALWAYS EQUAL** to the gravitational acceleration i.e. **9,8 m.s^{-2}** downwards, since **ONLY** the gravitational force is acting on the projectile in free – fall.

The gravitational force acting on the projectile can be represented in a free – body diagram:



KEY:
F_g / w = gravitational force **or** weight

Newtons 2nd Law of motion

From Newton's second law of motion, it can be deduced:

$$F_{net} = ma$$
$$F_g = ma$$
$$mg = ma \quad (\div m \text{ on both sides})$$
$$\therefore g = a$$

PRO-TIPS

Tips for drawing free-body diagrams:

- A dot is used to represent the object.
- The arrow which represents the force must always point **AWAY** from the dot.
- A key must be drawn.

COMMON SCENARIOS IN VERTICAL PROJECTILE MOTION

Scenario 1: An object is dropped from a height, h, above the ground.

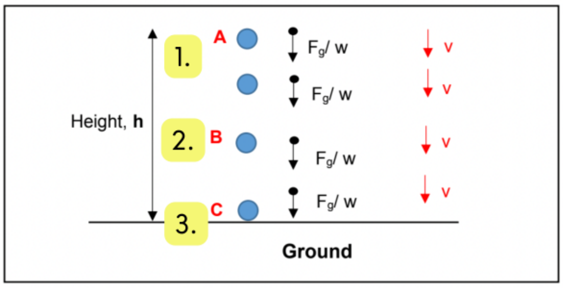


Figure 1

NOTE:
Throughout the motion the acceleration of the object remains constant at 9,8 $m.s^{-2}$ downwards. If the object is **thrown/ shot vertically** downwards with a certain initial velocity, then the initial velocity is **NOT** 0 $m.s^{-1}$

Figure 1 above represents motion vertically downwards.

1. **Point A:** The object is dropped from a height, h above the ground. The initial velocity of the object is 0 $m.s^{-1}$. The word "dropped" or "released" usually means that the initial velocity of the object is 0 $m.s^{-1}$

2. As object moves vertically downwards, in the same direction as the gravitational force, towards **point B**, the velocity of the object **INCREASES** uniformly.

3. **Point C:** The velocity at which the object strikes the ground at point C, is its maximum velocity during the object's motion.

Scenario 2: An object is thrown/shot vertically upwards from the ground.

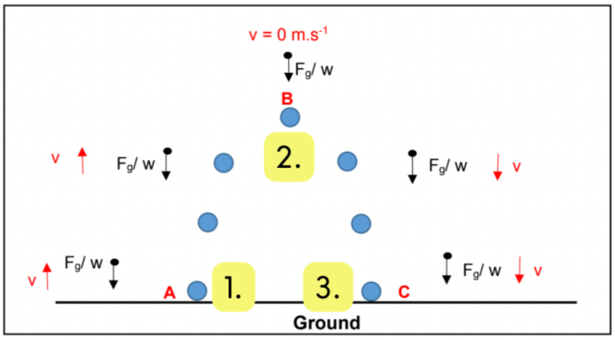


Figure 2

NOTE:
Throughout the motion the acceleration of the object remains constant at 9,8 $m.s^{-2}$ downwards, even at the **highest point**, because the **gravitational** force is still acting on the object at the highest point.

Figure 2 represents symmetrical motion, since the start and end points of motion are the same.

1. **Point A:** The object is given a certain initial upwards velocity, and is then in free – fall. As the object moves vertically upwards against the gravitational force towards point B, the velocity of the object **DECREASES** uniformly.

2. **Point B:** The object reaches the **HIGHEST POINT** in its motion, the velocity at the highest point is 0 $m.s^{-1}$, as the object is **CHANGING DIRECTION** at this point. The gravitational acceleration at the highest point is still equal to 9,8 $m.s^{-2}$ downwards, as the gravitational force is still acting on the object.

3. As the object moves downwards (**from point B to C**), in the same direction as the gravitational force, the velocity of the object **INCREASES** uniformly.
The velocity at which the object strikes the ground is the same as the initial velocity with which it was projected, but, in the opposite direction (i.e., the object strikes the ground at the same speed) as this is symmetrical motion.

Scenario 3: Object is projected vertically upwards, with a certain initial velocity, from a height, h.

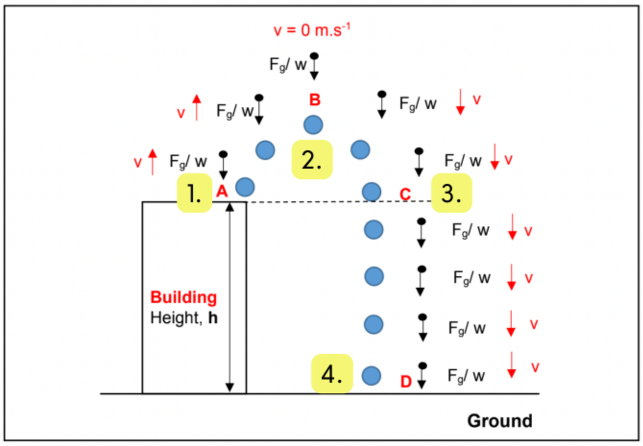


Figure 3

NOTE:

- Usually, this scenario involves the object being projected from the top of a building, of height, h.
- Throughout the motion the acceleration of the object remains constant at 9,8 $m.s^{-2}$ downwards, even at the highest point, because the gravitational force is still acting on the object at the highest point.

1. **Point A:** The object is given a certain initial upwards velocity, and is then in free – fall.
As object moves vertically upwards against the gravitational force towards point B, the velocity of the object **DECREASES** uniformly.

2. **Point B:** The object reaches the **HIGHEST POINT** in its motion, the velocity at the highest point is 0 $m.s^{-1}$, as the object is **CHANGING DIRECTION** at that point. The gravitational acceleration is still 9,8 $m.s^{-2}$ downwards, as the gravitational force is still acting on the object.
As the object moves downwards (from point B to C), in the same direction as the gravitational force, the velocity **INCREASES** uniformly.

3. **Point C:** The velocity at point C is the same as the initial velocity at point A, but in the opposite direction (downwards) (i.e., the same speed), as the motion from point A to C is symmetrical.

4. **Point D:** The object strikes the ground downwards with a maximum velocity.

CALCULATIONS: VERTICAL PROJECTILE MOTION

VERTICAL PROJECTILE MOTION INVOLVES:

- motion is a straight line (upwards and/or downwards)
- motion at a constant acceleration (9,8 m.s^{-2} downwards)
- Therefore, equations of motion can be used to do calculations involving scenarios of vertical projectile motion.

PRO-TIPS

- Each equation of motion has 4 variables, you **need 3** variables to do a calculation.
- Always state your sign convention (or positive direction). This can either be **UPWARDS** or **DOWNWARDS**.

FOUR EQUATIONS OF MOTION

1. $v_f = v_i + a\Delta t$
2. $v_f^2 = v_i^2 + 2a\Delta y$ **OR** $v_f^2 = v_i^2 + 2a\Delta x$
3. $\Delta y = v_i\,\Delta t + \frac{1}{2}a\Delta t^2$ **OR** $\Delta x = v_i\,\Delta t + \frac{1}{2}a\Delta t^2$
4. $\Delta y = \left(\frac{vi+vf}{2}\right)\Delta t$ **OR** $\Delta x = \left(\frac{vi+vf}{2}\right)\Delta t$

What do these variables mean and what are the SI units?

v_f = final velocity in m.s^{-1}
v_i = initial velocity in m.s^{-1}
a = gravitational acceleration (for motion in free – fall) in m.s^{-2}, 9,8 m.s^{-2} downwards.

If downwards is taken as positive: a = +9,8 m.s^{-2}
If upwards is taken as positive: a = - 9,8 m.s^{-2}
Δt = time in s (seconds)
Δy = displacement in m (metres)

Tip:
conversion:
km.h^{-1} → m.s^{-1} ÷ 3,6

NOTE WHAT THE QUESTION IS ASKING YOU TO CALCULATE:

- **Displacement** (Δy or Δx): Change in position. It is the magnitude and direction of the straight line drawn from the starting point to the end point of motion. Displacement is a vector quantity (it has magnitude/ size and direction).
- **Distance** is the total path length travelled.
  Distance is a scalar quantity (it has magnitude only), therefore the magnitude of the displacement from one point to another is the distance travelled.
- **Velocity** (v) is a vector quantity.
- **Speed** is a scalar quantity, and is equal to the magnitude of the velocity.

Worked examples

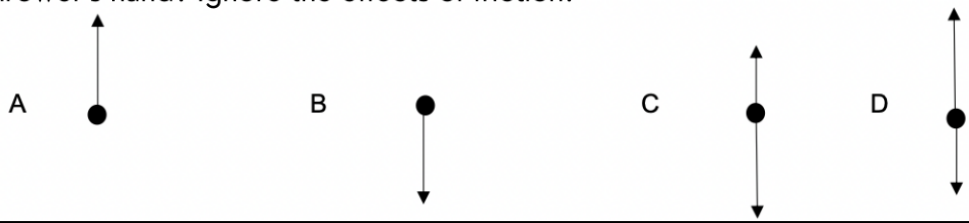
Multiple choice questions

C + A

PRO-TIPS

- Multiple choice questions involve the application of knowledge.
- Cancel out the very obvious incorrect options first.

1.1 A ball is thrown vertically upwards. Which **ONE** of the following diagrams best represents the force(s) acting on the ball immediately after the ball leaves the thrower's hand? Ignore the effects of friction.



Answer: B
The ball is in free – fall, therefore the only force acting on the ball is the gravitational force, and the gravitational force always acts downwards.

1.2 A ball is thrown vertically upwards at x $m.s.^{-1}$ If air friction is ignored, what would the acceleration and velocity of the ball be ONE second after leaving the thrower's hand?

	Acceleration	**Velocity**
A	9,8$m.s^{-2}$ downwards	x + 9,8
B	9,8$m.s^{-2}$ upwards	x - 9,8
C	(x- 9,8) $m.s^{-2}$downwards	x + 9,8
D	9,8$m.s^{-2}$downwards	x - 9,8

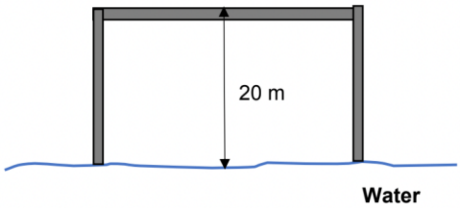
Answer: D
Acceleration: The ball is in free – fall, therefore the acceleration is always 9,8 $m.s^{-2}$ downwards (equal to gravitational acceleration).

Velocity: The ball is in free - fall. This means the velocity of the ball changes by 9,8 $m.s^{-1}$ every second. Since the ball is initially moving vertically upwards (against the gravitational force) during the **FIRST** second that it is in motion, the velocity of the object will decrease by 9,8 $m.s^{-1}$, therefore, the velocity after being **ONE** second in motion is the initial velocity with which the ball was projected, minus 9,8 $m.s^{-1}$

Worked example

C + A

1. A man drops a stone from the top of a bridge that is 20 m above the water. Ignore the effects of air resistance.



1.1 Write down the acceleration of the stone for the time that the stone is moving in air. Give a reason for the answer. (2)

1.2 Calculate:

1.2.1 the velocity at which the stone hits the water. (3)

1.2.2 how long it takes the stone to reach the water, from the moment that it is dropped. (3)

1.1 9,8 $m.s^{-2}$ downwards. When the stone is moving through the air, the only force acting on stone is the gravitational force, since air resistance is negligible (or ignored).

1.2.1

DOWNWARDS AS POSITIVE

Let downwards be positive

$v_f^2 = v_i^2 + 2a\Delta y$ 1.

$v_f^2 = (0)^2 + 2(+9,8)(+20)$ 2.

$v_f^2 = 392$

$\sqrt{vf^2} = \pm\sqrt{393}$

$V_f = 19,80\ m.s^{-1}$ or $\cancel{v_f = -19,80\ m.s^{-1}}$ 3.

$\therefore v_f = 19,80\ m.s^{-1}$ downwards

Data (Points chosen: Start and end point)

vi = 0 $m.s^{-1}$ (Stone was dropped)

Δy = +20 m

(Tip: Draw a line from start to end, do you see the arrow points downwards?)

a = +9,8 $m.s^{-2}$

(Gravitational acceleration is 9,8 $m.s^{-2}$ downwards, and downwards was taken as positive)

vf = ? (This is the velocity with which the stone strikes the ground, after it has travelled 20 m, with an initial velocity of 0 $m.s^{-1}$)

MARKING NOTES:

1. 2. A mark is awarded for the correct formula and correct substitution. Use brackets when substituting.

3. Since the equation is quadratic, to calculate v_f there are **TWO** possible answers. However, since downwards was taken as positive, and the stone strikes the ground downwards, the **POSITIVE** answer is the only valid answer in this calculation (since it means that the stone strikes the ground in the positive direction).

OR

UPWARDS AS POSITIVE

Let upwards be positive

$v_f^2 = v_i^2 + 2a\Delta y$ —— 1.

$v_f^2 = (0)^2 + 2(-9{,}8)(-20)$ —— 2.

$v_f^2 = 392$

$\sqrt{vf^2} = \pm\sqrt{393}$

$v_f = 19{,}80\ m.s^{-1}$ or

$v_f = -19{,}80\ m.s^{-1}$ —— 3.

$\therefore v_f = 19{,}80\ m.s^{-1}$ downwards

Data (Points chosen: Start and end point)

$v_i = 0\ m.s^{-1}$ (Stone was dropped)

$\Delta y = -20\ m$

(Tip: Draw a line from start to end, do you see the arrow points downwards?)

$a = -9{,}8\ m.s^{-2}$

(Gravitational acceleration is $9{,}8\ m.s^{-2}$ downwards, but upwards was taken as positive)

$vf = ?$ (This is the velocity with which the stone strikes the ground, after it has travelled 20 m, with an initial velocity of $0\ m.s^{-1}$)

MARKING NOTES:

1. 2. A mark is awarded for the correct formula and correct substitution.
Use brackets when substituting.

3. Since the equation is quadratic to calculate v_f there are **TWO** possible answers. However, since upwards was taken as positive, and the stone strikes the ground downwards, the **NEGATIVE** answer is the only valid answer in this calculation (since it means that the stone strikes the ground in the negative direction).

Remember that **velocity** is a vector quantity, therefore, it has **magnitude** and **direction**. The final answer **MUST** be written with both magnitude and direction, unless stated otherwise.

1.2.2

DOWNWARDS AS POSITIVE

Let downwards be positive

$v_f = v_i + a\Delta t$ —— 1.

$(+19{,}80) = (0) + (+9{,}8)\Delta t$ —— 2.

$\Delta t = 2{,}02\ s$ —— 3.

Data (Points chosen: Start and end point)

$vi = 0m.s^{-1}$ (Stone was dropped)

$a = +9{,}8m.s^{-2}$

(Gravitational acceleration is $9{,}8\ m.s^{-2}$ downwards, and downwards was taken as positive)

$vf = +19{,}80m.s^{-1}$

(Calculated in question 1.2.1).

$\Delta t = ?$

(How long is the time taken from the moment the stone is dropped, until it reaches the water).

PRO-TIPS

- **How long** = **time taken**.
- In this question, the time calculated is the **time taken** from the **moment** the stone is dropped, to when the stone reaches the water.

OR

UPWARDS AS POSITIVE

Let upwards be positive

$v_f = v_i + a\Delta t$ 1.

$(-19{,}80) = (0) + (-9{,}8)\Delta t$ 2.

$\Delta t = 2{,}02\ s$ 3.

PRO-TIPS

- **How long** = **time taken.**
- In this question, the time calculated is the **time taken** from the **moment** the stone is dropped, to when the stone reaches the water.

Data (Points chosen: Start and end point)

v_i = 0 $m.s^{-1}$ (Stone was dropped)

a = - 9,8 $m.s^{-2}$

(Gravitational acceleration is 9,8 $m.s^{-2}$ downwards, but upwards was taken as positive)

v_f = - 19,80 $m.s^{-1}$

(Calculated in question 2.2.1).

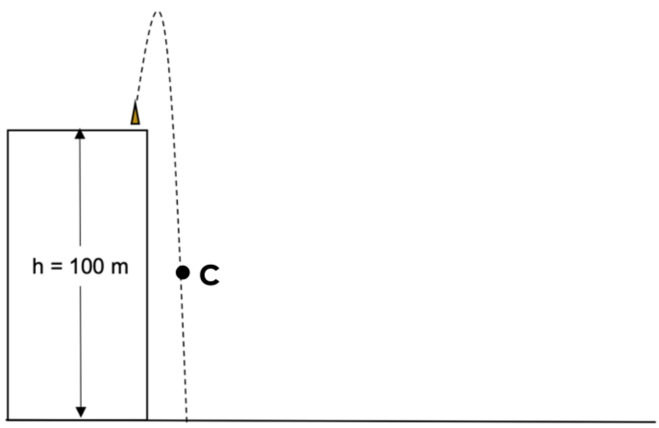
Δt = ?

(How long is the time taken from the moment the stone is dropped, until it reaches the water).

Worked example

A

2. A bullet is shot vertically upwards from the top of a building which is 100 m high at a velocity of 98 $m.s^{-1}$



Calculate the:

2.1 maximum height above the ground which the bullet will reach. (4)

2.2 time the bullet takes to reach the highest point. (3)

2.3 speed at which the bullet reaches point **C**, 50 m above the ground. (3)

2.1 There are two calculations needed to solve this question:

1. Calculate the distance from start to highest point.
2. To calculate the maximum height above the ground that the bullet will reach: Add 100m to the distance from the start to the highest point (calculated in point **1**), since the bullet was initially 100 m above the ground.

DOWNWARDS AS POSITIVE

Let downwards be positive

$v_f^2 = v_i^2 + 2a\Delta y$ — 1.

$(0)^2 = (-98)^2 + 2(+9{,}8)\Delta y$ — 2.

$\Delta y = -490$ m — 3.

$\therefore$Distance from start to highest point = 490 m

Maximum height above the ground = 490 + 100
= 590 m

Data (Points chosen: Start to highest point)

vi = -98 m.s^{-1} (Bullet was shot with an initial velocity upwards, but downwards is taken as the positive direction)

vf = 0 m.s^{-1} (At the highest point in the motion, the velocity is 0 m.s^{-1})

a = +9,8 m.s^{-2} (Gravitational acceleration is 9,8 m.s^{-2} downwards, and downwards was taken as positive)

Δy =? (This is the displacement from the start [At the top of the building] to the highest point).

OR

UPWARDS AS POSITIVE

Let upwards be positive

$v_f^2 = v_i^2 + 2a\Delta y$ — 1.

$(0)^2 = (+98)^2 + 2(-9{,}8)\Delta y$ — 2.

$\Delta y = 490$ m — 3.

$\therefore$Distance from start to highest point = 490 m

Maximum height above the ground = 490 + 100
= 590 m

Data (Points chosen: Start to highest point)

vi = +98 m.s^{-1} (Bullet was shot with an initial velocity upwards)

vf = 0 m.s^{-1} (At the highest point in the motion, the velocity is 0 m.s^{-1})

a = -9,8 m.s^{-2} (Gravitational acceleration is 9,8 m.s^{-2} downwards, but upwards was taken as positive)

Δy =? (This is the displacement from the start [At the top of the building] to the highest point).

MARKING NOTES:

1. 2. A mark is awarded for the correct **formula** and correct **substitution**. Use brackets when substituting.

3. **Displacement** is a vector quantity; distance is a **scalar** quantity. The magnitude of the displacement from the start to the highest point, is equal to the distance from the **start** to the **highest** point.

2.2

DOWNWARDS AS POSITIVE

Let downwards be positive

$v_f = v_i + a\Delta t$ — 1.

$(0) = (-98) + (+9,8)\Delta y$ — 2.

$\Delta t = 10$ s — 3.

Data (Points chosen: Start to highest point)

vi = -98 m.s^{-1}

(Bullet was shot with an initial velocity upwards, but downwards is taken as the positive direction)

vf = 0 m.s^{-1}

(At the highest point in the motion, the velocity is 0 m.s^{-1})

a = +9,8 m.s^{-2}

(Gravitational acceleration is 9,8 m.s^{-2} downwards, and downwards was taken as positive)

Δt =?

(This is the time taken from the start to the highest point)

OR

UPWARDS AS POSITIVE

Let upwards be positive

$v_f = v_i + a\Delta t$ — 1.

$(0) = (+98) + (-9,8)\Delta y$ — 2.

$\Delta t = 10$ s — 3.

Data (Points chosen: Start to highest point)

vi = +98 m.s^{-1}

(Bullet was shot with an initial velocity upwards)

vf = 0 m.s^{-1}

(At the highest point in the motion, the velocity is 0 m.s^{-1})

a = -9,8 m.s^{-2}

(Gravitational acceleration is 9,8 m.s^{-2} downwards, but upwards was taken as positive)

Δt =?

(This is the time taken from the start to the highest point)

MARKING NOTES:

1. 2. A mark is awarded for the correct **formula** and correct **substitution**. Use brackets when substituting.

3. **Time** is a scalar quantity and must **ALWAYS** be **positive**. If you calculate a negative time value, go back to the question and check that your chosen sign convention was properly applied.

2.3

DOWNWARDS AS POSITIVE

Let downwards be positive

$v_f^2 = v_i^2 + 2a\Delta y$ — 1.

$v_f^2 = (-98)^2 + 2(+9,8)(+50)$ — 2.

$v_f^2 = 10584$

$$\sqrt{vf^2} = \pm\sqrt{10584}$$

$v_f = 102,88$ m.s^{-1} or $v_f = \cancel{-102,88}$ m.s^{-1} — 3.

Speed of bullet 50 m above the ground = 102,88 m.s^{-1}

Data (Points chosen: Start to 50m above the ground)

vi = -98 m.s^{-1} (Bullet was shot with an initial velocity upwards, but downwards is taken as the positive direction)

Δy = +50 m (Tip: Draw a line from start to 50 m above the ground, do you see the arrow points downwards?)

a = +9,8 m.s^{-2} (Gravitational acceleration is 9,8 m.s^{-2} downwards, and downwards was taken as positive)

vf =? (This is the velocity 50 m above the ground, which is over a 50 m displacement, from the start)

OR

UPWARDS AS POSITIVE

Let upwards be positive

$v_f^2 = v_i^2 + 2a\Delta y$ — 1.

$v_f^2 = (98)^2 + 2(-9,8)(-50)$ — 2.

$v_f^2 = 10584$

$\sqrt{vf^2} = \pm\sqrt{10584}$

$v_f = 102,88\ m.s^{-1}$ or $v_f = \cancel{-102,88}\ m.s^{-1}$ — 3.

Speed of bullet 50 m above the ground = 102,88 $m.s^{-1}$

Data (Points chosen: Start to 50m above the ground)

vi = +98 $m.s^{-1}$ (Bullet was shot with an initial velocity upwards, but downwards is taken as the positive direction)

Δy = -50 m (Tip: Draw a line from start to 50 m above the ground, do you see the arrow points downwards?)

a = -9,8 $m.s^{-2}$ (Gravitational acceleration is 9,8 $m.s^{-2}$ downwards, but upwards was taken as positive)

vf =? (This is the velocity 50 m above the ground, which is over a 50 m displacement, from the start)

MARKING NOTES:

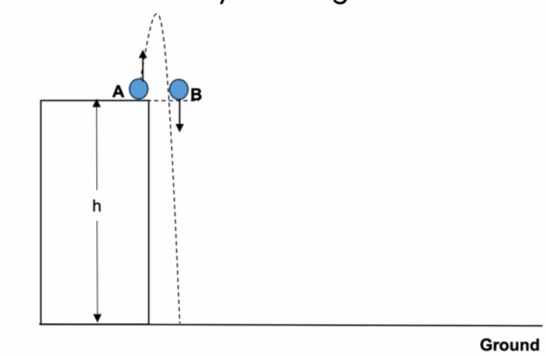
1. 2. A mark is awarded for the correct **formula** and correct **substitution**. Use brackets when substituting.

3. **Speed** is a scalar quantity, with magnitude only, therefore **ONLY** the **positive** answer is accepted, and **NO direction** must be indicated.

Worked example

A + P

3. A ball, **A**, is thrown vertically upward from a height, h, with a speed of 15 $m.s^{-1}$. ONE second later, a second identical ball, **B**, is dropped from the same height as ball **A** as shown in diagram.
Both balls undergo free fall and eventually hit the ground.



Calculate the distance between ball **A** and ball **B** when ball **A** is at its maximum height. (7)

ANSWER:

There is very little guidance given in this question. The following must be considered and calculated:

1. What is the maximum height reached by ball **A**? (from the top of the building).
2. What is the time taken for ball **A** to reach the maximum height?
3. Can this time be used to calculate how far ball **B** has travelled? Factoring in that ball **B** was only dropped 1 second after ball **A** was in motion?
4. The distance between ball **A** and **B** can now be calculated by adding the distances.
   NOTE: ball **A** is moving upwards and ball **B** is moving downwards.

<u>Ball A:</u> Maximum height reached by ball A

DOWNWARDS AS POSITIVE

Let downwards be positive

$v_f^2 = v_i^2 + 2a\Delta y$ — 1.

$(0)^2 = (-15)^2 + 2(+9,8)\Delta y$ — 2.

$\Delta y = -11,48$ m

Distance from the top of the building to the highest point = 11,48 m — 3.

Data (Points chosen: Start to highest point)

vi = -15 m.s^{-1} (Ball A was projected with an initial velocity upwards, but downwards is taken as the positive direction)

vf = 0 m.s^{-1} (The velocity at the highest point is 0 m.s^{-1}).

a = +9,8 m.s^{-2} (Gravitational acceleration is 9,8 m.s^{-2} downwards, and downwards was taken as positive)

Δy =? (This is the displacement, from the start to the highest point)

OR

UPWARDS AS POSITIVE

Let upwards be positive

$v_f^2 = v_i^2 + 2a\Delta y$ — 1.

$(0)^2 = (15)^2 + 2(-9,8)\Delta y$ — 2.

$\Delta y = 11,48$ m

Distance from the top of the building to the highest point = 11,48 m — 3.

Data (Points chosen: Start to highest point)

vi = +15 m.s^{-1} (Ball A was projected with an initial velocity upwards)

vf = 0 m.s^{-1} (The velocity at the highest point is 0 m.s^{-1})

a = -9,8 m.s^{-2} (Gravitational acceleration is 9,8 m.s^{-2} downwards, but upwards was taken as positive)

Δy =? (This is the displacement, from the start to the highest point)

MARKING NOTES:

1. 2. A mark is awarded for the correct **formula** and correct **substitution**. Use brackets when substituting.

3. **Displacement** is a vector quantity; distance is a **scalar** quantity. The magnitude of the displacement from the start to the **highest point**, is the distance from the start to the highest point.

Ball A: **Time taken for ball A to reach maximum height**

DOWNWARDS AS POSITIVE

Let downwards be positive

$v_f = v_i + a\Delta t$ — 1.

$(0) = (-15) + (+9,8)\Delta y$ — 2.

$\Delta t = 1,53$ s — 3.

Data (Points chosen: Start to highest point)

vi = -15 m.s⁻¹ (Ball A was projected with an initial velocity upwards, but downwards is taken as the positive direction)

vf = 0 m.s⁻¹ (At the highest point in the motion, the velocity is 0 m.s⁻¹)

a = +9,8 m.s⁻² (Gravitational acceleration is 9,8 m.s⁻² downwards, and downwards was taken as positive)

Δt =? (This is the time taken from the start to the highest point)

OR

UPWARDS AS POSITIVE

Let upwards be positive

$v_f = v_i + a\Delta t$ — 1.

$(0) = (+15) + (-9,8)\Delta y$ — 2.

$\Delta t = 1,53$ s — 3.

Data (Points chosen: Start to highest point)

vi = +15 m.s⁻¹ (Ball A was projected with an initial velocity upwards)

vf = 0 m.s⁻¹ (At the highest point in the motion, the velocity is 0 m.s⁻¹)

a = -9,8 m.s⁻² (Gravitational acceleration is 9,8 m.s⁻² downwards, but upwards was taken as positive)

Δt =? (This is the time taken from the start to the highest point)

MARKING NOTES:

1. 2. A mark is awarded for the correct **formula** and correct **substitution**. Use brackets when substituting.

3. **Time** is a scalar quantity and must **ALWAYS** be **positive**. If you calculate a negative time value, go back to the question and check that your chosen sign convention was properly applied.

Ball B

Ball B was only dropped 1 second **AFTER** ball **A** was projected upwards, and therefore was only in motion for 0,53 s when ball **A** reached the highest point – this can be used to calculate the distance travelled by ball **B** for the 0,53 s it is in motion:

Ball B

DOWNWARDS AS POSITIVE

Let downwards be positive

$\Delta y = v_i \Delta t + \frac{1}{2} a\Delta t^2$ ——— 1.

$\Delta y = (0)(0,53) + \frac{1}{2}(+9,8)(0,53)^2$ ——— 2.

$\Delta y = 1,38$ m

Distance travelled by ball B during the 0,53s = 1,38 m ——— 3.

Data (Points chosen: Start to 0,53 s in motion)

$v_i = 0$ m.s^{-1} (Ball B was dropped, and started from rest).

$\Delta t = 0,53$ s (Ball B is in motion for 0,53s as ball A is moving towards the highest point).

$a = +9,8$ m.s^{-2} (Gravitational acceleration is 9,8 m.s^{-2} downwards, and downwards was taken as positive)

$\Delta y = ?$ (This is the displacement, from the starting point to 0,53s in motion).

OR

UPWARDS AS POSITIVE

Let upwards be positive

$\Delta y = v_i \Delta t + \frac{1}{2} a\Delta t^2$ ——— 1.

$\Delta y = (0)(0,53) + \frac{1}{2}(-9,8)(0,53)^2$ ——— 2.

$\Delta y = -1,38$ m

Distance travelled by ball B during the 0,53s = 1,38 m ——— 3.

Data (Points chosen: Start to 0,53 s in motion)

$v_i = 0$ m.s^{-1} (Ball B was dropped, and started from rest).

$\Delta t = 0,53$ s (Ball B is in motion for 0,53s as ball A is moving towards the highest point).

$a = -9,8$ m.s^{-2} (Gravitational acceleration is 9,8 m.s^{-2} downwards, but upwards was taken as positive)

$\Delta y = ?$ (This is the displacement, from the starting point to 0,53s in motion).

Distance between ball **A** and **B** when ball **B** is at the highest point = 11,48 + 1,38

Distance between ball **A** and **B** when ball **B** is at the highest point = 12,86 m

MARKING NOTES:

1. 2. A mark is awarded for the correct **formula** and correct **substitution**. Use brackets when substituting.

3. **Displacement** is a vector quantity; distance is a **scalar** quantity. The magnitude of the displacement from the start to 0,53 s in motion, is the distance from the start to 0,53 s in motion.

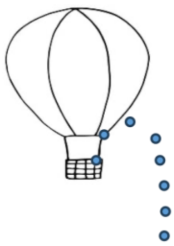
HOT AIR BALLOON – TYPE PROBLEMS

Commonly in these problems, the hot air balloon is moving at a **constant** velocity ($\therefore a = 0$ m.s^{-2}).

- An object is either dropped **OR** projected from the moving hot air balloon.
- The **initial** velocity of the object is measured relative to the velocity of the balloon, and can be calculated by **finding** the **vector sum** of the velocity of the **balloon** and the velocity with which the **object** is released or projected.

Below are common scenarios encountered in these "hot air balloon" problems:

Scenario 1: Object is dropped from a hot air balloon moving vertically **upwards** at a **constant velocity**.



- The initial velocity of the object is **NOT** zero in this case, but rather equal to the velocity of the hot air balloon. Due to the object's inertia, the object will initially move vertically upwards.

Scenario 2: Object is thrown vertically upwards from a hot air balloon moving vertically **upwards** at a **constant velocity**.

- The object will now have a greater initial upwards velocity - the initial velocity of the object is the **vector sum** of the velocity with which the object is initially projected upwards, and the velocity of the hot air balloon.

Scenario 3: Object is thrown vertically upwards from a hot air balloon moving vertically downwards at a constant velocity.

- **Assuming the upwards velocity of the object is greater than the downwards velocity of the hot air balloon**, the object will initially move vertically upwards with a velocity less than the velocity with which the object is projected.

i.e. $v_{i\ object} < v_{object\ projected}$

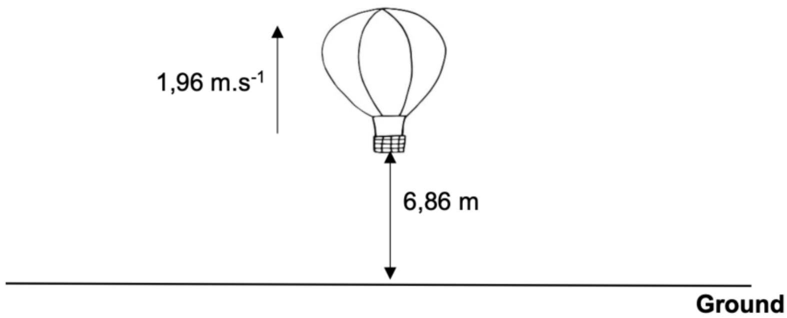
PRO-TIPS

- These hot air balloon type problems do not only apply to hot air balloons – it can be any object moving vertically **upwards** or **downwards** at a constant speed e.g., helicopter, plane etc.
- These problems occur again in **graphs of motion!**

Worked example

C + A + P

1. Vuyo is in a hot air balloon that is moving at a constant velocity of 1,96 $m.s^{-1}$ upwards. As he leans over to look below, his watch falls off. At this point he is 6,86 m above the ground.



1.1 Write down the initial velocity of the watch at the moment it falls off. Give a reason for the answer. (2)

Calculate:

1.2 the maximum height reached by the watch above the point where it fell off. (3)

1.3 the distance between the hot air balloon and the watch, 1 second after the watch has fallen.
Assume that the hot air balloon continues moving at the same constant velocity. (6)

ANSWER:

1.1 1,96 $m.s^{-1}$ upwards. The initial velocity of the watch is the same as that of the hot air balloon, due to the inertia of the watch.

1.2 **DOWNWARDS AS POSITIVE**
Let downwards be positive

$v_f^2 = v_i^2 + 2a\Delta y$ — 1.
$(0)^2 = (-1,96)^2 + 2(+9,8)\Delta y$ — 2.
$\Delta y = -0,196$ m

Maximum height reached by the watch = 0,196 m — 3.

Data (Points chosen: Start to highest point)

$v_i = -1,96\ m.s^{-1}$ (Watch was projected with an initial velocity upwards, but downwards is taken as the positive direction)

$v_f = 0\ m.s^{-1}$ (The velocity at the highest point is 0 $m.s^{-1}$).

$a = +9,8\ m.s^{-2}$ (Gravitational acceleration is 9,8 $m.s^{-2}$ downwards, and downwards was taken as positive)

$\Delta y = ?$ (This is the displacement, from the starting point to the highest point)

OR

UPWARDS AS POSITIVE

Let upwards be positive

$v_f^2 = v_i^2 + 2a\Delta y$ —— 1.

$(0)^2 = (1{,}96)^2 + 2(-9{,}8)\Delta y$ —— 2.

$\Delta y = 0{,}196$ m

Maximum height reached by the watch = 0,196 m —— 3.

MARKING NOTES:

1. 2. A mark is awarded for the correct **formula** and correct **substitution**. Use brackets when substituting.
3. **Displacement** is a vector quantity; distance is a **scalar** quantity. The magnitude of the displacement from the start to the **highest point**, is the distance from the start to the highest point, in this case.

Data (Points chosen: Start to highest point)

vi = +1,96 m.s^{-1} (Watch was projected with an initial velocity upwards)

vf = 0 m.s^{-1} (The velocity at the highest point is 0 m.s^{-1}).

a = -9,8 m.s^{-2} (Gravitational acceleration is 9,8 m.s^{-2} downwards, but upwards was taken as positive)

Δy =? (This is the displacement, from the starting point to the highest point.)

1.3 A few things to consider in this question:

- The hot air balloon continues to move at the **same** constant velocity for that 1 second – this can be used to **calculate the distance** it travels in that one second.
- The watch will be at its downwards motion for a portion of the 1 second.
- The distance between the hot air balloon and the watch can be calculated by **adding** the **distances** calculated **after** 1 second.

HOT AIR BALLOON

1.3 DOWNWARDS AS POSITIVE

Let downwards be positive

$\Delta y = v_i \Delta t + \frac{1}{2} a \Delta t^2$

$\Delta y = (-1{,}96)(1) + \frac{1}{2}(0)(1)^2$

$\Delta y = -1{,}96$ m

Distance travelled by hot air balloon for 1s = 1,96 m

Data (Points chosen: Start to 1 s in motion)

vi = -1,96 m.s^{-1} (The hot air ballon has an initial velocity upwards, but downwards was taken as positive).

Δt = 1 s (Hot air balloon is in motion for 1 second that displacement needs to be determined).

a = 0 m.s^{-2} (The hot air balloon is moving at a constant velocity)

Δy =? (This is the displacement for 1 second in motion).

OR

HOT AIR BALLOON

1.3 UPWARDS AS POSITIVE

Let upwards be positive

$\Delta y = v_i \Delta t + \frac{1}{2} a \Delta t^2$

$\Delta y = (1{,}96)(1) + \frac{1}{2}(0)(1)^2$

$\Delta y = 1{,}96$ m

Distance travelled by hot air balloon for 1s = 1,96 m

Data (Points chosen: Start to 1 s in motion)

vi = +1,96 m.s^{-1} (The hot air ballon has an initial velocity upwards).

Δt = 1 s (Hot air balloon is in motion for 1 second that displacement needs to be determined).

a = 0 m.s^{-2} (The hot air balloon is moving at a constant velocity)

Δy =? (This is the displacement for 1 second in motion).

WATCH

1.3 DOWNWARDS AS POSITIVE

Let downwards be positive

$$\Delta y = v_i \Delta t + \frac{1}{2} a\Delta t^2$$

$$\Delta y = (-1{,}96)(1) + \frac{1}{2}(9{,}8)(1)^2$$

$$\Delta y = 2{,}94 \text{ m}$$

Downwards distance travelled by watch after 1 s = 2,94 m

OR

WATCH

1.3 UPWARDS AS POSITIVE

Let upwards be positive

$$\Delta y = v_i \Delta t + \frac{1}{2} a\Delta t^2$$

$$\Delta y = (+1{,}96)(1) + \frac{1}{2}(-9{,}8)(1)^2$$

$$\Delta y = -2{,}94 \text{ m}$$

Downwards distance travelled by watch after 1 s = 2,94 m

Data (Points chosen: Start to 1 s in motion)

$v_i = -1{,}96$ m.s^{-1} (The initial velocity of the watch is the same as that of the hot air balloon)

$\Delta t = 1$ s (The watch is in motion for 1 second).

$a = +9{,}8$ m.s^{-2} (Gravitational acceleration is 9,8 m.s^{-2} downwards, and downwards was taken as positive)

$\Delta y = ?$ (This is the displacement, from the starting point to 1 s in motion).

Data (Points chosen: Start to 1 s in motion)

$v_i = +1{,}96$ m.s^{-1} (The initial velocity of the watch is the same as that of the hot air balloon)

$\Delta t = 1$ s (The watch is in motion for 1 second).

$a = -9{,}8$ m.s^{-2} (Gravitational acceleration is 9,8 m.s^{-2} downwards, but upwards was taken as positive)

$\Delta y = ?$ (This is the displacement, from the starting point to 1 s in motion).

Distance between **hot air balloon** and the **watch** after 1 s = 1,96 + 2,94
Distance between **hot air balloon** and the **watch** after 1 s = 4,90 m

Worked example

A + P

2. A bird carries a stick while flying vertically upwards at a constant velocity. The bird lifts its beak and projects the stick upwards at with an additional velocity of 0,1 m.s^{-1}.
The stick travels for 4s before reaching the ground.

During this time, the bird flies a further 2m. Take the ground as the reference point. Calculate the height above the ground reached by the bird, the moment the stick reaches the ground.

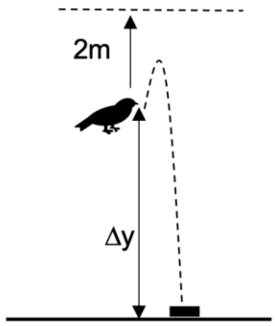
ANSWER:

A few things to consider in this question:

1. The bird **continues** moving at the same constant velocity for 4s, and covers a distance of 2m during this time, this can be used to calculate the velocity of the bird. The initial velocity of the stick is the vector sum of the initial velocity of the bird and the velocity with which the stick is projected.
2. Calculating the **displacement** of the stick from start to end will be the **same** as the initial distance of the bird above the ground.
3. The **distance** that the bird is above the ground can be **determined** by adding the 2m to the initial distance above the ground.

2. DOWNWARDS AS POSITIVE

Let downwards be positive



TIP : DRAW AN IMAGE OF THE SCENARIO

Bird (moving at a constant v; a = 0 m.s^{-2})

$\Delta y = v_i \Delta t + \frac{1}{2} a\Delta t^2$

$(-2) = v_i(4) + \frac{1}{2}(0)(4)^2$

$v_i = -0{,}50 \text{ m.s}^{-1}$

Stick (in free – fall; a = +9,8 m.s^{-2})

Initial velocity of the stick = - 0,5 + (-0,1)
Initial velocity of the stick = - 0,60 m.s^{-1}

$\Delta y = v_i \Delta t + \frac{1}{2} a\Delta t^2$

$\Delta y = (-0{,}60)(4) + \frac{1}{2}(9{,}8)(4)^2$

$\Delta y = 76 \text{ m}$

NOTE: The magnitude of the displacement of the stick = distance from point stick released to the ground.

Height reached by bird = 76 + 2
Height reached by bird = 78 m

Data (Points chosen: Start to 4s in motion)

Δy = -2m (The displacement of the bird is 2m upwards after 4s, this is -2m as downwards is taken as positive)

Δt = 4 s (The bird is in motion for 4 second).

a = 0 m.s^{-2} (The bird is moving at a constant velocity ∴ a = 0 m.s^{-2})

vi =? (This is the initial velocity of the bird, which is the also the constant velocity with which the bird moves)

Data (Points chosen: Start to 4s in motion i.e., ground)

vi = -0,60 m.s^{-1} (The velocity of the stick i.e. the vector sum of the velocity of the bird and the velocity with which the stick is projected upwards)

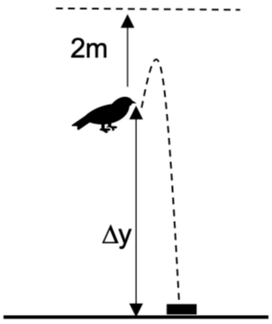
Δt = 4 s (The stick is in motion for 4 seconds when it reaches the ground).

a = +9,8 m.s^{-2} (The stick is in free- fall, therefore its acceleration is 9,8 m.s^{-2} downwards)

Δy =? (This is the displacement from the point where the stick is projected, until it reaches the ground).

2. UPWARDS AS POSITIVE

Let upwards be positive



HINT : DRAW AN IMAGE OF THE SCENARIO

Bird (moving at a constant v; a = 0 m.s^{-2})

$\Delta y = v_i \Delta t + \frac{1}{2} a\Delta t^2$

$(2) = v_i(4) + \frac{1}{2}(0)(4)^2$

$v_i = 0{,}50 \text{ m.s}^{-1}$

Stick (in free – fall; a = -9,8 m.s^{-2})

Initial velocity of the stick = 0,5 + (+0,1)
Initial velocity of the stick = +0,60 m.s^{-1}

$\Delta y = v_i \Delta t + \frac{1}{2} a\Delta t^2$

$\Delta y = (0{,}60)(4) + \frac{1}{2}(-9{,}8)(4)^2$

$\Delta y = -76 \text{ m}$

The magnitude of the displacement of the stick = distance from ground from which the stick was released.

Height reached by bird = 76 + 2
Height reached by bird = 78 m

Data (Points chosen: Start to 4s in motion)

Δy = +2m (The displacement of the bird is 2m upwards after 4s.)

Δt = 4 s (The bird is in motion for 4 second).

a = 0 m.s^{-2} (The bird is moving at a constant velocity ∴ a = 0 m.s^{-2})

vi =? (This is the initial velocity of the bird, which is the also the constant velocity with which the bird moves)

Data (Points chosen: Start to 4s in motion i.e., ground)

vi = 0,60 m.s^{-1} (The velocity of the stick i.e. the vector sum of the velocity of the bird and the velocity with which the stick is projected upwards)

Δt = 4 s (The stick is in motion for 4 seconds when it reaches the ground).

a = -9,8 m.s^{-2} (The stick is in free- fall, therefore its acceleration is 9,8 m.s^{-2} downwards, however upwards was taken as positive).

Δy =? (This is the displacement from the point where the stick is projected, until it reaches the ground).

VERTICAL PROJECTILE MOTION: GRAPHS OF MOTION

In vertical projectile motion, there are **THREE** types of graphs of motion that will be covered:

1. **Velocity–time** graph.
2. **Position–time** graph.
3. **Acceleration – time** graph.

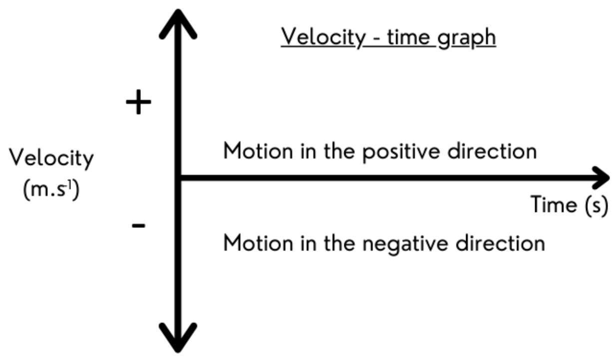
PRO-TIPS

- When **sketching/ drawing graphs**, choosing a sign convention is important.
- This should be the **same** sign convention as the sign convention chosen for your **calculations**.

Let's discuss each of these graphs in detail:

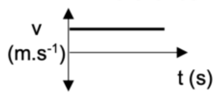
1. VELOCITY – TIME GRAPH

This graph represents how the velocity of an object changes with time. The axes are labelled as follows:



Velocity is a vector quantity; therefore, a positive velocity value represents motion in the positive direction, this motion is represented **above** the x – axis. A negative velocity value represents motion in the negative direction, this motion is represented **below** the x – axis.

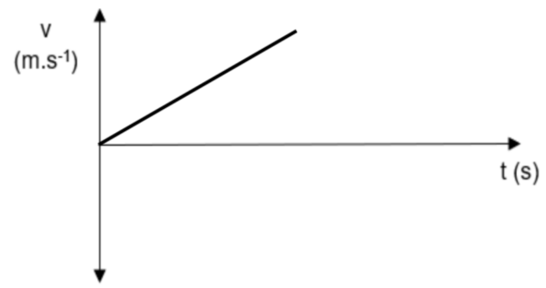
GRADIENT AND THE AREA UNDER THE VELOCITY: TIME GRAPH

Gradient	Area under the velocity- time graph
Gradient = $\frac{\Delta y}{\Delta x}$ Gradient = $\frac{\Delta v}{\Delta t}$ or $\frac{change\ in\ velocity}{change\ in\ time}$ **Gradient = acceleration** For vertical projectile motion, the acceleration is constant, therefore, the gradient of the graph is also constant.	Reference:  Area under the velocity – time graph = l x b Area under the velocity – time graph = Δt x v Area under the velocity – time graph = displacement

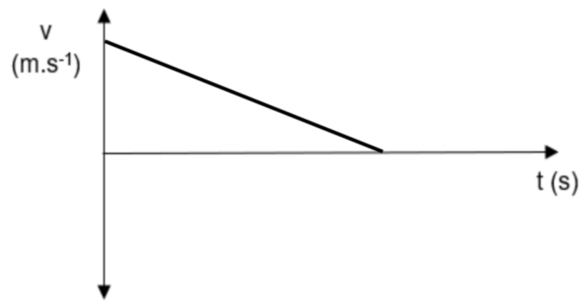
REPRESENTING MOTION ON A VELOCITY: TIME GRAPH

The velocity time graph is a straight line graph due to constant acceleration.

Velocity increases uniformly in the positive direction

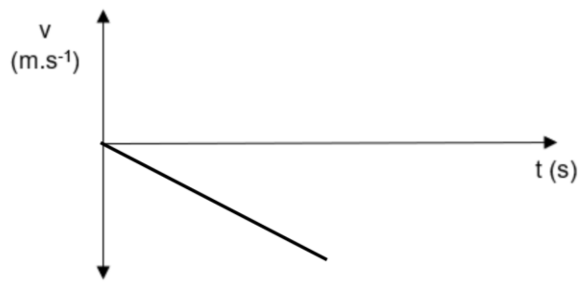


Velocity decreases uniformly in the positive direction

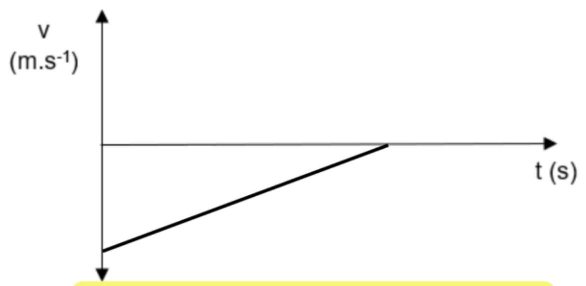


Note: The graph approaches zero!

Velocity increases uniformly in the negative direction

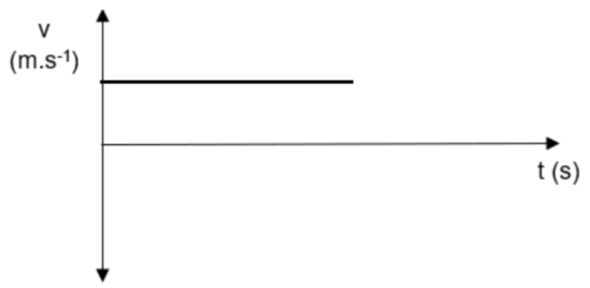


Velocity decreases uniformly in the negative direction

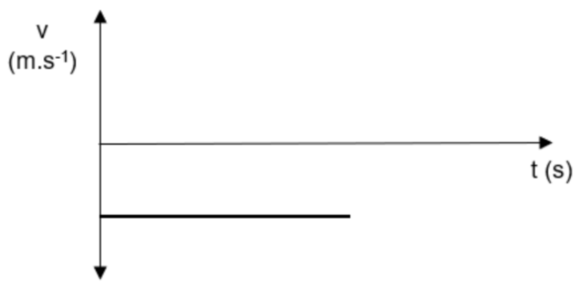


Note: The graph approaches zero!

Constant velocity in the positive direction

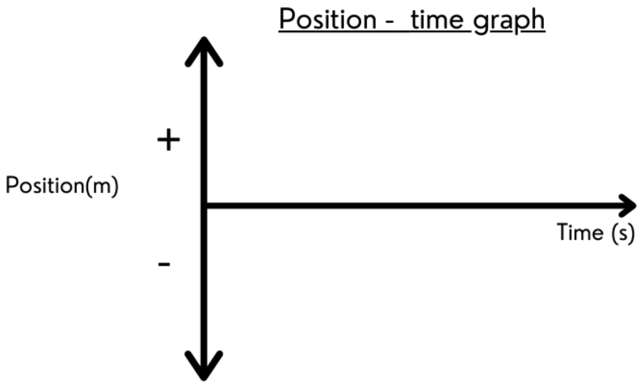


Constant velocity in the negative direction.



2.POSITION – TIME GRAPH

This graph represents how the position of an object changes with time. The axes are labelled as follows:



Position is a vector quantity; **it is measured relative to (from) a reference point** (**starting point**). Usually the ground is taken as the reference point (starting point), but this is **NOT** always the case – sometimes even the top of the building is taken as the reference point!

GRADIENT OF THE POSITION – TIME GRAPH

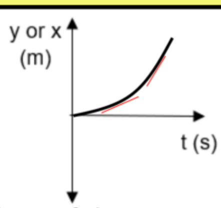
Gradient
Gradient = $\frac{\Delta y}{\Delta x}$ Gradient = $\frac{\Delta y}{\Delta t}$ or $\frac{change\ in\ position}{change\ in\ time}$ **Gradient = velocity** For vertical projectile motion, the acceleration is constant, due to the velocity changing by a constant amount ($9{,}8\ m.s^{-2}$) every second.

REPRESENTING MOTION ON A POSITION – TIME GRAPH

NOTE:
The motion is **INITIALLY** represented above or below the x – axis, depending on whether the position is in the positive or negative direction.

PRO-TIPS

- The position-time graph is a curved graph when representing **accelerated** motion.
- Draw a **tangent** to the curve (a line that touches the graph at **one** point **ONLY**) to determine the shape of the curve.

<table>
<tr><th>Velocity increases uniformly in the positive direction</th><th>Velocity decreases uniformly in the positive direction</th></tr>
<tr><td>


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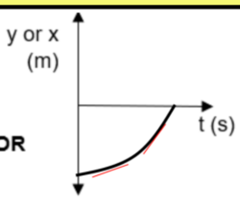
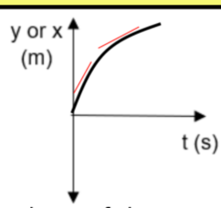


Gradient of the tangent is **positive** (velocity in the positive direction) and **increasing** (velocity increasing).
</td><td>


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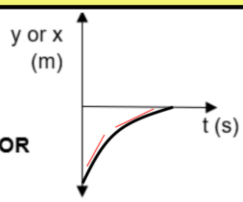
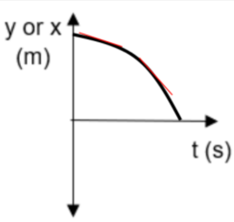


Gradient of the tangent is **positive** (velocity in the positive direction) and **decreasing** (velocity decreasing).
</td></tr>
<tr><th>Velocity increases uniformly in the negative direction</th><th>Velocity decreases uniformly in the negative direction</th></tr>
<tr><td>


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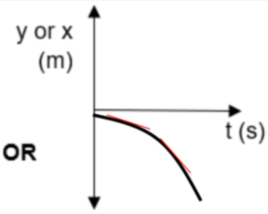
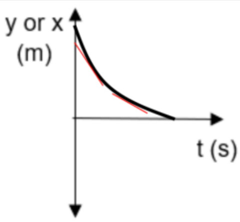


Gradient of the tangent is **negative** (velocity in the negative direction) and **increasing** (velocity increasing).
</td><td>


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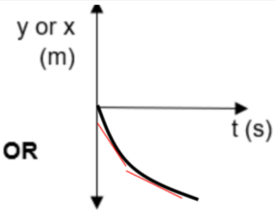
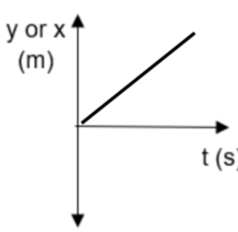
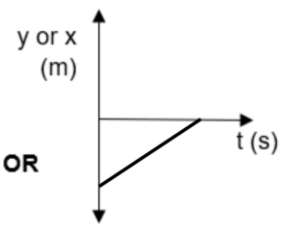
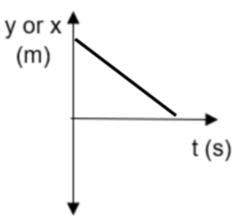
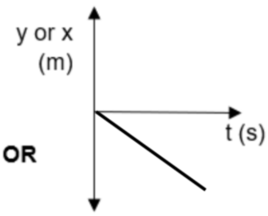


Gradient of the tangent is **negative** (velocity in the negative direction) and **decreasing** (velocity decreasing).
</td></tr>
<tr><th>Constant velocity in the positive direction</th><th>Constant velocity in the negative direction</th></tr>
<tr><td>


OR


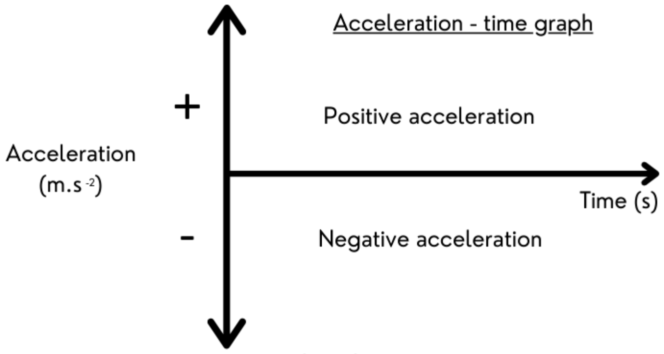
Note: The position – time graph is a straight- line graph with a constant gradient when the object moves at a constant velocity.
</td><td>


OR


Note: The position – time graph is a straight- line graph with a constant gradient when the object moves at a constant velocity.
</td></tr>
</table>

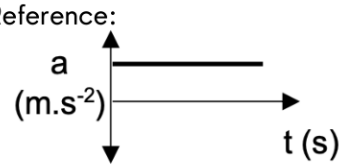
3.ACCELERATION – TIME GRAPH

This graph represents the acceleration of an object with respect to time. The axes are labelled as follows:



Acceleration is a vector quantity; it can either be positive, negative or zero. For a projectile, the acceleration is 9,8 m.s^{-2} downwards.

AREA UNDER THE ACCELERATION – TIME GRAPH

Area under the acceleration – time graph	
Reference: 	Area under the acceleration – time graph = l x b Area under the velocity – time graph = Δt x a Area under the velocity – time graph = change in velocity (for that time interval).

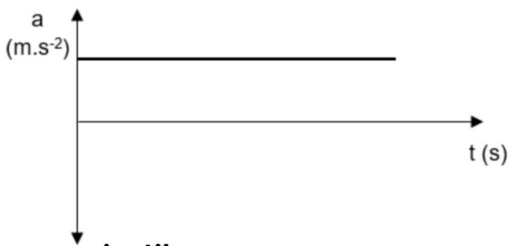
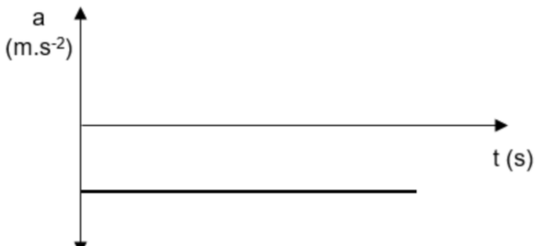
REPRESENTING MOTION ON AN ACCELERATION– TIME GRAPH

NOTE:
The acceleration of the object can be represented above **OR** below the x – axis, depending on whether the **acceleration** is in the **positive** or **negative** direction.

PRO-TIPS

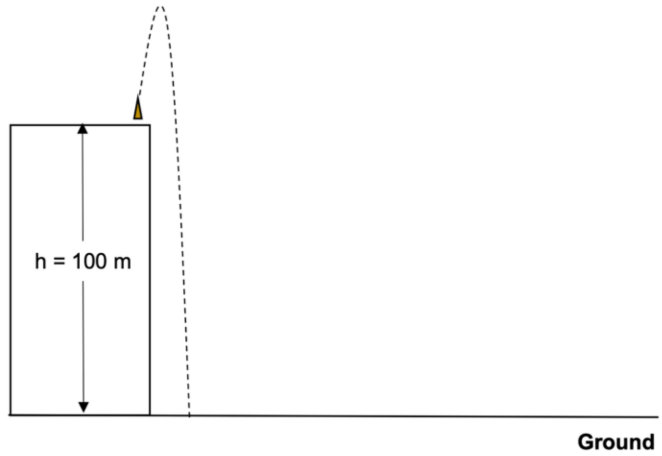
In this section of graphs of motion, you will be expected to:

- **Sketch** or draw graphs of velocity-time/ position -time/ acceleration – time.
- **Interpret** graphs of velocity-time/position- time/ acceleration – time.

Positive acceleration	Negative acceleration
	
For a projectile: a = 9,8 m.s^{-2} downwards. If downwards is taken as positive, this can be represented as +9,8 m.s^{-2}, and the acceleration of the projectile is represented above the x - axis as shown in the above acceleration – time graph.	**For a projectile:** a = 9,8 m.s^{-2} downwards. If upwards is taken as positive, this can be represented as -9,8 m.s^{-2}, and the acceleration of the projectile is represented below the x - axis as shown in the above acceleration – time graph.

Worked example

1. A bullet is shot vertically upwards from the top of a building which is 100 m high at a velocity of 98 $m.s^{-1}$. The maximum height reached by the bullet is 590 m and it takes the bullet 10 s to reach the maximum height. The bullet strikes the ground at a velocity of 107,54 $m.s^{-1}$ after 20,97 s.



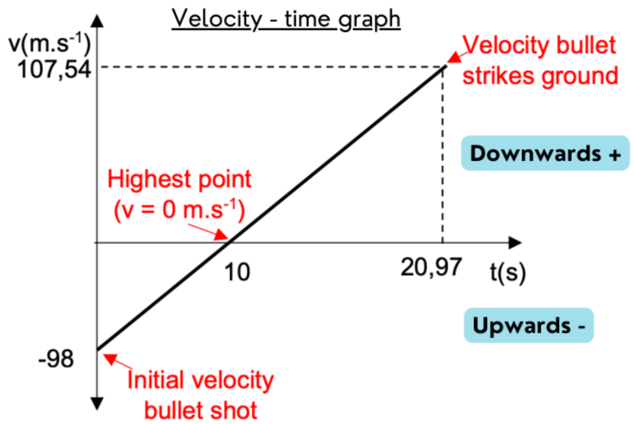
Sketch the following graphs representing the entire motion of the bullet. Include all relevant values. Take the ground as the zero reference.

1.1 Velocity – time graph. (3)
1.2 Position – time graph. (3)
1.3 Acceleration – time graph. (2)

Answer:
Note: These are all sketch graphs, therefore the graph does not need to be drawn to scale and only the relevant values need to be indicated.

1.1 DOWNWARDS AS POSITIVE

Downwards was taken as positive, therefore the initial velocity of the bullet is -98 $m.s^{-1}$, as it was shot upwards. The bullet strikes the ground downwards at 107, 54 $m.s^{-1}$

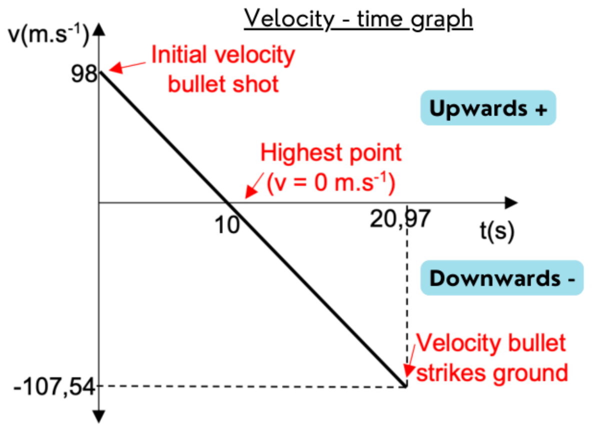


NOTE: The graph is not symmetrical as most of the motion happened downwards therefore the start and end point are not the same.

OR

UPWARDS AS POSITIVE

Upwards was taken as positive, therefore the initial velocity of the bullet is +98 $m.s^{-1}$, as it was shot upwards. The bullet strikes the ground downwards,at -107, 54 $m.s^{-1}$.

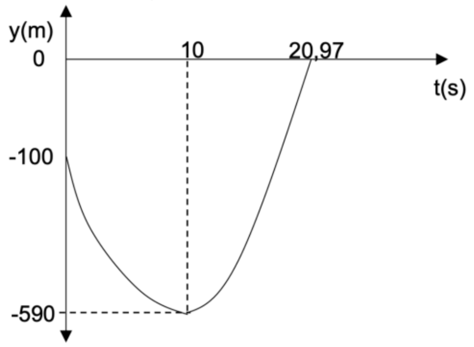


NOTE: The graph is not symmetrical as most of the motion happened downwards, and the start and end point are not the same.

1.2 DOWNWARDS AS POSITIVE **OR** UPWARDS AS POSITIVE

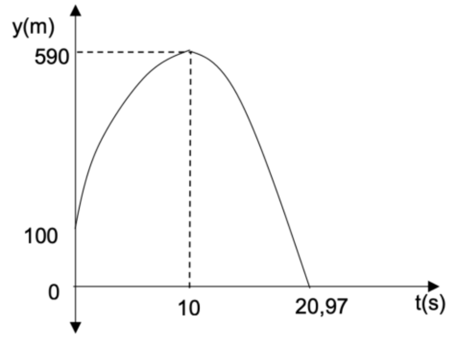
DOWNWARDS AS POSITIVE

Downwards was taken as positive, therefore the initial position from the ground (reference point) is 100 m upwards, which is -100 m, the object reaches the maximum height from the ground which is -590 m and then moves downwards towards the ground and reaches the final position of 0 m after 20,97s.



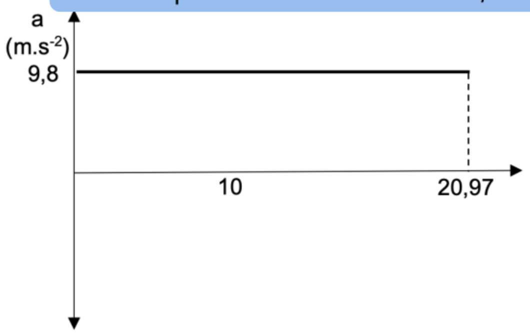
UPWARDS AS POSITIVE

Upwards was taken as positive, therefore the initial position from the ground (reference point) is 100 m upwards, which is +100 m, the object reaches the maximum height from the ground which is +590 m and then moves downwards towards the ground and reaches the final position of 0 m after 20,97s.



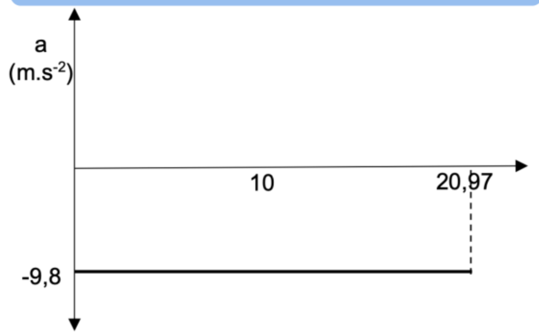
1.3) DOWNWARDS AS POSITIVE

Constant positive acceleration of 9,8 $m.s^{-2}$



UPWARDS AS POSITIVE

Constant negative acceleration of -9,8 $m.s^{-2}$



Worked example

2. A baseball player throws a ball vertically upwards in the air and after 4 seconds, catches it again at the same position from which the ball was thrown. Ignore the effects of air resistance.

2.1 Calculate the velocity of the ball when the player throws it up in the air. (4)

2.2 Sketch a velocity – time graph, showing the motion of the ball from the moment it leaves the baseball players hand to when it returns to the baseball players' hand.
Indicate the following on the graph:

- Initial velocity with which the ball leaves the player's hand.
- Velocity at the highest point.
- Time taken to reach the highest point.
- Velocity at which the player catches the ball.
- Time taken for player to catch the ball. (4)

2.1 DOWNWARDS AS POSITIVE

Let downwards be positive

$v_f = v_i + a\Delta t$ —— 1.

$(0) = v_i + (9,8)(2)$ —— 2.

$v_i = -19,60\ m.s^{-1}$

$v_i = 19,60\ m.s^{-1}$ upwards —— 3.

NOTE: This is symmetrical motion (start and end point are the same), therefore the initial and final velocity are equal in magnitude, but opposite in direction.

Data (Points chosen: Start to highest point in motion)

vf = 0 m.s^{-1} (At the highest point, the velocity is 0 m.s^{-1})

Δt = 2 s (Symmetrical motion, therefore the time taken from start to highest point, is half the total time that the ball is in motion).

a = +9,8 m.s^{-2} (Gravitational acceleration is 9,8 m.s^{-2} downwards, and downwards was taken as positive)

vi =? (This is the initial velocity of the ball, when it leaves the throwers hand)

OR

UPWARDS AS POSITIVE

$v_f = v_i + a\Delta t$ —— 1.

$(0) = v_i + (-9,8)(2)$ —— 2.

$v_i = 19,60\ m.s^{-1}$ upwards —— 3.

Note: This is symmetrical motion (start and end point are the same), therefore the initial and final velocity are equal in magnitude, but opposite in direction.

Data (Points chosen: Start to highest point in motion)

vf = 0 m.s^{-1} (At the highest point, the velocity is 0 m.s^{-1})

Δt = 2 s (Symmetrical motion, therefore the time taken from start to highest point, is half the total time that the ball is in motion).

a = -9,8 m.s^{-2} (Gravitational acceleration is 9,8 m.s^{-2} downwards, but upwards was taken as positive)

vi =? (This is the initial velocity of the ball, when it leaves the throwers hand)

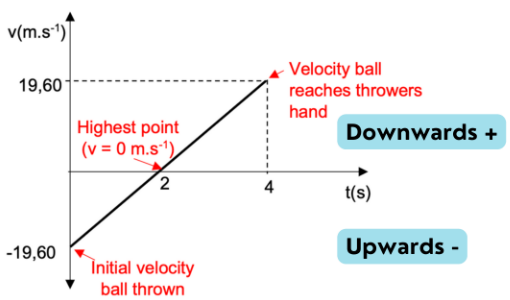
MARKING NOTES:

1. 2. A mark is awarded for the correct **formula** and correct **substitution**. Use brackets when substituting.

3. Velocity is a vector quantity and must always be represented with both magnitude and direction. If a negative answer is calculated, this cannot be left as a negative answer.

2.2) DOWNWARDS AS POSITIVE

Downwards was taken as positive, therefore the initial velocity of the ball is -19,60 m.s^{-1}, as it was thrown upwards. The ball reaches the throwers hand after 4s at the same speed, but in the opposite direction, at a velocity of 19,60 m.s^{-1}.

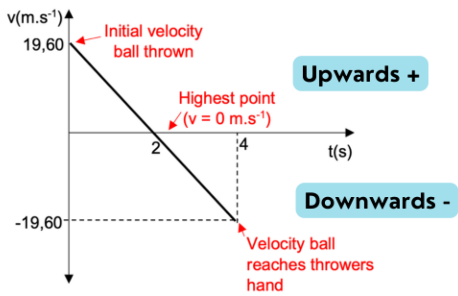


NOTE: The graph is symmetrical as the starting and end point are the same.

OR

UPWARDS AS POSITIVE

Upwards was taken as positive, therefore the initial velocity of the ball is +19,60 m.s^{-1}, as it was thrown upwards. The ball reaches the throwers hand after 4s at the same speed, but in the opposite direction, at a velocity of -19,60 m.s^{-1}.



NOTE: The graph is symmetrical as the starting and end point are the same.

GRAPHS OF MOTION: GRAPH INTERPRETATION

PRO-TIPS

1. **Determine** the positive direction of motion, if this is not given.
2. **Analyze** the motion of the projectile – **identify** on the **graph** relevant points of motion and values of time, position OR velocity OR acceleration.
3. **Calculations** usually involve using the graph, **NOT** equations of motion, unless stated otherwise.

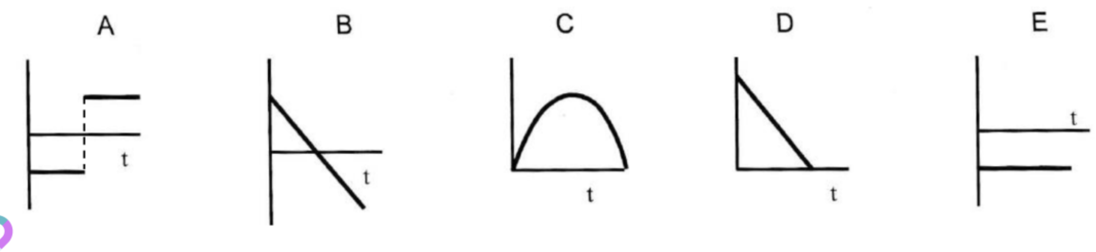
Worked examples

Multiple choice questions

C + A

The information below applies to QUESTIONS 1.1 – 1.3.
A ball is projected vertically upwards and returns to the point of projection in 10 s. In the graphs that follow, only the time axis is marked. Take upwards as positive.

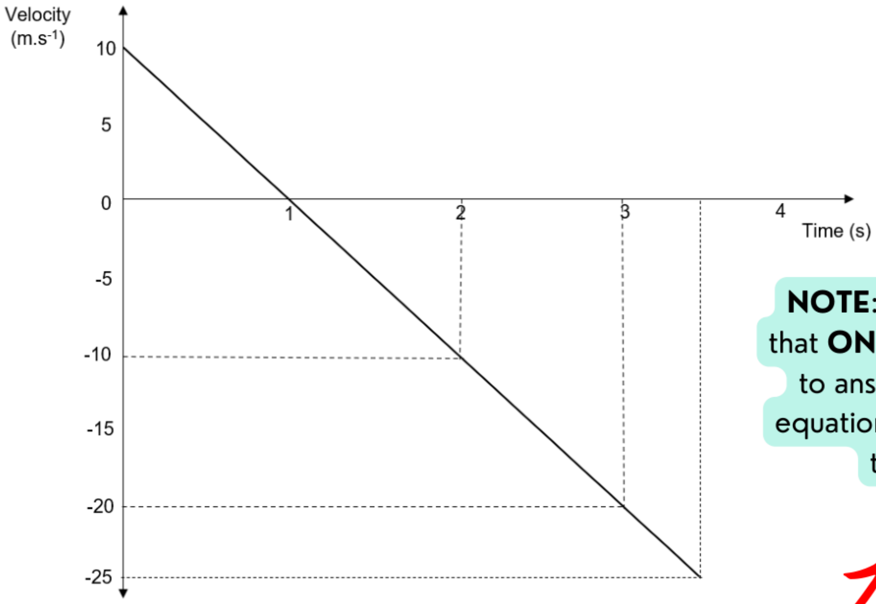
1.1 Which graph represents the position – time graph?
1.2 Which graph represents the velocity – time graph?
1.3 Which graph represents the acceleration – time graph?



Answer:
1.1 C
Hint: Look for the "curved" graph.
Initially as the ball moves upwards, its velocity decreases uniformly, at the highest point, the ball changes direction and during the downwards motion, the velocity of the ball increases uniformly.
1.2 B
Hint: Look for the "straight- line diagonal graph" Same explanation as question 1.1.
1.3 E
Hint: Look for the "flat, straight-line graph that is parallel to the x – axis."
The acceleration of the ball is $9,8\ m.s^{-2}$ downwards, due to upwards being taken as positive, this is $-9,8\ m.s^{-2}$, namely negative acceleration for the entire motion of the ball.

Worked example

A boy stands at the edge of a high cliff. He throws a stone vertically upwards. The stone strikes the ground after 3,5 s. The velocity – time graph below represents the motion of the stone. Upwards is taken as positive.



NOTE: 'Using the graph' means that **ONLY** the graph can be used to answer these questions, no equations of motion may be used to do calculations.

Using the graph, answer the following questions:

1.1 Calculate the acceleration of the stone between t = 2 s and t = 3 s. (4)

1.2 After how many seconds does the stone reach its highest point? (1)

1.3 Calculate the height of the cliff. (4)

1.1 The gradient of the velocity – time graph represents the acceleration.

$a = \text{gradient} = \frac{\Delta y}{\Delta x}$

$a = \text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$

$a = \text{gradient} = \frac{(-20)-(-10)}{(3)-(2)}$

$a = \text{gradient} = -10$

$\therefore a = 10\ m.s^{-2}$ downwards

NOTE: In this example, the gravitational acceleration was rounded up to 10 $m.s^{-2}$ downwards.

1.2 1 second.

NOTE: The velocity at the highest point is 0 $m.s^{-1}$, from the graph, this is at 1 s.

1.3 By calculating the displacement from start to end, using the area under the graph, the magnitude of the displacement will represent the height of the cliff, since the stone started at the top of the cliff and ended on the ground.

Δy = Area under the velocity – time graph

$\Delta y = \frac{1}{2}b \times \perp h + (-\frac{1}{2}b \times \perp h)$

$\Delta y = \frac{1}{2}(1)(10) + [-\frac{1}{2}(2,5)(25)]$

$\Delta y = -26,25$ m

$\therefore$Height of the cliff = 26,25 m.

NOTE: Height is distance, which is a scalar quantity, therefore, it has **magnitude** only.

VERTICAL PROJECTILE MOTION: BALL BOUNCING SCENARIOS

When an object, for example a ball, strikes the ground, it bounces or rebounds. One common example is a tennis ball striking the ground. Due to its elastic material, it rebounds.

INELASTIC VS ELASTIC COLLISIONS

Inelastic collision: Collision where the kinetic energy is not conserved.
Elastic collision: Collision where the kinetic energy is conserved.

- Most collisions in real-life are inelastic collisions, because when an object collides it sometimes **deforms** or changes shape, produces **sound** or gives off **heat** energy.
- This explains why the ball rebounds to a height that is **LESS** than the initial height from which it was projected/ released, because some of the mechanical energy of the ball is converted into other forms of energy such as sound or heat energy.
- **HOWEVER**, elastic collisions can occur when the object is hard, and does not easily deform or change shape.

BALL BOUNCING SCENARIOS: ELASTIC VS INELASTIC COLLISIONS

Scenario 1: Inelastic collision

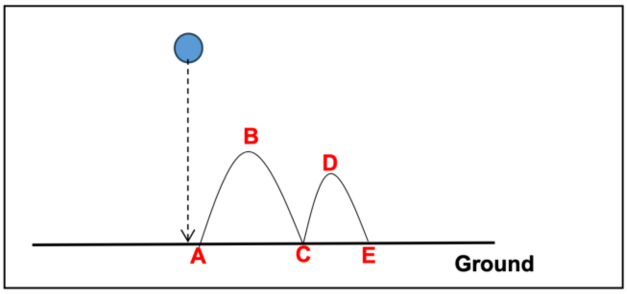


Figure 4

Figure 4 represents an inelastic collision. When a ball bounces off the ground (floor) the maximum height reached by the ball is initially the greatest and decreases with each bounce.

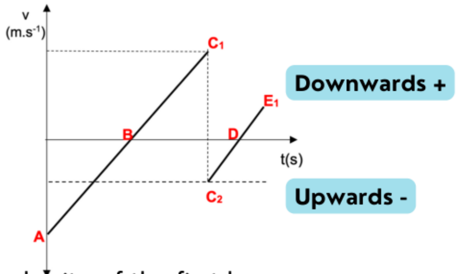
1. **Point A – B:** The ball strikes the ground at point **A** and rebounds upwards (due to the upwards force of the ground on the ball) at a velocity that is less than the velocity at which the ball strikes the ground. (vi bounce **<** v strikes ground). As the ball moves towards point **B**, its velocity **decreases** uniformly.
2. **Point B:** The velocity at the highest point (maximum height in the first bounce) is **ZERO**. The ball changes direction at this point.
3. **Point B - C:** the velocity of the ball increases **UNIFORMLY** during the downwards motion.
4. **Point C:** The ball reaches the ground (floor) at a maximum velocity for the downward motion, this velocity is equal in magnitude **BUT** opposite in direction to the velocity at which the ball rebounds (i.e. same speed), because the motion is symmetrical motion.
5. **Point C - D:** The ball strikes the ground at point **C** and rebounds upwards at a velocity that is less than the velocity at which the ball strikes the ground (Vi bounce **<** Vstrikes ground). The rebound velocity in the second bounce is less than the rebound velocity in the first bounce. As the ball moves towards point **D,** its velocity decreases uniformly.
6. **Point D:** The ball reaches the highest point (maximum height in the second bounce) and the velocity at this point is zero. The ball changes direction at this point.
7. **Point D - E**: the velocity of the ball increases uniformly during the downwards motion.
8. **Point E:** The ball reaches the ground (floor) at a maximum velocity for the downward motion, this velocity is equal in magnitude but opposite in direction to the velocity at which the ball rebounds (in the second bounce), because the motion is **SYMMETRICAL** motion.

SCENARIO 1: INELASTIC COLLISION: GRAPHS OF MOTION

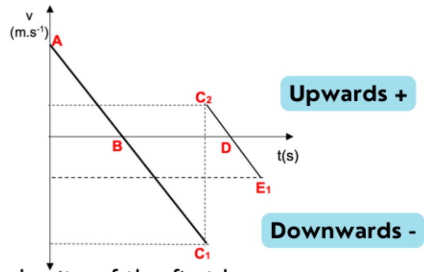
1. Velocity–time graph

Note: The two lines are parallel as the gradients are the same, which represents the constant gravitational acceleration.

Let downwards be positive	Let upwards be positive
The corresponding velocity – time graph represents the motion of the ball from the first bounce(time between bounces ignored).	The corresponding velocity – time graph represents the motion of the ball from the first bounce(time between bounces ignored)



A: Initial velocity of the first bounce.
B: Velocity at the highest point (0 m.s^{1}) in the first bounce.
C_1 : Velocity at which the ball strikes the ground during the first bounce (same speed as at A).
C_2: Initial velocity of the second bounce.
D: Velocity at the highest point (0 m.s^{1}) in the second bounce.
E_1: Velocity at which the ball strikes the ground during the second bounce.

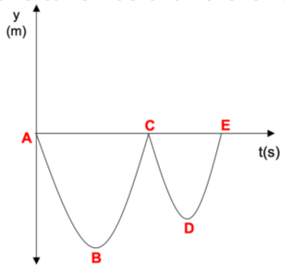


A: Initial velocity of the first bounce.
B: Velocity at the highest point (0 m.s^{-1}) in the first bounce.
C_1: Velocity at which the ball strikes the ground during the first bounce (same speed as at A).
C_2: Initial velocity of the second bounce.
D: Velocity at the highest point (0 m.s^{-1}) in the second bounce.
E_1: Velocity at which the ball strikes the ground during the second bounce.

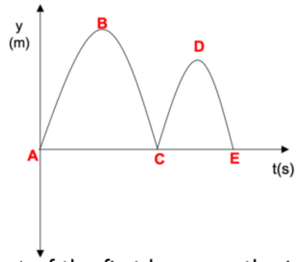
2. Position–time graph

Note: The time that the ball is airborne decreases with each bounce.

Let downwards be positive	Let upwards be positive
The corresponding position – time graph represents the motion of the ball from the first bounce (time between bounces ignored). The ground is taken as the reference point.	The corresponding position – time graph represents the motion of the ball from the first bounce (time between bounces ignored). The ground is taken as the reference point.



A: At the start of the first bounce, the initial position is 0 m.
B: Maximum height reached during the first bounce.
C: Ball strikes the ground during the first bounce, and rebounds during the second bounce.
D: Maximum height reached during the second bounce. This is less than the maximum height reached during the first bounce.
E: Ball strikes the ground.

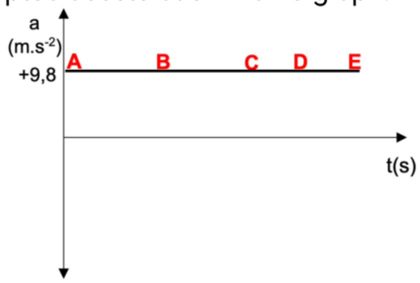


A: At the start of the first bounce, the initial position is 0 m.
B: Maximum height reached during the first bounce.
C: Ball strikes the ground during the first bounce, and rebounds during the second bounce.
D: Maximum height reached during the second bounce. This is less than the maximum height reached during the first bounce.
E: Ball strikes the ground during the second bounce.

3. Acceleration–time graph

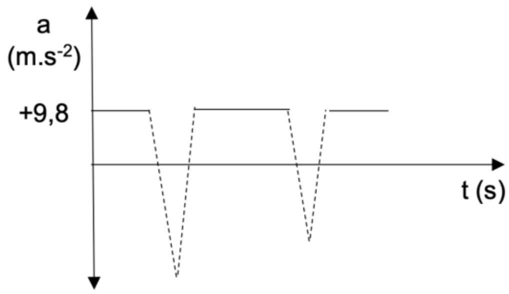
Let downwards be positive

The corresponding acceleration – time graph represents the acceleration of the ball from the first bounce (time between bounces ignored).
Accepted acceleration – time graph:



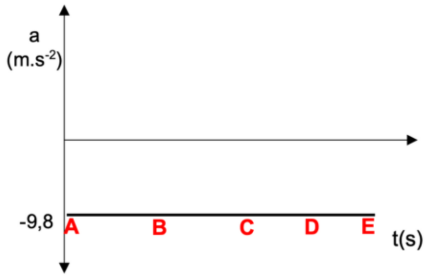
A – E: The acceleration of the ball is constant for the entire motion.

Realistic acceleration – time graph:
In reality, the ball also experiences an upwards acceleration during its contact with the ground. This is as a result of the force that the ground exerts on the ball, and only takes place during the time of contact. It is this upwards force which causes the ball to rebound. A more correct acceleration – time graph that takes upward acceleration into account:



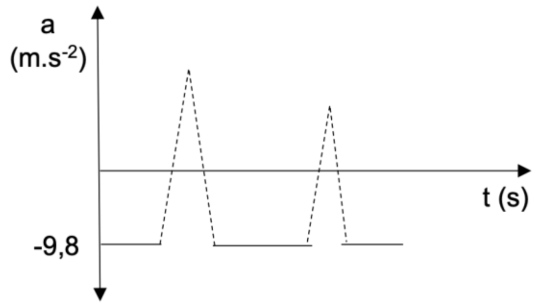
Let upwards be positive

The corresponding acceleration– time graph represents the acceleration of the ball from the first bounce (time between bounces ignored).
Accepted acceleration – time graph:



A – E: The acceleration of the ball is constant for the entire motion.

Realistic acceleration – time graph:
In reality, the ball also experiences an upwards acceleration during its contact with the ground. This is as a result of the force that the ground exerts on the ball, and only takes place during the time of contact. It is this upwards force which causes the ball to rebound. A more correct acceleration – time graph that takes upward acceleration into account:



A LESS COMMON SCENARIO IS REPRESENTED BELOW:
SCENARIO 2: ELASTIC COLLISION

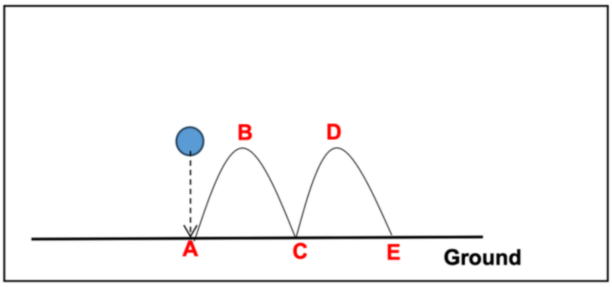


Figure 5

Figure 5 represents an elastic collision. When a ball bounces off the ground (floor) the maximum height reached by the ball is the same as from where it was dropped, and with each bounce, the maximum height reached remains constant.

- **Point A – B:** The ball strikes the ground at point A and rebounds upwards (due to the upwards force of the ground on the ball) at a velocity that is equal to the velocity at which the ball strikes the ground (vi bounce = vstrikes ground). As the ball moves towards point B, its velocity decreases uniformly.
- **Point B:** The velocity at the highest point (maximum height in the first bounce) is zero. The ball changes direction at this point.
- **Point B - C:** the velocity of the ball increases uniformly during the downwards motion.
- **Point C:** The ball reaches the ground (floor) at a maximum velocity for the downward motion, this velocity is equal in magnitude but opposite in direction to the velocity at which the ball rebounds, because the motion is symmetrical motion.
- **Point C - D:** The ball strikes the ground at point C and rebounds upwards at a velocity that is equal the velocity at which the ball strikes the ground (vi bounce = vstrikes ground). The rebound velocity in the second bounce is equal to the rebound velocity in the first bounce. As the ball moves towards point D, its velocity decreases uniformly.
- **Point D:** The ball reaches the highest point (maximum height in the second bounce) and the velocity at this point is zero. The ball changes direction at this point.
- **Point D - E:** the velocity of the ball increases uniformly during the downwards motion.
- **Point E**: The ball reaches the ground (floor) at a maximum velocity for the downward motion, this velocity is equal in magnitude but opposite in direction to the velocity at which the ball rebounds (in the second bounce), because the motion is symmetrical motion.

NOTE:
- The fact that the ball reaches the same height in the second bounce as that of the first bounce and from where it was dropped/thrown, indicates that the mechanical energy was conserved in this scenario and it is therefore an elastic collision.

Worked examples

Multiple choice questions

C + A

1.1 A ball is dropped and strikes the ground with a speed 2**x**. The ball rebounds with a speed **x** and reaches the highest point in **y** seconds.
Which **ONE** of the following is **CORRECT**? Take downwards as positive.

	Type of collision	Maximum height during the bounce in terms of **x** and **y**
A	Elastic	$\Delta y = -2xy + 4{,}9y^2$
B	Inelastic	$\Delta y = -2xy + 4{,}9y^2$
C	Elastic	$\Delta y = -xy + 4{,}9y^2$
D	Inelastic	$\Delta y = -xy + 4{,}9y^2$

Answer: D

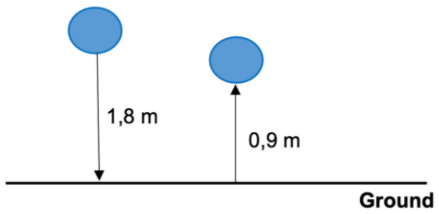
The type of collision is an inelastic collision because the velocity with which the ball rebounds is less than the velocity with which the ball strikes the ground (**x** < 2**x**).
To determine the maximum height reached during the bounce, the rebound velocity, must be used, since downwards is taken as positive this is **-x**. From the equation of motion:

$$\Delta y = v_i\Delta t + \tfrac{1}{2}a\Delta t^2$$
$$\Delta y = -xy + \tfrac{1}{2}(9{,}8)y^2$$
$$\Delta y = -xy + 4{,}9y^2$$

Worked example

C + A

A ball bounces to a vertical height of 0,9 m when it is released from a height of 1,8 m above the ground. The ball rebounds immediately after it strikes the ground. The effects of air resistance are negligible.



2.1 Calculate how long it takes for the ball to hit the ground after it has been dropped. (3)
2.2 Determine the speed at which the ball rebounds from the ground. (4)
2.3 Calculate how long it takes the ball to reach its maximum height during its bounce? (3)
2.4 Sketch a position – time graph for the motion of the ball from the time that it is dropped to the time when it rebounds to 0,9 m.

Clearly show the following on the graph. Take the ground as the zero reference.
- The time when the ball hits the ground.
- The time when it reaches 0,9 m.
- Relative position values. (4)

2.1 DOWNWARDS AS POSITIVE

Let downwards be positive

$$\Delta y = v_i\Delta t + \frac{1}{2}a\Delta t^2$$
$$(1,8) = (0)\Delta t + \frac{1}{2}(9,8)\Delta t^2$$
$$\Delta t = 0,61 \text{ s}$$

Data (Points chosen: Start to striking the ground)

vi = 0 m.s^{-1} (The ball was released, therefore the initial velocity is 0 m.s^{-1})

y = +1,8 m (Tip: Draw an arrow from the start to the end point in motion, notice it points downwards?)

a = +9,8 m.s^{-2} (Gravitational acceleration is 9,8 m.s^{-2} downwards, and downwards was taken as positive)

Δt =?(This is the initial velocity of the ball, when it leaves the throwers hand)

OR

UPWARDS AS POSITIVE

Let upwards be positive

$$\Delta y = v_i\Delta t + \frac{1}{2}a\Delta t^2$$
$$(-1,8) = (0)\Delta t + \frac{1}{2}(-9,8)\Delta t^2$$
$$\Delta t = 0,61 \text{ s}$$

Data (Points chosen: Start to striking the ground)

vi = 0 m.s^{-1}(The ball was released, therefore the initial velocity is 0 m.s^{-1})

y = -1,8 m (Tip: Draw an arrow from the start to the end point in motion, notice it points downwards?)

a = -9,8 m.s^{-2} (Gravitational acceleration is 9,8 m.s^{-2} downwards, but upwards was taken as positive)

Δt =?(This is the initial velocity of the ball, when it leaves the throwers hand)

2.2 DOWNWARDS AS POSITIVE

$v_f^2 = v_i^2 + 2a\Delta y$ —— 1.

$(0)^2 = v_i^2 + 2(+9,8)(-0,9)$ —— 2.

$v_i = 4,20$ m.s^{-1} or $v_i = -4,20$ m.s^{-1}

N/A

Speed at which ball rebounds = 4,20 m.s^{-1} —— 3.

Data (Points chosen: initial point ball bounces to highest point in bounce)

vf = 0 m.s^{-1}(The velocity at the highest point in the bounce is 0 m.s^{-1}).

Δy = -0,9 m (This is the displacement, from the start of the bounce to the highest point in the bounce).

a = +9,8 m.s^{-2}(Gravitational acceleration is 9,8 m.s^{-2} downwards, and downwards was taken as positive)

vi =? (This is the initial velocity of the bounce)

OR

UPWARDS AS POSITIVE

$v_f^2 = v_i^2 + 2a\Delta y$ —— 1.

$(0)^2 = v_i^2 + 2(-9,8)(+0,9)$ —— 2.

$v_i = 4,20$ m.s^{-1} or $v_i = -4,20$ m.s^{-1}

N/A

Speed at which ball rebounds = 4,20 m.s^{-1} —— 3.

Data (Points chosen: initial point ball bounces to highest point in bounce)

vf = 0 m.s^{-1}(The velocity at the highest point in the bounce is 0 m.s^{-1}).

Δy = +0,9 m (This is the displacement, from the start of the bounce to the highest point in the bounce).

a = -9,8 m.s^{-2}(Gravitational acceleration is 9,8 m.s^{-2}downwards, but upwards was taken as positive)

vi =? (This is the initial velocity of the bounce)

MARKING NOTES (QUESTION 2.2):

1. 2. A mark is awarded for the correct **formula** and correct **substitution**. Use brackets when substituting.

3. Since the equation is quadratic to calculate vf, there are **TWO** possible answers. However, since speed is a scalar quantity, only the magnitude is accepted.

NOTE: In question 2.2 on page 65, only the information regarding the **BOUNCE** of the ball is used in this question, as the initial velocity of the bounce is being determined.

2.3 DOWNWARDS AS POSITIVE

Let downwards be positive

$v_f = v_i + a\Delta t$ — 1.

$(0) = (-4{,}2) + (+9{,}8)\Delta y$ — 2.

$\Delta t = 0{,}43$ s — 3.

OR

UPWARDS AS POSITIVE

Let upwards be positive

$v_f = v_i + a\Delta t$ — 1.

$(0) = (+4{,}2) + (-9{,}8)\Delta y$ — 2.

$\Delta t = 0{,}43$ s — 3.

Data (Points chosen: initial point ball bounces to highest point in bounce)	**Data** (Points chosen: initial point ball bounces to highest point in bounce)
vi = -4,2 m.s^{-1} (Calculated in question 2.2; ball rebounds upwards but downwards was taken as the positive direction).	**vi = +4,2 m.s^{-1}** (Calculated in question 2.2; ball rebounds upwards and upwards was taken as the positive direction).
vf = 0 m.s^{-1} (At the highest point in the motion, the velocity is 0 m.s^{-1})	**vf = 0 m.s^{-1}** (At the highest point in the motion, the velocity is 0 m.s^{-1})
a = +9,8 m.s^{-2} (Gravitational acceleration is 9,8 m.s^{-2} downwards, and downwards was taken as positive)	**a = -9,8 m.s^{-2}** (Gravitational acceleration is 9,8 m.s^{-2} downwards, but upwards was taken as positive)
Δt =? (This is the time taken from the start of the bounce to the highest point)	**Δt =?** (This is the time taken from the start of the bounce to the highest point)

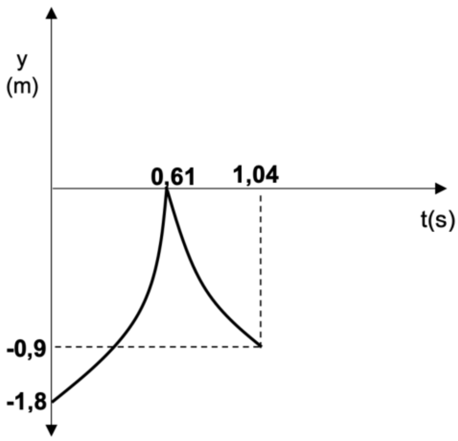
MARKING NOTES:

1. 2. A mark is awarded for the correct **formula** and correct **substitution**. Use brackets when substituting.

3. **Time** is a scalar quantity and must **ALWAYS** be **positive**. If you calculate a negative time value, go back to the question and check that your chosen sign convention was properly applied.

2.4 DOWNWARDS AS POSITIVE

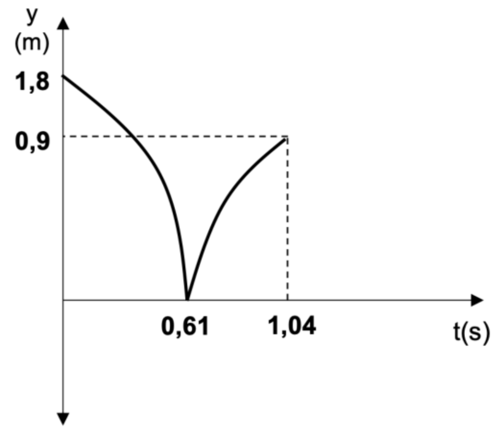
Let downwards be positive



OR

UPWARDS AS POSITIVE

Let upwards be positive



NOTE:
- The 1,04 s is calculated by adding the time t(s) taken for the ball to reach the ground (0,61 s) to the time taken for the ball to reach the highest point in the bounce (0,43 s).
- Total time = 0,61 + 0,43
  Total time = 1,04 s

REMINDER :QUESTION DIFFICULTY

C — COMPREHENSION AND RECALL QUESTIONS

These are common questions which include definitions and calculation questions that look similar to the questions covered in class. Approximately **50%** of Paper 1 (Physics) will include questions on this level.

A — ANALYSIS AND APPLICATION QUESTIONS

These are more complex questions which involves applying the knowledge and skills learned in this chapter. Approximately **40%** of Paper 1 (Physics) will include questions on this level.

P — PROBLEM- SOLVING QUESTIONS

These are questions that require critical thinking and being able to make connections between different representations of information and integrating different topics. They are not familiar questions, but are able to be solved through critical analysis. Approximately **10%** of Paper 1 (Physics) will include questions on this level.

THE DOPPLER EFFECT

(WAVES, SOUND AND LIGHT)

The **Doppler effect** is a phenomenon that was named after **Christian Doppler**, an Austrian Mathematician and Physicist. He observed that when a sound source moves relative to an observer i.e., at a different velocity to the observer, a change in the **pitch** of the sound is observed, this change in the pitch (or frequency) of the sound observed by the listener (or observer) is known as the **Doppler effect.**

Did you know? The Doppler effect is a common event that takes place. Recall a time when an emergency vehicle or police car with a blearing siren was moving towards you.

You may have noticed that as the moving vehicle was moving towards you, sound of a high pitch is observed (**whee whee!**) and as the moving vehicle is moving away from you, sound of a lower pitch (**wheeeeeeee wheeeeeeeeee**) is observed.

This is the Doppler effect of sound. It is **not** the volume (or loudness) of the sound that is changing, but rather the pitch, i.e., how high or low the sound is.

Definitions

Doppler effect: Change in the frequency (or pitch) of the sound detected by a listener because the sound source and the listener have different velocities relative to the medium of sound propagation.

Revision of sound waves

Sound is caused by **vibrations**. This results in variations in the pressure of the **medium**. Sound requires a medium (material) such as a **solid**, **liquid** or **gas** to pass through.

Areas of high pressure where the wave particles are closer together are called **compressions**.
Areas of low pressure where the particles are further apart are called **rarefactions**.
Sound is an example of a **longitudinal wave.**

The Doppler effect can be observed in **all types of waves,** therefore it can be observed in both **sound and light waves.**

NOTE: A longitudinal wave is a wave in which the particles in the medium move **parallel** to the direction of motion of the wave.

Figure 1 below: Longitudinal wave represented in a slinky. A Longitudinal wave is generated by moving the slinky back and forth (or to and fro).

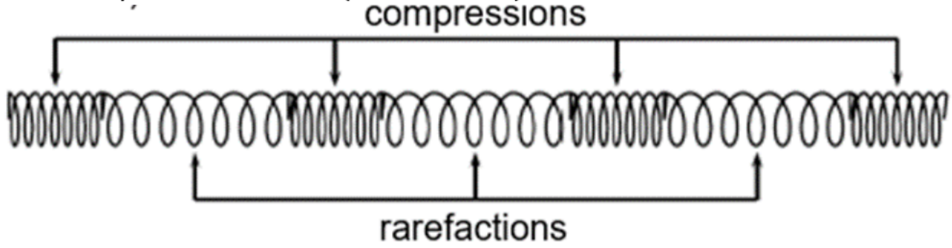
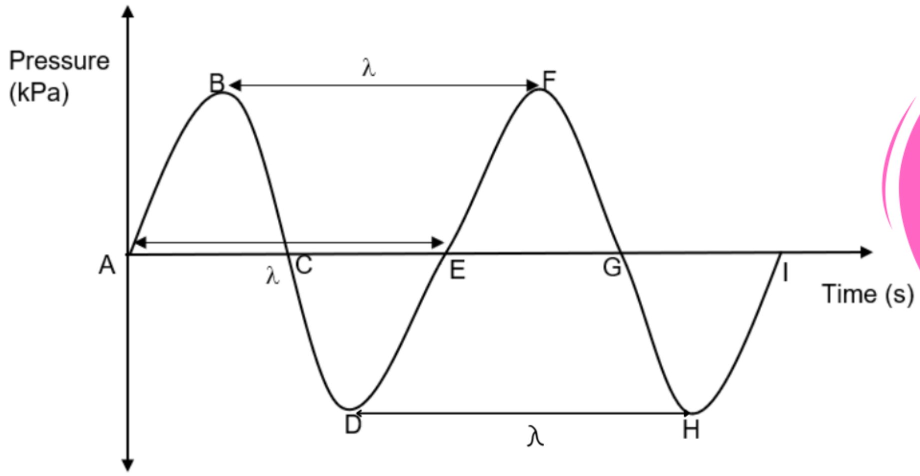


Figure 2 below: A longitudinal wave can be represented in a pressure - time graph.



PRO-TIPS

In a pressure - time graph, graph interpretation skills can be tested, such as:

- **Drawing** information off the graph.
- Doing **calculations** based on the information in the graph.

Wave terminology and graph interpretation

- A pressure - time graph represents how the pressure in the medium varies with time.
- Points **A, C, E, G** and **I** represent the rest or equilibrium position, where there is no disturbance in the medium.
- Points **B** and **F** represent compressions and points **D** and **H** represent rarefactions.
- One wavelength (λ) is the distance between two consecutive points that are in phase i.e., two consecutive compressions, or two consecutive rarefactions or two consecutive rest positions.
- **Period** of the wave (T): Time taken to complete one full wave or vibration. From the graph, this is the time from point **A** to point **E**.
- **Frequency** of the wave (f): Number of wave cycles or vibrations **per second.**

$$f = \frac{1}{T}$$

Speed of sound waves

The speed of sound depends on the following factors:

1. The **type of medium and the density of the medium**. Sound travels fastest in a solid and slowest in a gas, because in a gas the particles are further apart, therefore there will be fewer vibrations per second.
2. **Temperature.** Temperature is a measure of the average kinetic energy of the particles. At higher temperatures, sound travels faster as this results in more vibrations per second.
3. **Altitude.** Altitude affects air pressure. The higher the altitude, the lower the air pressure, the slower the speed of sound.

Calculating wave speed (speed of sound)

Wave speed can be calculated using the **Universal wave equation:**

$$v = f\lambda$$

v = wave speed in $m.s^{-1}$
f = frequency in hertz (Hz)
λ= wavelength in metres (m)

PRO- TIP

The speed of sound (wave speed) **only changes** if:

- The type (and density of the medium) changes.
- Temperature changes.
- Altitude (air pressure) changes

The speed of sound in air varies between 330 $m.s^{-1}$ to 340 $m.s^{-1}$ depending on temperature and altitude.

Note:
Frequency is inversely proportional to wavelength provided wave speed remains constant.

Pitch of sound

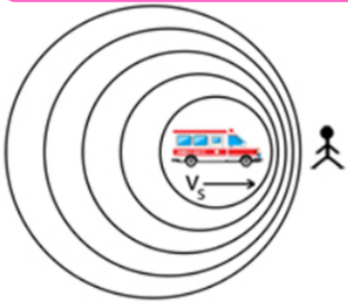
When the Doppler effect occurs, a change in the pitch (and frequency) of the sound is observed.

What is the pitch of sound?
Pitch refers to **how high or how low** the sound is.
Pitch is **NOT** the volume (or loudness) of the sound.
Imagine a choir of singers. You get a variety of different pitches: Bases and altos have a lower pitch, whereas sopranos and descants (which can reach the higher notes) have a higher pitch.

- The **pitch of sound depends on the frequency of the sound waves,** therefore a change in the pitch of the sound also results in a change in the frequency (and wavelength) of the sound waves (assuming wave speed remains constant).
- The **higher the pitch, the higher the frequency** of the sound waves, the shorter the wavelength.
- The **lower the pitch, the lower the frequency** of the sound waves, the longer the wavelength.

COMMON SCENARIOS IN THE DOPPLER EFFECT

Scenario 1: Sound source moves towards a stationary listener.

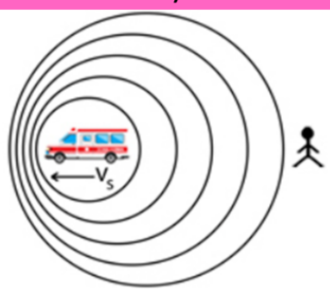


Note for all scenarios:

The sound waves move in ALL directions at the same speed, even if they are closer together or further apart because the type of medium, temperature and altitude remains constant

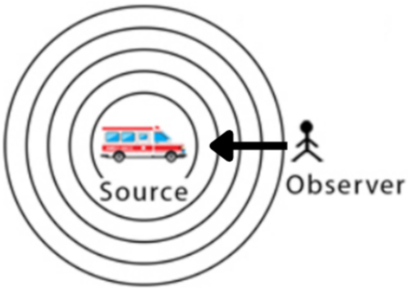
- When the sound source (e.g. a vehicle with a siren emitting sound) moves **TOWARDS** a stationary listener ($v_L = 0$ m.s^{-1}), the sound waves **IN FRONT** of the sound source are **CLOSER** together and have a shorter wavelength. This is because the sound source is moving towards previously emitted sound waves and "catching up" with the previously emitted sound waves.
- From the universal wave equation ($v = f\lambda$) if the wave speed remains constant, and the wavelength **DECREASES**, the frequency of the sound detected by the listener will **INCREASE** (since frequency is inversely proportional to wavelength).
- Therefore, sound of a **HIGHER** frequency and **HIGHER** pitch will be observed by the listener (i.e., the observed frequency of the listener will be greater than the frequency of the sound source ($f_L > f_s$)

Scenario 2: Sound source moves away from a stationary listener.



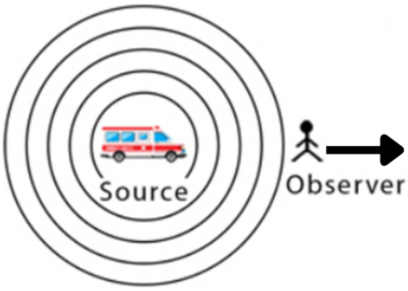
- When the sound source (e.g. a vehicle with a siren emitting sound) moves **AWAY** a stationary listener ($v_L = 0$ m.s^{-1}), the sound waves **BEHIND** the sound source are **FURTHER** together and have a longer wavelength.
  This is because the sound source is moving away previously emitted sound waves while emitting new sound waves.
- From the universal wave equation $v = f\lambda$) if the wave speed remains constant, and the wavelength **INCREASES**, the frequency of the sound detected by the listener will **DECREASE** (since frequency is inversely proportional to wavelength).
- Therefore, sound of a **LOWER** frequency and **LOWER** pitch will be observed by the listener (i.e., the observed frequency of the listener will be less than the frequency of the sound source ($f_L < f_s$)

Scenario 3: Listener moves towards a stationary sound source



- When the listener moves **TOWARDS** the stationary sound source (v_s = 0 m.s^{-1}), the listener (observer) observes **more** vibrations (sound waves) **per second.**
- Therefore, sound of a **HIGHER** frequency and **HIGHER** pitch will be observed by the listener (i.e., the observed frequency of the listener will be greater than the frequency of the sound source ($f_L > f_s$)

Scenario 4: Listener moves away from a stationary sound source



- When the listener moves **AWAY FROM** the stationary sound source (v_s = 0 m.s^{-1}), the listener (observer) observes **fewer** vibrations (sound waves) **per second.**
- Therefore, sound of a **LOWER** frequency and **LOWER** pitch will be observed by the listener (i.e., the observed frequency of the listener will be less than the frequency of the sound source ($f_L < f_s$)

PRO-TIPS

The Doppler effect (i.e., a change in the observed frequency) will **not** be observed in the following scenarios:

1. The sound source and the listener have the **SAME** velocity (i.e., they are both at rest or they move at the same speed in the same direction). In this scenario there is no relative motion between the sound source and the observer.
2. The sound source passes the listener. The sound waves observed are neither closer together or further apart as the sound source passes the listener, as the listener is neither in in front of, nor behind the sound source.
3. The driver of the sound source will **NOT** experience the Doppler effect, as the driver is moving at the same velocity as the sound source.

DOPPLER EFFECT CALCULATIONS: THE DOPPLER EFFECT EQUATION

$$f_L = \frac{v \pm v_L}{v \pm v_s} f_s$$

What do these variables mean and what are the SI units?

v = speed of sound in air in $m.s^{-1}$
v_L = speed of listener in $m.s^{-1}$
f_L = observed frequency of listener in Hz.
v_s = speed of source in $m.s^{-1}$
f_s = frequency of the sound source (true frequency)

PRO- TIPS

- Speed is a **scalar quantity** with magnitude (size) only.
- The speed of sound in air usually varies between **330 $m.s^{-1}$ to 343 $m.s^{-1}$.**
- This is either given or calculated, **never assumed!**
- Always write down the **full Doppler effect equation** before simplifying it for a calculation.

If $\frac{v \pm v_L}{v \pm v_s}$ **>1, then fL > fs**

If $\frac{v \pm v_L}{v \pm v_s}$ **<1, then fL < fs**

If $\frac{v \pm v_L}{v \pm v_s}$ **= 1, then fL = fs**

$\boxed{\frac{v \pm v_L}{v \pm v_s}}$ →

Note:

This **fraction** in the Doppler effect equation represents the **factor** by which the observed frequency of the listener (f_L) differs from the **true frequency of the sound source** (f_s), this is equal to $\frac{f_L}{f_s}$

When is + or - used in the Doppler effect equation?

From scenario 1: Sound source moves towards a stationary listener.

- v_L = 0 $m.s^{-1}$ since the listener is at rest, and v_s> 0 since the sound source is moving.
- The sound source is moving towards the stationary listener, therefore $f_L > f_s$ and the factor $\frac{v \pm v_L}{v \pm v_s}$ is **greater than 1.**
- Sign in the denominator: Mathematically, for the factor to be greater than 1, the **denominator** must be **smaller than the numerator**, therefore a MINUS sign is used in the denominator i.e., $v - v_s$.
  The Doppler effect equation is simplified to: $f_L = \frac{v}{v - v_s} f_s$

From scenario 2: Sound source moves away from a stationary listener.

- v_L = 0 $m.s^{-1}$ since the listener is at rest, and v_s> 0 since the sound source is moving.
- The sound source is moving away the stationary listener, therefore $f_L < f_s$ and the factor $\frac{v \pm v_L}{v \pm v_s}$ is **smaller than 1.**
- Sign in the denominator: Mathematically, for the factor to be smaller than 1, the **denominator** must be **bigger than the numerator**, therefore a PLUS sign is used in the denominator
  i.e., $v + v_s$.
  The Doppler effect equation is simplified to: $f_L = \frac{v}{v + v_s} f_s$

From scenario 3: Listener moves towards a stationary sound source

- $v_s = 0\ m.s^{-1}$ since the sound source is at rest, and $v_L > 0$ since the listener is moving.
- The listener is moving towards the stationary sound source, therefore $f_L > f_s$ and the factor $\frac{v \pm v_L}{v \pm v_s}$ is **greater than 1.**
- Sign in the numerator: Mathematically, for the factor to be greater than 1, the **numerator** must be **bigger than the denominator**, therefore a PLUS sign is used in the numerator i.e., $v + v_L$.
  The Doppler effect equation is simplified to: $f_L = \frac{v + v_L}{v} f_s$

From scenario 4: Listener moves away from a stationary sound source

- $v_s = 0\ m.s^{-1}$ since the sound source is at rest, and $v_L > 0$ since the listener is moving.
- The listener is moving away the stationary sound source, therefore $f_L < f_s$ and the factor $\frac{v \pm v_L}{v \pm v_s}$ is **smaller than 1.**
- Sign in the numerator: Mathematically, for the factor to be smaller than 1, the **numerator** must be **smaller than the denominator**, therefore a MINUS sign is used in the numerator i.e., $v - v_L$.
  The Doppler effect equation is simplified to: $f_L = \frac{v - v_L}{v} f_s$

Summary:

$$\boxed{f_L = \frac{v \pm v_L}{v \pm v_s} f_s}$$

Numerator:

+ listener moves towards source

- Listener moves away from source

Denominator:

+ Source moves away from listener

- Source moves towards listener

OTHER USEFUL FORMULAE IN WAVES, SOUND AND LIGHT & DOPPLER EFFECT

$$v = f\lambda$$

$$f = \frac{1}{T} \quad \textbf{OR} \quad T = \frac{1}{f}$$

Worked examples

Multiple choice questions

C + A

1. Which ONE of the following statements is INCORRECT regarding the Doppler effect?

A It occurs as the sound source moves towards the listener.
B It occurs as the listener moves towards the sound source.
C It occurs as the sound source moves away from the listener.
D It occurs when the listener moves with the sound source.

Answer: D
The Doppler effect i.e., the change in the observed frequency and pitch of the sound is only observed by a listener if the listener and the sound source have **DIFFERENT** velocities, either by moving at different speeds or in different directions. Therefore, if the listener moves with the sound source (at the same velocity), the Doppler effect will **NOT** be observed by the listener.

2. A sound source approaches a stationary observer at a constant velocity. Which ONE of the following best describes the **observed** frequency and wavelength in comparison to the sound of the source?

	Frequency	**Wavelength**
A	Greater than	Greater than
B	Smaller than	Smaller than
C	Greater than	Smaller than
D	Smaller than	Greater than

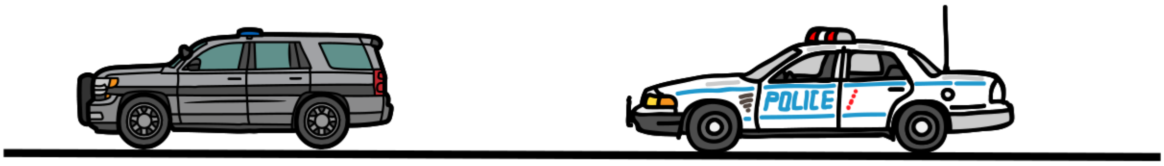
Answer: C
NOTE: Even though the sound source is moving towards the listener and sound of a higher pitch is observed, the speed of the sound waves remains constant because temperature, altitude and the density of the medium remains constant.

Due to sound source approaching the stationary listener, therefore sound of a **higher frequency and pitch will be observed by the listener.** From the formula, $v = f\lambda$ when wave speed remains constant, $\lambda \propto \frac{1}{f}$ (or wavelength is inversely proportional to the frequency), therefore the waves in front of the sound source will have a **SHORTER (SMALLER)** wavelength, compared to the sound waves of the sound source.

Worked example

C + A

1. A policeman chasing a speeder along a straight section of the highway. Both the speeder and the policeman are moving at 144 km·h^{-1}. The siren on the police car has a frequency of 2 x 10^{3} Hz. Take the speed of sound in air to be 340 m.s^{-1}



1.1 Determine the frequency of the sound waves detected by:

1.1.1 the policeman (1)

1.1.2 the speeder. Explain the answer. (2)

1.2 Calculate the frequency and wavelength of the sound waves detected by an observer standing at the side of the road as the police car approaches the observer. (5)

1.1.1 f = 2 x 10^{3}Hz.

Note: The policeman is in sound source (the police car) and therefore moves at the same velocity as the sound source (police car). The observed frequency of the policeman will be the same as the true frequency of the sound source (the Doppler effect will not be observed by the policeman).

1.1.2 f = 2 x 10^{3} Hz.

The speeder (listener) is moving at the **same speed and in the same direction** as the police car (sound source). Therefore the speeder (listener) and the police car (sound source) are moving at the **same velocity.** From the Doppler effect equation, there will be no change in the observed frequency because there is no relative motion between the sound source and the observer.

Proof: A calculation using the Doppler effect equation can be done to prove the answer in **question 1.2:**

$$f_L = \frac{v \pm v_L}{v \pm v_s} f_s$$

$$f_L = \frac{v - v_L}{v - v_s} f_s$$

$$f_L = \left(\frac{340-40}{340-40}\right)(2 \times 10^3)$$

$$f_L = 2 \times 10^3 \text{ Hz}$$

The proof shown on the left is to prove the answer using a calculation. However, the original question asks to **explain the answer,** this means that you must use words in your explanation and a calculation will not be accepted, unless the questions asks to prove the answer using a calculation

1.2 Frequency of the sound waves detected by stationary listener

$f_L = \frac{v \pm v_L}{v \pm v_s} f_s$ ---------------------- 1

$f_L = \frac{v + v_L}{v - v_s} f_s$ ---------------------- 2

$f_L = \left(\frac{340+0}{340-40}\right)(2 \times 10^3)$ ---------------------- 3

$f_L = 2266{,}67$ Hz ---------------------- 4

Data

f_L = ?

v = 340 m.s^{-1}

v_L = 0 m.s^{-1}

v_s = 144 ÷ 3,6

v_s = 40 m.s^{-1}

f_s = 2 x 10^3 Hz

km.h^{-1} → m.s^{-1} ÷ 3,6

1.2 (continued) Wavelength of the sound waves observed by the listener

$v = f\lambda$ ---------------------- 1

$(340) = (2266{,}67)\lambda$ -------------------- 3

$\lambda = 0{,}15$ m --------------------- 4

MARKING NOTES:

1. 2. A mark is awarded for the correct formula (taken off the data sheet) and the correct substitution.

3. Use brackets when substituting.

4. Answer and units. Direction is only necessary if it is a **VECTOR QUANTITY** e.g., velocity

Data

v = 340 m.s^{-1}

(The speed of sound in air does not change as temperature, altitude and the density of the medium remains constant)

f = 2266,67 Hz

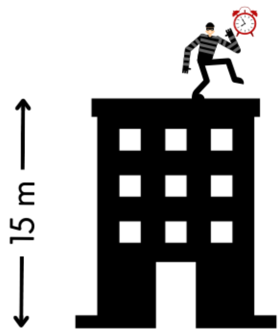
(This is the observed frequency of the listener, which is higher than the true frequency of the sound source)

λ = ?

Worked example

C + A

2. A robber steals an alarm clock from the roof of a building that is 15 m high. The alarm starts to ring at a frequency of 1500 Hz. The intruder realizes the police can hear the alarm clock and drops it at the side of the building. At which frequency will a policeman on the ground hear the alarm clock ring if it is 5 m above the ground? Take the speed of sound in air as 340 m.s^{-1}. Ignore the effects of air resistance. (7)



Note the following in this question:

- The alarm clock is free - falling when it is dropped from the top of the building.
- The alarm clock is moving **TOWARDS** the policeman, however it is not moving at a constant speed during its free - fall, therefore the observed frequency heard by the policeman will constantly change, namely increase as the alarm clock moves towards him.

How to approach this question:

- Calculate the speed of the alarm clock using **equations of motion** when it is 5 m above the ground. Remember if the building is 15 m high (above the ground), then the alarm clock was in free - fall for 10 m when it reaches a point that is 5 m above the ground.
- Using the **Doppler effect equation**, calculate the observed frequency heard by the policeman (listener) at this speed.

PRO-TIPS

Revise vertical projectile motion before starting this question!

LET DOWNWARDS BE POSITIVE

$v_f^2 = v_i^2 + 2a\Delta y$

$v_f^2 = (0)^2 + 2(9{,}8)(10)$

$v_f = 14$ m.s^{-1} or $\cancel{v_f = -14 \text{ m.s}^{-1}}$

Speed of alarm clock 5 m above the ground = 14 m.s^{-1}

Data (Points chosen: start to end)

v = 0 m.s^{-1}

(Drops implies that the alarm clock was initially at rest)

y = +10 m

(The alarm clock was only in free - fall for 10 m, when it is 5 m above the ground, dropped from a building of 15 m high. Since downwards is taken as positive, it has changed its position by 10 m downwards, therefore +10m)

a = +9,8 m.s^{-2}

(The alarm clock is in free - fall, since downwards is taken as positive and gravitational acceleration is 9,8 m.s downwards, therefore it is +9,8 m.s^{-2}

v = ?

OR

LET UPWARDS BE POSITIVE

$v_f^2 = v_i^2 + 2a\Delta y$

$v_f^2 = (0)^2 + 2(-9{,}8)(-10)$

$v_f = 14$ m.s^{-1} or $\cancel{v_f = -14 \text{ m.s}^{-1}}$

Speed of alarm clock 5 m above the ground = 14 m.s^{-1}

Data (Points chosen: start to end)

v_i = 0 m.s^{-1}

(Drops implies that the alarm clock was initially at rest)

Δy = -10 m

(The alarm clock was only in free - fall for 10 m, when it is 5 m above the ground, dropped from a building of 15 m high. Since downwards is taken as positive, it has changed its position by 10 m downwards, therefore -10m)

a = -9,8 m.s^{-2}

(The alarm clock is in free - fall, since downwards is taken as positive and gravitational acceleration is 9,8 m.s^{-2} downwards, therefore it is -9,8 m.s^{-2}

v_f = ?

Now, using the Doppler effect equation, the observed frequency of the listener can be calculated...

$$f_L = \frac{v \pm v_L}{v \pm v_s} f_s$$

$$f_L = \frac{v + 0}{v - v_s} f_s$$

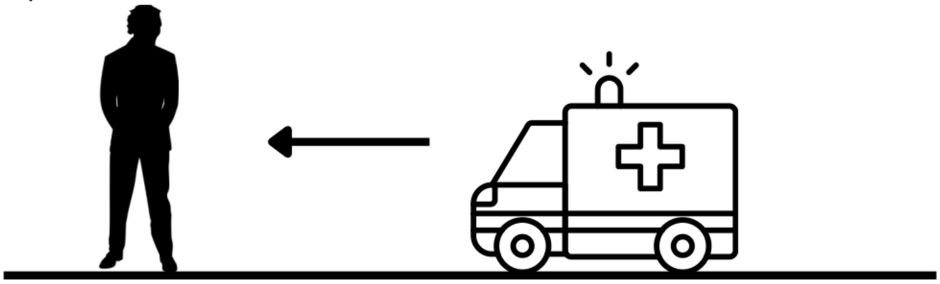
$$f_L = \left(\frac{340 + 0}{340 - 14}\right)(1500)$$

$$f_L = 1564{,}42 \text{ Hz}$$

Worked example

C + A + P

3. An emergency vehicle travelling down a road at constant speed emits sound waves from its siren. A man stands on the side of the road with a detector which registers sound waves at a frequency of 2000 Hz as the vehicle approaches him. After moving away from him at the same constant speed, sound waves of frequency 1300 Hz are registered. Assume that the speed of sound in air is 330 $m \cdot s^{-1}$.



3.1 State the Doppler Effect in words. (2)

3.2 Explain why the observed frequency of the siren decreases as it moves away from the man. (2)

3.3 Calculate the speed at which the emergency vehicle is moving. (7)

3.4 Calculate the frequency at which the siren emits the sound waves. (3)

3.1 The Doppler effect is the **change** in the **frequency** (or pitch) of the sound detected by a listener because the sound source and the listener have different **velocities** relative to the medium of sound propagation.

3.2 As the emergency vehicle moves away from the man, the <u>wave fronts (sound waves) behind the sound source are further apart, therefore fewer wave fronts are detected per second.</u>

3.3 **Note: This is a problem - solving question.**
No information about the speed or distance covered by the emergency vehicle is given, therefore equations of motion **cannot** be used. Another approach has to be taken: the **Doppler effect equation**! (Remember this equation has the **speed** of the **sound source** as one of its variables **AND** the **observed frequencies** as the emergency vehicle approaches and moves away from the listener is given).

PRO-TIPS

If the equation has **TWO unknowns**, set up an second equation and solve the equations using **simultaneous equations.**

Sound source moves <u>TOWARDS</u> the stationary listener

$$f_L = \frac{v \pm v_L}{v \pm v_s} f_s$$

$$f_L = \frac{v + 0}{v - v_s} f_s$$

$$(2000) = \left(\frac{330 + 0}{330 - v_s}\right) f_s$$

$(2000)(330 - v_s) = 330\ f_s$ ----------------- 1

Sound source moves <u>AWAY FROM</u> the stationary listener

$$f_L = \frac{v \pm v_L}{v \pm v_s} f_s$$

$$f_L = \frac{v + 0}{v + v_s} f_s$$

$$(1300) = \left(\frac{330 + 0}{330 + v_s}\right) f_s$$

$(1300)(330 + v_s) = 330\ f_s$ ----------------- 2

1 = 2: $(2000)(330 - v_s) = (1300)(330 + v_s)$

$660\,000 - 2000v_s = 429\,000 + 1300v_s$

$660\,000 - 429\,000 = 2000v_s + 1300v_s$

$231\,000 = 3300v_s$

$v_s = 70\ m.s^{-1}$

NOTE:

- This question is asking the **TRUE** frequency of the sound source (f_s) you can use any of the equations set up in question 3.3 to answer this question as you have the constant velocity of the sound source (as calculated in question 3.3)

<u>From equation 1:</u>

$(2000)(330 - v_s) = 330\ f_s$

$(2000)(330 - 70) = 330\ f_s$

$f_s = 1575{,}76$ Hz

OR

<u>From equation 2:</u>

$(1300)(330 + v_s) = 330\ f_s$

$(1300)(330 + 70) = 330\ f_s$

$f_s = 1575{,}76$ Hz

REMINDER :QUESTION DIFFICULTY

C — COMPREHENSION AND RECALL QUESTIONS

These are common questions which include definitions and calculation questions that look similar to the questions covered in class. Approximately **50%** of Paper 1 (Physics) will include questions on this level.

A — ANALYSIS AND APPLICATION QUESTIONS

These are more complex questions which involves applying the knowledge and skills learned in this chapter. Approximately **40%** of Paper 1 (Physics) will include questions on this level.

P — PROBLEM- SOLVING QUESTIONS

These are questions that require critical thinking and being able to make connections between different representations of information and integrating different topics. They are not familiar questions, but are able to be solved through critical analysis. Approximately **10%** of Paper 1 (Physics) will include questions on this level.

DOPPLER EFFECT & GRAPHS

An important skill that is tested in Doppler effect is **graph interpretation** and even in some cases, **graph sketching**.
The type of graphs that you could encounter include, but are not limited to:

- **Position - time graph**

Gradient of the position - time graph = velocity

- **Speed or velocity - time graph**

Gradient of the velocity - time graph = acceleration

- **Pressure - time graph**

This graph represents how the pressure in the medium varies with wavelength. It is in the shape of a sin or cos graph.

- **Pressure - distance graph**

This graph represents how the pressure in the medium varies with distance. It is in the shape of a sin or cos graph.

- **Frequency - time graph (NEW!)**

Scenario 1: The sound source is moving and the listener is stationary

This graph represents how the **OBSERVED** frequency of the listener changes as the **sound source moves** towards, passes and moves away from the stationary listener.

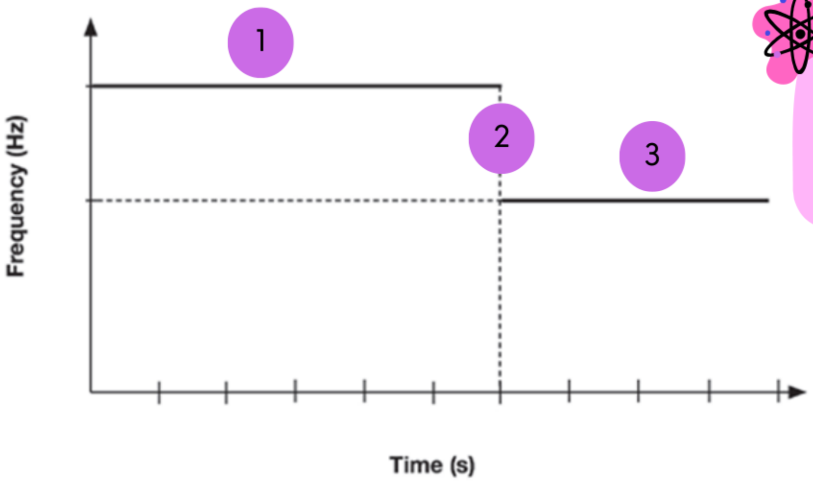
NOTE:
It is assumed that the sound source is moving at the constant velocity, otherwise the observed frequency will constantly change as the sound source moves towards the listener and away from the listener.

Scenario 2: The listener is moving and the sound source is stationary

This graph represents how the **OBSERVED** frequency of the listener changes as the **listener moves towards**, passes and moves away from the stationary sound source.

NOTE: It is assumed that the sound source is moving at the constant velocity, otherwise the observed frequency will constantly change as the sound source moves towards the listener and away from the listener.

A **frequency - time graph** is represented below.



NOTE: that the shape of the graph remains constant and parallel to the x - axis as long as the velocity of the sound source or the listener remains constant.

1 The sound source is moving at a constant velocity TOWARDS the listener
OR
The listener is moving at a constant velocity TOWARDS the sound source.
The initial observed frequency is GREATER than the true frequency of the sound source

2 The sound source moves past the listener, this time interval is not factored in, therefore a dotted vertical line is used to indicate that the time taken to pass the listener is not factored in.
OR
The listener moves past the sound source, this time interval is not factored in, therefore a dotted line is used to indicate that the time taken to pass the sound source is not factored in.

NOTE: Although the time period for which the source moves past the listener is ignored, in reality, the observed frequency will be EQUAL to the frequency of the sound source during this instant. However, this instant is usually ignored, therefore the time period over which this instant takes place, is not factored in.

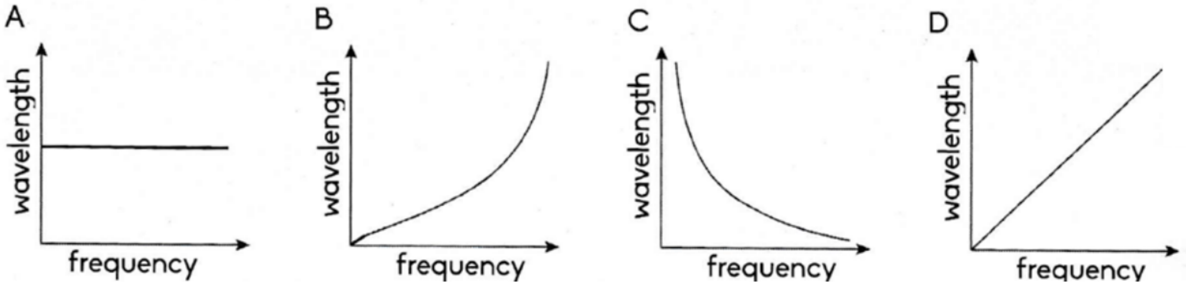
3 The sound source is moving at a constant velocity AWAY FROM the listener
OR
The listener is moving at a constant velocity AWAY FROM the sound source.
The observed frequency is LESS THAN than the true frequency of the sound source.

Worked examples

Multiple choice question

C + A

1. Which ONE of the following graphs represents the relationship between wavelength and the frequency of a sound?



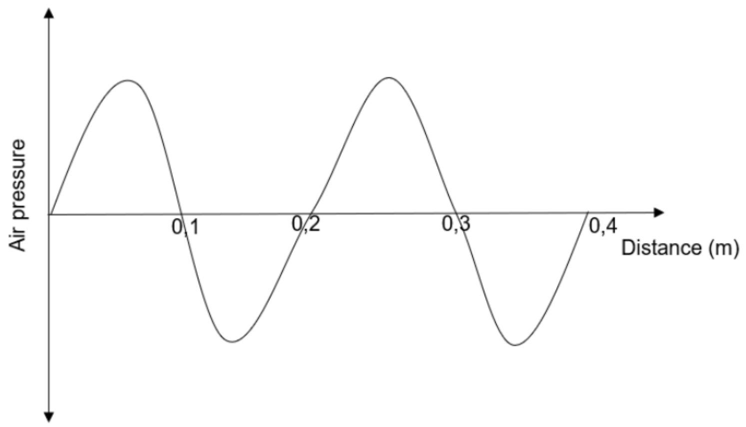
Answer: C

From the formula v = f λ the relationship between f and λ is an inversely proportional relationship, provided the speed of sound remains constant. An inversely proportional relationship is represented as a hyperbola.

Worked example

C + A

1. An owner runs directly towards its stationary dog at constant velocity. The dog constantly emits sound waves at a frequency of 1650 Hz.
The owner hears a change in pitch as the owner moves towards the dog.
The air pressure versus distance graph below represents the waves detected by the owner as the owner moves towards its dog. The speed of sound in air is 340 $m.s^{-1}$.



Using the information on the graph and the information above, calculate the:

1.1 frequency of the sound waves detected by the owner. (3)

1.2 magnitude of the velocity at which the owner moves. (5)

1.3 The speed at which the owner runs increases. How will this affect the frequency of the sound waves observed by the owner? Write down only INCREASES, DECREASES or REMAINS THE SAME. (1)

1.1 **NOTE:** The pressure- distance graph represents distance covered by the wave as the pressure in the medium (air) varies. It is evident that the wavelength of the sound waves are constant at 0,2 m with each oscillation (vibration). The speed of sound in air is constant.

$$v = f\lambda$$
$$(340) = f(0{,}2)$$
$$f = 1700 \text{ Hz}$$

Note that the observed frequency of the owner (1700 Hz) is **GREATER** than the true frequency of the dog's bark (1650 Hz)

1.2 The Doppler effect equation can be used to determine the constant speed at which the owner runs.

$$f_L = \frac{v \pm v_L}{v \pm v_s} f_s$$

$$f_L = \frac{v + v_L}{v + 0} f_s$$

$$(1700) = \left(\frac{340 + v_L}{340}\right)(1650)$$
$$v_L = 10{,}30 \text{ m.s}^{-1}$$

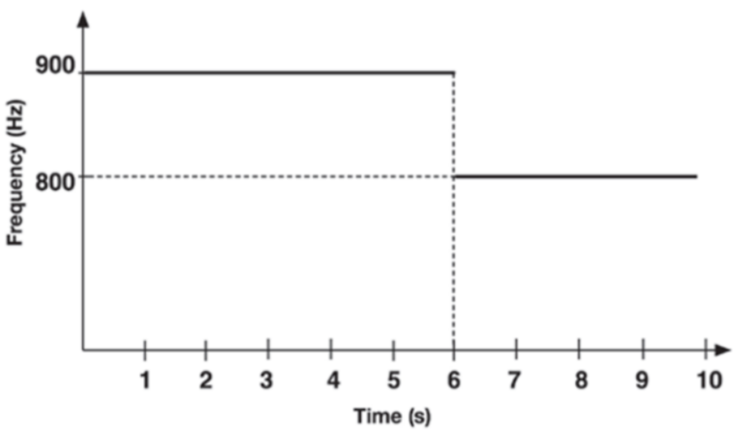
1.3 INCREASES.

The greater the speed at which the owner approaches the barking dog the greater the numerator in the Doppler effect equation, therefore increasing the factor by which the true frequency and the observed frequency differ.

Worked example

A + P

2. The siren of a stationary (not moving) ambulance emits sound waves at a frequency of 850 Hz. An observer (person witnessing this) who is travelling in a car at a constant speed in a straight line, begins measuring the frequency of the sound waves emitted by the siren when he is at a distance **x** from the ambulance. The observer continues measuring the frequency as he approaches, passes, and moves away from the ambulance. The results obtained are shown in the graph below.



2.1 The observed frequency suddenly changes at t = 6 s. Give a reason for this sudden change in frequency. (1)

2.2 Calculate:

2.2.1 The speed of the car (Take the speed of sound in air as 340 $m\cdot s^{-1}$). (4)

2.2.2 Distance **x** between the car and the ambulance when the observer BEGINS measuring the frequency. (4)

2.3 Write down the frequency of the sound waves heard by the ambulance driver. (1)

2.1 The observer **passes the stationary sound source at t = 6 s,** therefore there is a sudden change in frequency.

2.2.1

$$f_L = \frac{v \pm v_L}{v \pm v_s} f_s$$

$$f_L = \frac{v + v_L}{v + 0} f_s$$

$$(900) = \left(\frac{340 + v_L}{340}\right)(850)$$

$v_L = 20\ m.s^{-1}$

OR

$$f_L = \frac{v \pm v_L}{v \pm v_s} f_s$$

$$f_L = \frac{v - v_L}{v + 0} f_s$$

$$(800) = \left(\frac{340 - v_L}{340}\right)(850)$$

$v_L = 20\ m.s^{-1}$

2.2.2 **NOTE:** The observer begins measuring the frequency at t = 0s. It takes the observer 6s to **just** pass the stationary ambulance, and in question 2.2.1 the constant velocity of the observer as it approaches the sound source was calculated. An equation of motion is needed to do this question (as it is motion in a straight line and at a constant acceleration), or even the average velocity formula!

Let towards the sound source be positive

$\Delta x = v_i\Delta t + \frac{1}{2}a\Delta t^2$

$\Delta x = (20)(6) + \frac{1}{2}(0)(6)^2$

$\Delta x = 120$ m

OR

Let towards the sound source be positive

$\Delta x = \left(\frac{v_i + v_f}{2}\right)\Delta t$

$\Delta x = \left(\frac{(20)+(20)}{2}\right)(6)$

$\Delta x = 120$ m

OR

$V_{ave} = \frac{\Delta x}{\Delta t}$

$(20) = \frac{\Delta x}{(6)}$

$\Delta x = 120$ m

The observer in the car is moving at a constant velocity, therefore a = 0 $m.s^{-2}$

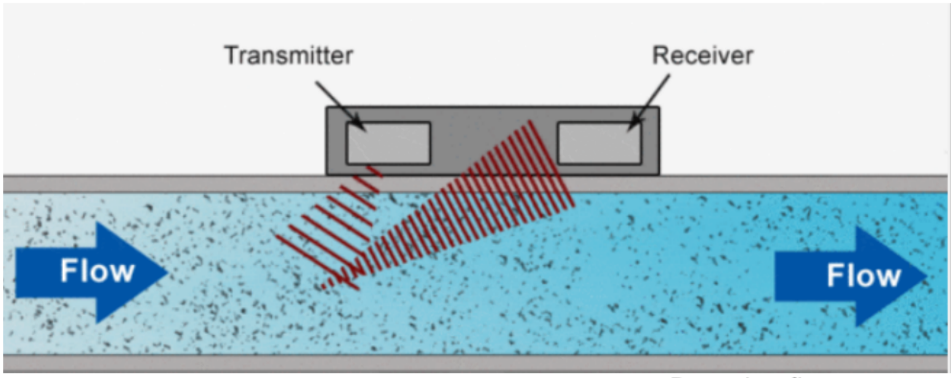
2.3 850 Hz.

The driver of the ambulance (sound source) is also at rest and therefore has the same velocity as the ambulance. There is no relative motion between the sound source and the listener, therefore, the Doppler effect will not be observed by the driver of the ambulance **or** there will be no change in the frequency of the sound observed by the driver of the ambulance.

REAL LIFE APPLICATIONS OF THE DOPPLER EFFECT

MEDICAL USES OF THE DOPPLER EFFECT

1. The **Doppler flow meter** used in medical science emits and receives continuous ultrasound waves (f > 20 000 Hz) and then measures the change in the frequency and wavelength. It is used to measure how fast or slow (the speed/velocity at which) blood is moving through arteries and veins, which can indicate a circulatory problem. It measures the speed at which blood is flowing by looking at the Doppler shift (the observed change in the frequency).



Doppler flow meter

2. A **foetal heart Doppler** is used to measure the heartbeat of an unborn foetus in the womb.

ANOTHER INTERESTING USE OF THE DOPPLER EFFECT

Speed cameras

The doppler effect can be used in speed traps to determine the speed of a vehicle.
A radar device transmits microwaves to the moving vehicle. The waves are reflected and received again. The change in frequency can be used to calculate the speed at which the vehicle is moving.

PRO-TIPS

If the question asks:
"Write down ONE real life application of the Doppler effect"
Answer: Doppler flow meter.
The Doppler flow meter is the most common and widely used application of the Doppler effect.

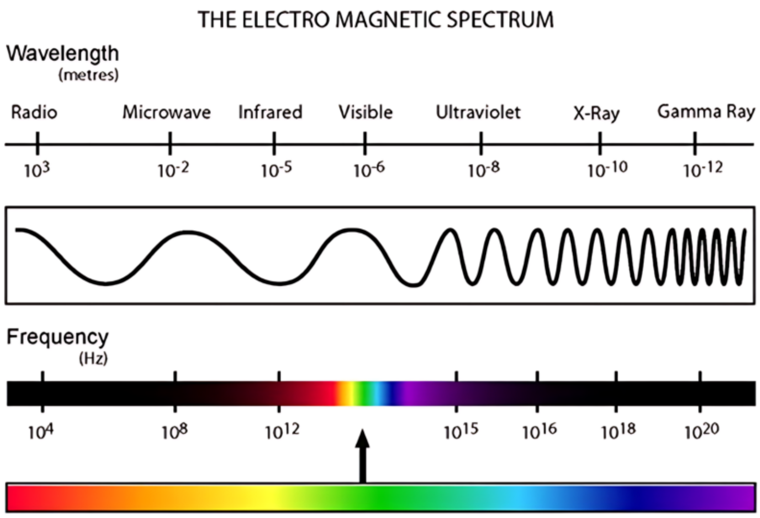
THE DOPPLER EFFECT AND LIGHT

The Doppler effect can be observed in all types of waves, specifically light waves, which is a type of **electromagnetic radiation.**

Recap from Grade 10:

- All types of electromagnetic radiation travels at the speed of light, **c**, namely $3{,}0 \times 10^{8}$ $m.s^{-1}$ in a vacuum or in air.
- To calculate the frequency or wavelength of the different types of electromagnetic waves, the formula: $c = f\lambda$
- Electromagnetic waves are examples of transverse waves, because the electric fields and magnetic fields oscillate at right angles to each other.

The electromagnetic spectrum in order of increasing frequency and decreasing wavelength:



PRO-TIPS

Did you notice that the **red end** of the visible light spectrum contains light colours of lower frequencies and the **blue end** of the visible light spectrum contains light colours of higher frequencies.

How will the Doppler effect be observed in visible light?

Visible light (white light) consists of a spectrum of colours; in order of increasing frequency and decreasing wavelength: red, orange, yellow, green, blue, indigo and violet.
The acronym **ROYGBIV** can be used to remember the visible light spectrum.

Therefore, change in the frequency of light will be observed as a change in the **colour** of the light.

The Doppler effect of light will only be observed if the light source or the observer is moving at very high (fast) speeds! Therefore, the Doppler effect is usually observed in **moving stars**.

Red shifts and blue shifts

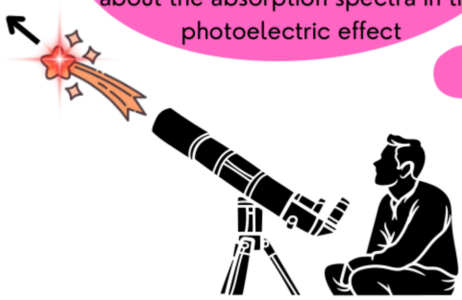
Astronomers and scientists have observed that stars and galaxies (A system of stars and planets) can either be moving **towards Earth** or **away from Earth**.
When this happens, a change in the colour of the light emitted by the star or galaxy is observed. Stars are made up of mostly hydrogen gas, below represents the **absorption spectra** of hydrogen gas, when the star is at rest:

PRO-TIPS

An absorption spectra is formed when white light is passed through a cold gas. It is very, very cold in outer space! The dark bands represent the light colours that are absorbed. You will learn more about the absorption spectra in the photoelectric effect

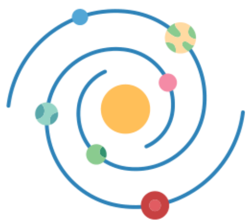
<u>Red shift</u> - stars and galaxies moving <u>AWAY</u> from earth

A red shift occurs when stars and galaxies move AWAY from the earth, therefore light colours of lower frequencies i.e., light colours towards the **red end** of the visible light spectrum are observed.



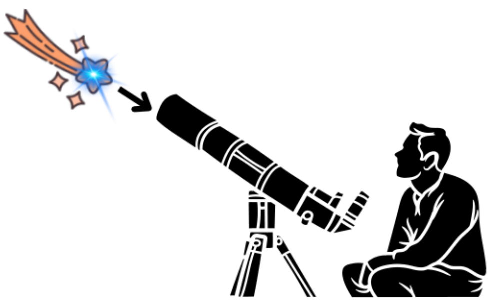
Light waves behind the light source (star) are further away, therefore light of a lower frequency is observed. The star appears to be more red in colour! The absorption spectra for a **red shift** shows the spectral lines are shifted towards the red end of the visible light spectrum:

Red shifts are evidence that the universe is expanding. Scientists are finding that more red shifts are taking place, proving that stars and galaxies are moving AWAY from Earth, and therefore that the universe is expanding.

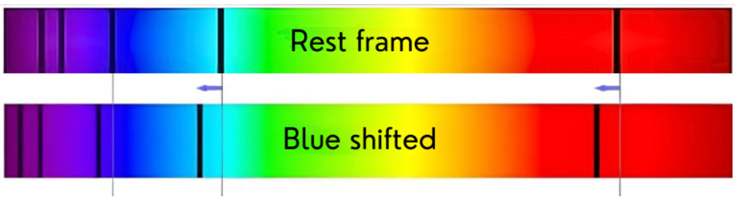


<u>Blue shift</u> - stars and galaxies moving <u>TOWARDS</u> earth

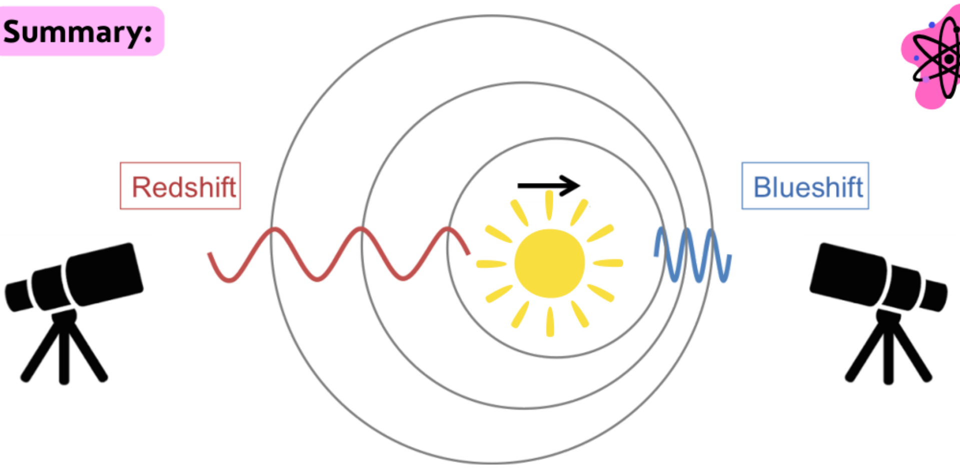
A blue shift occurs when stars and galaxies move TOWARDS the earth, therefore light colours of higher frequencies i.e., light colours towards the **blue end** of the visible light spectrum are observed:



Light waves in front of the light source (star) are closer together, therefore light of a higher frequency is observed. The star appears to be more blue in colour!
The absorption spectra for a **blue shift** shows the spectral lines are shifted towards the blue end of the visible light spectrum:



Summary:



Note: The greater the Doppler shift, the greater the speed at which the star moves towards or away from the Earth

Terminology:

Red shift: Spectral lines are shifted **towards the red** end of the visible spectrum. The **wavelengths** of the spectral lines are **longer** than expected.

Blue shift: Spectral lines are shifted **towards the blue** end of the visible spectrum. The **wavelengths** of the spectral lines are **shorter** than expected.

Worked example

1. Read the two observations below:
(a) Most stars and galaxies show a red shift.
(b) Stars and galaxies that are further away from us show a greater red shift.

1.1 Suggest what observation (a) tells us about our universe.
1.2 Suggest what observation (b) tells us about our universe.

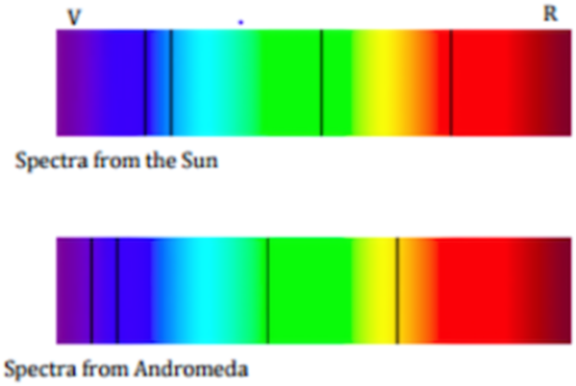
1.1. Observation (a) is evidence that the universe is expanding.

1.2. Observation (b) is evidence that the universe is expanding at a faster rate.

Worked example

C + P

1.Consider the spectral lines from the Sun and the Andromeda galaxy shown below:



PRO-TIPS

The sun, like stars, mainly consists of hydrogen gas. Therefore, the absorption spectra from the sun is the absorption spectra for hydrogen.

Is the Andromeda galaxy moving TOWARDS or AWAY FROM the earth? Explain the answer by referring to the spectral lines and the frequency and wavelength of the light waves emitted.

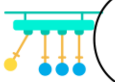
---

1. The spectral lines from the Andromeda galaxy show a blue shift – the spectral lines have shifted towards the blue end of the visible spectrum. This means that the light waves emitted from the Andromeda galaxy are moving towards the earth and have higher frequencies and shorter wavelengths.

MOMENTUM AND IMPULSE

Any object that is in motion, and therefore has a certain **velocity**, has momentum. 'Momentum' is a common term used in sport. For example, at a cricket match you might hear a spectator say "Wow, that ball has a lot of momentum", this is usually because the cricket ball is moving at a high velocity.

Definitions:Momentum: Product of an object's mass and its velocity.



We can calculate the momentum of an object using the following formula:

$$p = mv$$

<u>What do these variables mean and what are the SI units?</u>

m = mass (of the object) in kg
v = velocity (of the object) in $m.s^{-1}$
p= momentum (of the object) in $kg.m.s^{-1}$

PRO-TIPS

Do not forget your conversions of mass and velocity:

g --> kg $\div$ 1000
$km.h^{-1}$--> $m.s^{-1}$ $\div$ 3,6

From the momentum formula **p = mv**, the following can be conclusions can be drawn:

- $p \propto m$ (directly proportional) provided the velocity of the object remains constant. Therefore, the greater the mass of the object, the greater it's momentum.
- $p \propto v$ (directly proportional) provided the mass of the object remains constant. Therefore, the greater the velocity of the object, the greater it's momentum.

<u>Is momentum a scalar or vector quantity?</u>

Momentum is a **vector quantity**, which has both magnitude and direction.
The direction of the momentum of an object is in the same direction as the **VELOCITY** (motion) of the object.

Worked example

1. A body of mass **m**, with a velocity **v** has a momentum **p**. How will the momentum of the object be expressed in terms of **p** if:

1.1 the velocity of the object is doubled. (2)
1.2 the velocity of the object is decreased by a factor of 3. (2)
1.3 the mass of the object is increased by a factor of 4. (2)

1.1 From the formula p = mv, $p \propto v$ provided the mass of the object remains constant, therefore by doubling the initial velocity of the object, the momentum is also doubled. Double is the same as multiplying by two!
$\therefore$ 2**p**

1.2 The velocity being DECREASED by a factor of 3, means that it is now 3 times smaller, and mathematically this means it is $\frac{1}{3}$ of what it initially was.
From the formula p = mv, $p \propto v$ provided the mass of the object remains constant, therefore because the velocity is a $\frac{1}{3}$ of what it initially was, the momentum is also:
$\therefore \frac{1}{3}$**p**

1.3 From the formula p = mv, $p \propto m$ provided the velocity of the object remains constant, therefore by quadrupling of the initial mass of the object, the momentum is also quadrupled. Increasing by a factor of 4 is the same as quadrupling!
$\therefore$ 4**p**

CHANGE IN MOMENTUM

'Change in' also referred to as **delta,** Δ, is always the final amount minus the initial amount. Usually, a change in momentum is due to a change in velocity on a high school level, therefore, the mass of the object is kept constant.

$$\Delta p = p_f - p_i$$
$$\Delta p = mv_f - mv_i$$
$$\Delta p = m(v_f - v_i)$$
$$\Delta p = m\Delta v$$

Formula on the data sheet

PRO-TIPS

Always start off by writing down the formula on the data sheet and then adjust the formula accordingly (and if necessary)

What do these variables mean and what are the SI units?

Δp = change in momentum in $kg.m.s^{-1}$
m = mass of object in kg
v = final velocity in $m.s^{-1}$
v = initial velocity in $m.s^{-1}$

What is the direction of the <u>change in momentum</u>?

An object's momentum changes when it's velocity changes. Do you remember from Grade 11 what results in the velocity of an object changing? A NET FORCE!
Therefore, when net force acts on an object (From Newton's II law), the velocity of the object changes and therefore the momentum of the object also changes.
Remember an object which has a changing velocity is **accelerating.**

PRO-TIPS

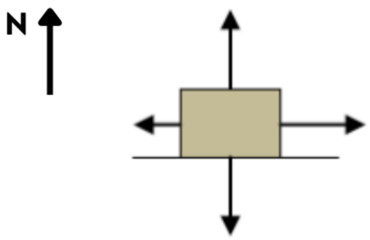
- The direction of the **momentum** of the object is in the direction of the velocity (motion) of the object.
- The direction of the **change in momentum** is in the direction of the net force acting on the object - this is in the same direction as the acceleration of the object, and the direction of the acceleration is not always in the direction of motion!

Worked examples

Multiple choice questions

C + A

1. The following forces act on a crate as shown below:



Which ONE of the following represents the direction of the change in momentum of the crate?

A East
B West
C North
D South

Answer: A
Tip: Remember that the direction of the **change** in **momentum** is in the **direction** of the **net force** - if the direction of the net force is determined, the direction of the change has also been determined.

From the diagram, there are **four forces** acting on the crate. The two vertical forces are in equilibrium, however, there is a **net** or resultant horizontal force, due to the **LARGER** force acting to the East. The net/ resultant force is therefore also acting **East**, and the direction of the change in **momentum** is also East.

2. A ball of mass **m**, moving at an initial velocity **v**, strikes the floor perpendicularly and rebounds with a velocity $\frac{1}{2}$**v**.

Which ONE of the following represents the MAGNITUDE of the change in momentum of the ball?

A $2mv$

B $3mv$

C $\frac{1mv}{2}$

D $\frac{3mv}{2}$

Answer: D

In this question, either UPWARDS or DOWNWARDS can be chosen as the positive direction:

OPTION 1: Let upwards be positive

$\Delta p = mv_f - mv_i$

$\Delta p = m(\frac{1}{2}v) - m(-v)$

$\Delta p = \frac{3mv}{2}$

OPTION 2: Let downwards be positive

$\Delta p = mv_f - mv_i$

$\Delta p = m(-\frac{1}{2}v) - m(+v)$

$\Delta p = -\frac{3mv}{2}$

$\therefore \Delta p = \frac{3mv}{2}$

Magnitude only

Note that the change in momentum is in the negative, upwards direction!

PRO-TIPS

When working with a **vector quantity**, always state your sign convention (i.e., choose a positive direction)

Worked example

C + A

1. A cricketer throws a ball of mass 100 g towards the batsman at an initial velocity of the 10 $m.s^{-1}$.
   The batsman hits the ball and the ball travels with a velocity of 3 $m.s^{-1}$ in the opposite direction.

1.1 Calculate the initial momentum of the ball. (3)

1.2 Calculate the final momentum of the ball. (3)

1.3 Calculate the change in momentum of the ball. (4)

NOTE:

- In this example, the direction in which the ball is initially moving is stated as "towards the batsman" and when the batsman hits the ball, the ball rebounds and moves in the opposite direction, this is away from the batsman.
- This is different to the usual directions given, such as left or right or upwards or downwards or West or East or North or South if cardinal points are used.

Tip: It is advised to stick to this convention and either let "towards the batsman" or "away from the batsman" be the positive direction. If a clear diagram, with clear directions is given, then directions such as left or right or upwards or downwards or West, East, North or South if cardinal points are indicated, can be used.

1.1 Remember: the **OBJECT** we are calculating the momentum of is the ball.

OPTION 1

Let towards the batsman be positive

p = mv

p = (0,1)(+10)

p = 1 $kg.m.s^{-1}$ towards the batsman ✓

Did you remember to convert g to kg by by ÷1000

OPTION 2

Let away from the batsman be positive

p = mv

p = (0,1)(-10)

p = -1 $kg.m.s^{-1}$

p = 1 $kg.m.s^{-1}$ towards the batsman

PRO-TIPS

Stick to your chosen positive direction in **ALL** the questions that follow.

1.2

OPTION 1

Let towards from the batsman be positive

$p = mv$

$p = (0,1)(\mathbf{-3})$

Remember the ball is now moving AWAY from the batsman, this is in the NEGATIVE direction.

$p = -0,3\ kg.m.s^{-1}$

$p = 0,3\ kg.m.s^{-1}$ away from the batsman

OPTION 2

Let away from the batsman be positive

$p = mv$

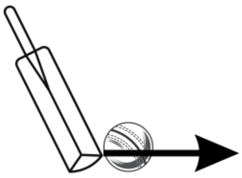
$p = (0,1)(+\mathbf{3})$

$p = 0,3\ kg.m.s^{-1}$ away from the batsman

Note that the direction of the momentum is in the direction of the velocity (motion)

1.3 **Remember:** the OBJECT we are calculating the change in momentum of is the **ball**.

A net force acting on the ball brings about a change in momentum, therefore, the force of the **bat on the ball** is the net force acting on the ball, and the direction of the net force and therefore the change in momentum is **away from the batsman**:



Force of bat on the ball

The initial momentum of the ball was calculated in question 1.1 and the final momentum of the ball in question 1.2 and therefore this does not have to be recalculated. However, the formula to calculate the change in momentum must be taken off the data sheet, and changed in the NEXT step:

OPTION 1

Let towards the batsman be positive

$\Delta p = mv - mv$

$\Delta p = p_f - p_i$

$\Delta p = (-0,3) - (+1)$

The formula can now be condensed after the formula from the data sheet is written.

$\Delta p = -1,3\ kg.m.s^{-1}$

$\Delta p = 1,3\ kg.m.s^{-1}$ away from the batsman

OPTION 2

Let away the batsman be positive

$\Delta p = mv_f - mv_i$

$\Delta p = p_f - p_i$

$\Delta p = (0,3) - (-1)$

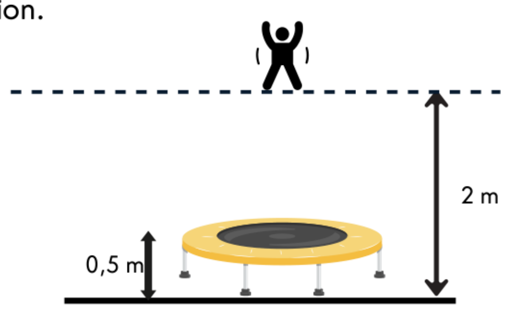
$\Delta p = 1,3\ kg.m.s^{-1}$ away from the batsman

The direction of the change in momentum is in the same direction as the net force

Worked example

A + P

2. A gymnast of mass 60 kg, falls vertically downwards from rest from a height of 2 m above the ground and lands on a trampoline that is 0,5 m above the ground. The gymnast rebounds. The magnitude of the change in momentum of the gymnast from the moment they strike the trampoline to rebounding is 500 $kg.m.s^{-1}$
Ignore the effects of friction.



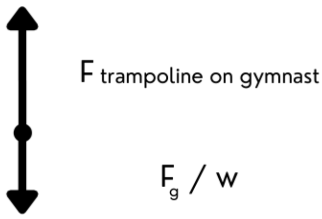
Calculate the velocity with which the gymnast will rebound from the trampoline. (7)

Guideline on how to approach this problem solving question:
In this question, the mass of the gymnast, magnitude of the change in momentum, and height of the gymnast and trampoline above the ground is given.

1. **Determine what is the direction of the change in momentum.**

This is the direction of the net force that results in a change in velocity. When the gymnast lands on the trampoline, gymnast exerts a downwards force on the trampoline, therefore due to Newton's third law of motion, the trampoline exerts an equal upwards force on the gymnast, resulting in the gymnast moving upwards (rebounding)

Free - body diagram showing the forces acting on the gymnast:



2. **Analyse the change in momentum formula and determine what information you have and need:**

$\Delta p = mv_f - mv_i$

Data

Δp = 500 kg.m.s^{-1} upwards

m = 60 kg

v_i = ? (this is the velocity with which the gymnast strikes the trampoline)

v_f = ? (needs to be calculated)

If the initial velocity with which the gymnast strikes the trampoline is determined, the final velocity with which the gymnast rebounds can be calculated. The initial velocity with which the gymnast strikes the trampoline can be determined using **equations of motion!**

OPTION 1

Let upwards be positive

$v_f^2 = v_i^2 + 2a\Delta y$

$v_f^2 = (0)^2 + 2(-9,8)(-1,5)$

v_f = 5,42 m.s^{-1} or v_f = -5,42 m.s^{-1}

Data (Points chosen: start to top of trampoline)

v_i = 0 m.s^{-1}

("From rest" means that initial velocity is zero)

Δy = -1,5 m

(The distance between the point the gymnast starts from and the top of the trampoline is calculated as:
Distance = 2 - 0,5 = 1,5 m. Therefore, to determine the displacement, draw a line from start to the top of the trampoline, notice the arrow points downwards)

a = -9,8 m.s^{-2}

(The gymnast is in free - fall, since upwards is taken as positive and gravitational acceleration is 9,8 m.s^{-2} downwards, therefore it is - 9,8 m.s^{-2})

v_f = ?

OR

OPTION 2

Let downwards be positive

$v_f^2 = v_i^2 + 2a\Delta y$

$v_f^2 = (0)^2 + 2(9,8)(1,5)$

v_f = 5,42 m.s^{-1} or v_f = -5,42 m.s^{-1}

Data (Points chosen: start to top of trampoline)

v_i = 0 m.s^{-1}

("From rest" means the initial velocity is zero)

Δy = 1,5 m

(The distance between the point the gymnast starts from and the top of the trampoline is calculated as:Distance = 2 - 0,5 = 1,5 m. Therefore, to determine the displacement, draw a line from start to the top of the trampoline, notice the arrow points downwards)

a = +9,8 m.s^{-2}

(The gymnast is in free - fall, since upwards is taken as positive and gravitational acceleration is 9,8 m.s^{-2} downwards, therefore it is + 9,8 m.s^{-2})

v_f = ?

3. **Substitute the information into the change in momentum formula to calculate the velocity with which the gymnast rebounds. Remember to choose a positive direction! Note: the gymnast rebounds UPWARDS.**

OPTION 1

Let upwards be positive

$\Delta p = mv_f - mv_i$

$(500) = (60)v_f - (60)(-5,42)$

v_f = 2,91 m.s^{-1} upwards

OR

OPTION 2

Let downwards be positive

$\Delta p = mv_f - mv_i$

$(-500) = (60)v_f - (60)(5,42)$

v_f = -2,91 m.s^{-1}

v_f = 2,91 m.s^{-1} upwards

NEWTON'S SECOND LAW OF MOTION IN TERMS OF MOMENTUM

You will recall that when a net force acts on an object of mass, this results in the velocity of the object **changing with time,** resulting in the object **accelerating.** This is Newton's second law of motion:

$$F_{net} = ma$$

This formula can be further expanded since acceleration is the **rate** of change of velocity:

$$F_{net} = m\frac{\Delta v}{\Delta t}$$

$$F_{net} = \frac{m\Delta v}{\Delta t}$$

$m\triangle v = \triangle p$:

$$F_{net} = \frac{\Delta p}{\Delta t}$$

PRO-TIPS

Rate is 'amount per second'. which can be written as:
rate = $\frac{1}{\Delta t}$

per = divided by

From this formula, it can be concluded that the net/ resultant force acting on an object is equal to the **rate of change of momentum** of the object, which brings us to Newton's second law of motion in terms of momentum:

Newton's second law in terms of momentum states that the resultant/net force acting on an object is equal to the **rate of change of momentum** of the object in the direction of the resultant/net force.

What do these variables mean and what are the SI units?

$\triangle p$ = change in momentum in $kg.m.s^{-1}$
F_{net} = **net or resultant** force in N.
$\triangle t$ = time period over which the net or resultant force acts on the object to bring about a change in momentum in s (seconds).

PRO-TIPS

A common multiple choice question:
The net/ resultant force is equal to the rate of change of momentum

The relationship between the variables in the formula:

$$F_{net} = \frac{\Delta p}{\Delta t}$$

$F_{net} \propto \triangle p$ (Net force is directly proportional to the change in momentum, provided the time taken remains constant).

- The **greater** the net **force** acting on the object, the **greater** the change in **momentum** (over the same time period).
- The **smaller** the net **force** acting on the object, the **smaller** the change in **momentum** (over the same time period).

$F_{net} \propto \frac{1}{\Delta t}$ (Net force is inversely proportional to the time taken, provided the change in momentum remains constant).

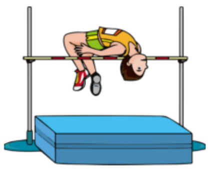
- The **greater** (longer) the **time** taken to bring about a change in **momentum**, the **smaller** the net **force** acting on the object.
- The **shorter** the **time** taken to bring about a change in **momentum**, the **greater** the net **force** acting on the object.

In summary: Increase the time taken (to bring about a change in momentum), decrease the force.

There are many real - life examples of the relationship between **net force and time**:

SPORT

A cricketer pulls his hands backwards when he catches a moving ball. In doing so, the velocity of the ball decreases to zero over a **GREATER TIME** period, Δt increases thereby reducing the force (of the ball on his hands) and making the catching process less painful. The change in momentum remains constant (because the mass of the ball and the change in velocity remains constant), the time taken to produce this change in momentum is just increased.



PRO-TIPS

The force exerted by one object on another object is always EQUAL in magnitude but OPPOSITE in direction due to Newton's Third Law of motion

In high jump, the athlete lands on a foam cushion instead of the hard ground. The foam cushion increases the time taken to bring about the **same** change in momentum, therefore decreasing the force acting on the athlete, therefore the athlete is able to safely land on the foam cushion.

SAFETY FEATURES IN VEHICLES

Modern vehicles are fitted with **crumple zones**, usually in the front and back of the vehicle. The bumper and rear of the vehicle is actually made of plastic instead of metal.
The crumple zone results in the area crumpling and deforming during a collision, which increases the time taken to bring about the **same** change in momentum, therefore decreasing the force experienced by the driver and passengers in the vehicle.

Air bags is another safety feature in modern vehicle. When the driver or passenger collides with the inflated airbag, this increases the time taken to bring about the same change in momentum, therefore decreasing the force experienced by the driver and passengers in the vehicle.

OTHER APPLICATIONS OF NEWTON'S SECOND LAW OF MOTION IN TERMS OF MOMENTUM:

Arrestor beds: Arrestor beds are safety features used to bring heavy duty vehicles such as trucks, safely to a rest, away from other traffic on the roads. The material that the arrestor bed is made of is usually crushable concrete (i.e., gravel).

How does an arrestor bed work?

An arrestor bed increases the time taken for the heavy duty to bring about the same change in momentum, therefore decreasing the force experienced by the driver and/or passengers.

Bending your legs when jumping from a high height:

When jumping from a high height, e.g., a table, a person usually bends their knees. In doing so, the time taken to bring about the **same** change in momentum is increased therefore the force experienced is decreased.

Worked examples

Multiple choice questions

A cyclist changes its velocity from **v** to 2**v** in time **t,** with a net force **F**. The cyclist now changes its velocity from **v** to 2**v** in time 4**t**. Which ONE of the following represents the net force acting on the cyclist?

A 4**F**

B 2**F**

C $\frac{1}{2}$**F**

D $\frac{1}{4}$**F**

Answer: D

In this question, the change in velocity from **v** to 2**v** remains constant, therefore the change in momentum is constant. However, the time taken to bring about the SAME change in momentum is 4 times greater (or increased by a factor of 4).

From the formula: $F_{net} = \frac{\Delta p}{\Delta t}$, if the change in momentum (△p) remains constant then $F_{net} \propto \frac{1}{\Delta t}$

Therefore, $F_{net} \propto \frac{1}{4}$, ∴ $\frac{1}{4}$**F.**

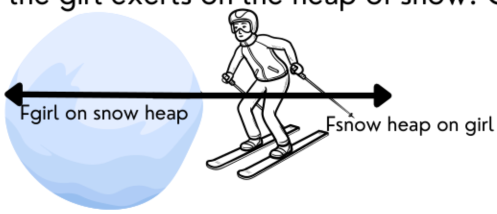
Worked example

C + A

1. A girl on ski's arrives at the base of a slope at a speed of 20 $m.s^{-1}$. She collides with a heap of snow and is brought to rest in the snow in 3 seconds. If the mass of the skier, with her equipment, is 60 kg:

1.1 Calculate the average force that the heap of snow exerts on her as she is brought to rest. (4)

1.2 Write down the force that the girl exerts on the heap of snow. Give a reason for the answer. (2)



PRO-TIPS

Draw a force diagram OR free - body diagram showing the forces acting on both objects that collide with each other.

1.1 **OPTION 1**

Let towards the heap of snow be positive

$F_{net} = \frac{\Delta p}{\Delta t}$

$F_{net} = \frac{m\Delta v}{\Delta t}$

$F_{net} = \frac{m(vf - vi)}{\Delta t}$

$F_{net} = \frac{(60)[(0)-(20)]}{(3)}$

F_{net} = -400 N

F_{net} = 400 N away from the heap of snow

OR

OPTION 2

Let away from the heap of snow be positive

$F_{net} = \frac{\Delta p}{\Delta t}$

$F_{net} = \frac{m\Delta v}{\Delta t}$

$F_{net} = \frac{m(vf - vi)}{\Delta t}$

$F_{net} = \frac{(60)[(0)-(-20)]}{(3)}$

F_{net} = 400 N away from the heap of snow

No diagram was provided, therefore "towards or away from" the heap of snow was used as the positive direction.

1.2 **NOTE:** The force that the girl and the heap of snow exert on each other are equal in magnitude but opposite in direction due to Newton's third law of motion:

F = 400 N **towards the heap of snow.** This is due to Newton's third law of motion.

Worked example

C + A

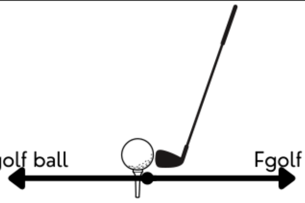
2. A club collides with a golf ball of mass 0,045 kg as shown below:

The force exerted by the club on the golf ball is 50 N. The club is in contact with the golf ball for 0,1 s.

2.1 Calculate the final velocity of the golf ball.

2.2 Calculate the change in momentum of the club.

2.1



The question is focused on the **GOLF BALL**. Remember that the golf ball is initially at rest, therefore the initial velocity of the golf ball is 0 m.s^{-1}. The ball experiences a net force to the **LEFT** due to the club and the club experiences a net force of the same magnitude to the **RIGHT**.

OPTION 1

Let left be positive

$F_{net} = \frac{\Delta p}{\Delta t}$

$F_{net} = \frac{m(vf - vi)}{\Delta t}$

$(+50) = \frac{(0{,}045)(vf-(0))}{(0{,}1)}$

v_f = 111,11 m.s^{-1} left

OR

OPTION 2

Let right be positive

$F_{net} = \frac{\Delta p}{\Delta t}$

$F_{net} = \frac{m(vf - vi)}{\Delta t}$

$(-50) = \frac{(0{,}045)(vf-(0))}{(0{,}1)}$

v_f = -111,11 m.s^{-1}

v_f = 111,11 m.s^{-1} left

2.2

OPTION 1

Let left be positive

$F_{net} = \frac{\Delta p}{\Delta t}$

$(-50) = \frac{\Delta p}{(0{,}1)}$

Δp = -5 kg.m.s^{-1}

Δp = 5 kg.m.s^{-1} right

OR

OPTION 2

Let right be positive

$F_{net} = \frac{\Delta p}{\Delta t}$

$(+50) = \frac{\Delta p}{(0{,}1)}$

Δp = 5 kg.m.s^{-1} right

PRO-TIPS

NOTE: The direction of the change in momentum of the club is the same direction as the net force acting on the club.

IMPULSE

What is the impulse acting on an object?

Impulse: the product of the resultant/net force acting on an object and the time the resultant/net force acts on the object.

From Newton's second law of motion in terms of momentum:

$$F_{net} = \frac{\Delta p}{\Delta t}$$

$$F_{net} = \frac{m\Delta v}{\Delta t}$$

Multiply both sides of the equation by Δt

$$F_{net}\Delta t = m\Delta v$$

$$\mathbf{F_{net}\Delta t = \Delta p}$$

Formula on the data sheet

Impulse - momentum theorem

The impulse-momentum theorem states that impulse is equal to the change in momentum. This can be seen from the equation above.

NOTE: Impulse has **no symbol.**

The impulse acting on an object can be calculated in one of TWO ways (depending on the information given):

1. $F_{net}\Delta t = \Delta p$
   $F_{net}\Delta t = m\Delta v$

$F_{net}\Delta t$ as a whole represents **IMPULSE** (since impulse has no symbol).

2. Impulse = $F_{net}\Delta t$

The above equation can be used if the net force and the time the net force acts on the object is known.

SI units for impulse

From the formula:

$F_{net}\Delta t = \Delta p$

N.s = kg.m. s^{-1}

The **SI units** for impulse is Newton seconds (N.s), which is equivalent to kilogram - metres per second. (1 kg.m.s^{-1})

1 N.s = 1 kg.m.s^{-1}

PRO-TIPS

The impulse- momentum formula can be further expanded:

$F_{net}\Delta t = \Delta p$

thus $F_{net}\Delta t = mv_f - mv_i$

thus $F_{net}\Delta t = m(v_f - v_i)$

thus $F_{net}\Delta t = m\Delta v$

Direction of impulse

The product of the net force and the interaction time is called the impulse.
Since net force is a vector quantity, impulse ($F_{net}\Delta t$) is also a **vector quantity**. The direction of the impulse is in the same direction as the **net force**, which is in the same direction as the change in momentum.

Impulse and Newton's Third Law of motion

$$\textbf{Impulse} = \mathbf{F_{net}\,\Delta t}$$

The force that one object (object **A**) exerts on a second object (object **B**) is equal in magnitude but opposite in direction.
The impulse that object **A** exerts on object **B** is equal in magnitude but opposite in direction to the impulse by object **B** on object **A**.
This follows Newton's Third Law of motion.

Worked examples

Multiple choice questions

Car **A** of mass **m** moving at an initial velocity **v** right, collides with car **B** of mass **2m.**
Car **A** rebounds with a velocity $\frac{1}{3}$**v.**
Which ONE of the following represents the impulse acting on car **B**?

A $\frac{4}{3}$mv left

B $\frac{4}{3}$ mv right

C mv left

D mv right

Answer: B

Information regarding the mass, initial velocity and final velocity of car **A** is known, this can be used to determine the impulse of car **A** since $F_{net}\Delta t = \Delta p$
The impulse of car **B** (as the question asks) can be then determined as it is equal in magnitude but opposite in direction to the impulse acting on car **A.**

Car A

Let left be positive

$F_{net}\Delta t = \Delta p$
$F_{net}\Delta t = m\Delta v$
$F_{net}\Delta t = m(v_f - v_i)$
$F_{net}\Delta t = m[\frac{1}{3}v - (-v)]$
$F_{net}\Delta t = \frac{4}{3}$ mv left

Let left be positive

$F_{net}\Delta t_B = -F_{net}\Delta t_A$
$F_{net}\Delta t_B = -\frac{4}{3}mv$
$F_{net}\Delta t_B = \frac{4}{3}$ mv right

The impulse acting on car B is equal in magnitude but opposite in direction to the impulse acting on car A.

Worked example

C + A

1. As a safety feature, modern car's front ends crumple on impact. A 2000 kg car travels left at a constant velocity of 10 $m.s^{-1}$ towards a stationary tree. It collides with the tree and comes to a stop in 0,3 s.

1.1 Calculate the impulse experienced by the car when it collides with the tree. (4)
1.2 What is the average net force exerted on the car? (3)
1.3 For the same impulse, calculate the average net force that the tree would exert on a car which stopped in 0,05 s, due to a bumper that does not crumple on impact. (3)

NOTE: When calculating impulse, always determine which information you have to calculate the impulse, since there are two formulae that can be be used to calculate impulse. In this question the initial and final velocity of the the car is known, this can be used to calculate the change in momentum of the car, which is equal to the impulse acting on the car.

1.1 **OPTION 1**

Let left be positive

$F_{net}\Delta t = \Delta p$
$F_{net}\Delta t = m\Delta v$
$F_{net}\Delta t = m\ (v_f - v_i)$
$F_{net}\Delta t = 2000[(0) - (10)]$
$F_{net}\Delta t = -20000$ N. s
$F_{net}\Delta t = 20000$ N.s right

OR

1.1 **OPTION 2**

Let right be positive

$F_{net}\Delta t = \Delta p$
$F_{net}\Delta t = m\Delta v$
$F_{net}\Delta t = m\ (v_f - v_i)$
$F_{net}\Delta t = 2000[(0) - (-10)]$
$F_{net}\Delta t = 20000$ N.s right

1.2 **OPTION 1**

Let left be positive

$F_{net} = \frac{\Delta p}{\Delta t}$
$F_{net} = \frac{(-20000)}{(0{,}3)}$
$F_{net} = -\ 66666{,}67$ N
$F_{net} = 66666{,}67$ N right

OR

1.2 **OPTION 2**

Let right be positive

$F_{net} = \frac{\Delta p}{\Delta t}$
$F_{net} = \frac{(20000)}{(0{,}3)}$
$F_{net} = 66666{,}67$ N right

1.3 **OPTION 1**

Let left be positive

$F_{net} = \frac{\Delta p}{\Delta t}$
$F_{net} = \frac{(-20000)}{(0{,}05)}$
$F_{net} = -\ 400\ 000$ N
$F_{net} = 400\ 000$ N right

OR

1.3 **OPTION 2**

Let right be positive

$F_{net} = \frac{\Delta p}{\Delta t}$
$F_{net} = \frac{(20000)}{(0{,}05)}$
$F_{net} = 400\ 000$ N right

Compare the net force in question 1.2 and question 1.3.
Notice that the shorter the time taken to bring about the same change in momentum (or impulse), the **GREATER** the net force acting on the car.

Worked example

A + P

2. A rifle with a mass of 6 kg fires a bullet with a mass of 0,002 kg at a velocity of 1200 m.s $^{-1}$.

2.1 Calculate the change in momentum of the bullet. (3)

2.2 Calculate the impulse that acts on the rifle. (3)

2.1 OPTION 1

Let right be positive

$\Delta p = mv_f - mv_i$

$\Delta p = (0,002)(1200) - (0,002)(0)$

$\Delta p = 2,40$ kg.m.s^{-1} right

OR

2.1 OPTION 2

Let left be positive

$\Delta p = mv_f - mv_i$

$\Delta p = (0,002)(-1200) - (0,002)(0)$

$\Delta p = -2,40$ kg.m.s^{-1}

$\Delta p = 2,40$ kg.m.s^{-1} right

2.2 OPTION 1

Let right be positive

$F_{net}\Delta t_{bullet} = \Delta p_{bullet}$

$F_{net}\Delta t_{bullet} = 2,40$ N.s right

$F_{net}\Delta t_{rifle} = -F_{net}\Delta t_{bullet}$

$F_{net}\Delta t_{rifle} = -2,4$ N.s

$F_{net}\Delta t_{rifle} = 2,4$ N.s left

OR

2.2 OPTION 2

Let left be positive

$F_{net}\Delta t_{bullet} = \Delta p_{bullet}$

$F_{net}\Delta t_{bullet} = -2,40$ N.s

$F_{net}\Delta t_{bullet} = 2,40$ N.s right

$F_{net}\Delta t_{rifle} = -F_{net}\Delta t_{bullet}$

$F_{net}\Delta t_{rifle} = -(-2,4)$

$F_{net}\Delta t_{rifle} = 2,4$ N.s left

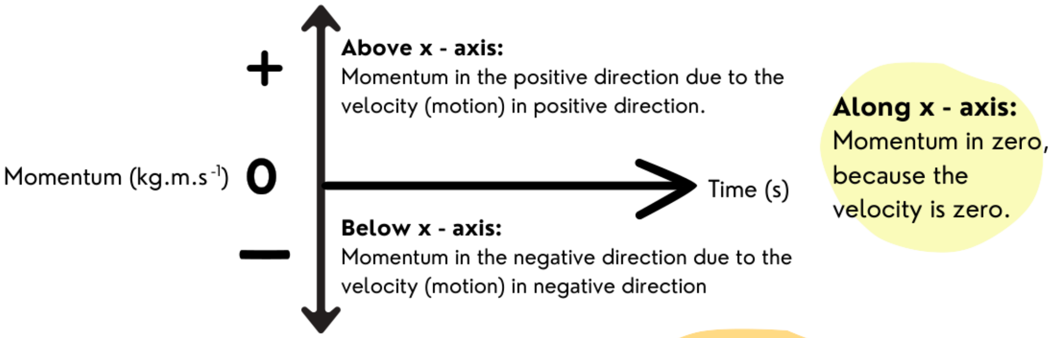
NOTE:

- The impulse experienced by the rifle is **equal in magnitude but opposite in direction** to the impulse experienced by the bullet.
- The bullet and the rifle exert a force of equal in magnitude but opposite in direction on each other (due to **Newton's third law** of motion) for the same time period.

MOMENTUM AND IMPULSE : COMMON GRAPHS

1. Momentum - time graph

A momentum - time graph is a rate graph that represents how the momentum of an object changes with time. It is similar in shape to a velocity - time graph, since momentum, **p = mv**.



Gradient of the momentum - time graph

Gradient = $\frac{\Delta y}{\Delta x}$

Gradient = $\frac{\Delta p}{\Delta t}$

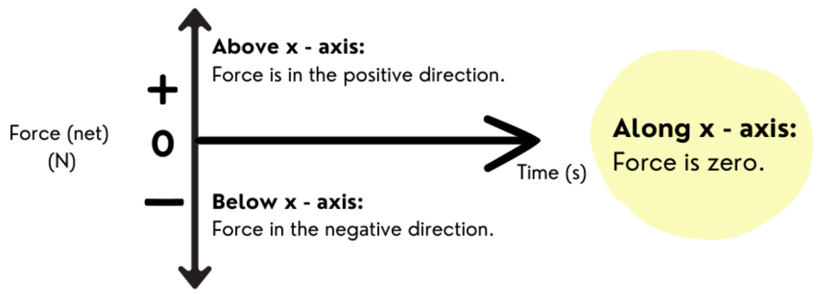
Gradient = F_{net}

The following variables can also be determined from the momentum - time graph

- Velocity at any time using the momentum formula, p = mv.
- The change in momentum (over a time period), therefore the impulse ($F_{net}\Delta t$) can be determined since $F_{net}\Delta t = \Delta p$.

2. Force - time graph

A force - time graph is a rate graph that represents how the (net) force of acting on an object changes with time.



Area under the force - time graph

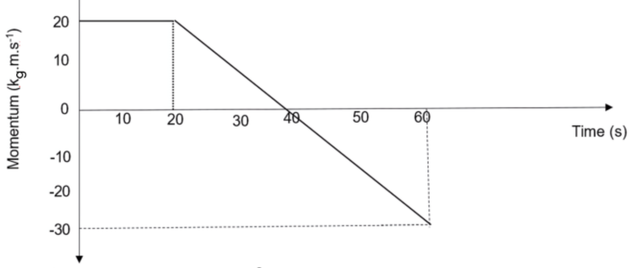
Area under the force - time graph = Net force x time
Area under the force - time graph = F_{net} x Δt
Area under the force - time graph = impulse
Area under the force - time graph = change in momentum (Δp)

The following variables can also be determined from the force - time graph

- Average acceleration, if the mass of the object is known, since $F_{net} = ma$.

Worked example

1. The momentum vs time graph shown below represents the motion an object of mass 30 kg initially moving NORTH.



Using the graph above,

1.1 Calculate the velocity of the object at time = 0 s. (3)

1.2 Determine the net force acting on the object between 0 and 20 s. (3)

1.3 Write down the change in momentum between 0 and 20 s. (1)

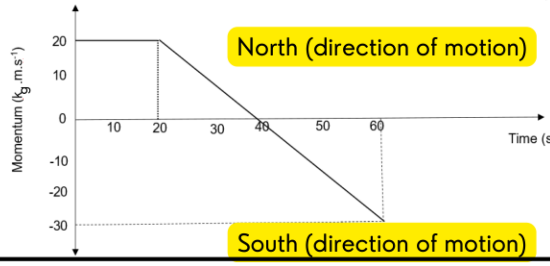
1.4 Calculate the change in momentum of the object between 20 and 40 s. (3)

1.5 Write down the momentum of the object at 60 s. (1)

1.6 Calculate the impulse acting on the object between 40 and 60 s. (3)

The graph is a momentum - time graph, therefore remember the following:

1. p = mv
2. gradient = net force



1.1 The momentum at t = 0s is 20 kg.m.s^{-1} and the mass of the object is 30 kg. Using the momentum formula, the velocity of the object at t = 0s can be determined. Remember to state your sign convention!

Let **North** be positive

From the graph: The object is initially travelling North.

p = mv

(20) = (30)v

v = 0,67 m.s^{-1} North

1.2 **OPTION 1 (By graph interpretation & application)**

Fnet = 0 N.

Between 0 and 20s, the momentum of the object is constant, therefore the velocity of the object is constant.

$\triangle$v = 0 m.s^{-2} , therefore, a = 0 m.s^{-1}.

From Newton's second law of motion:

F_{net} = ma

F_{net} = (30)(0)

F_{net} = 0 N.

OR

OPTION 2: The gradient of the momentum - time graph represents the net force:

Gradient = F_{net}

Gradient = $\frac{\Delta y}{\Delta x}$

Gradient = $\frac{(20)-(20)}{(20)-(0)}$

Gradient = 0

$\therefore F_{net}$ = 0 N

1.3 $\triangle$p = 0 kg.m.s^{-1}

Between 0 and 20s, the momentum of the object is constant, therefore the velocity of the object is constant.

$\triangle$v = 0 m.s^{-1} , therefore, $\triangle$p = 0 kg.m.s^{-1}

OR

1.3 **continued**

Let North be positive

$\triangle p = mv_f - mv_i$

$\triangle p = p_f - p_i$

$\triangle$p = (20) - (20)

$\triangle$p = 0 kg.m.s^{-1}

1.4 Let North be positive

$\Delta p = mv_f - mv_i$

$\Delta p = p_f - p_i$

$\Delta p = (0) - (20)$

$\Delta p = -20\ kg.m.s^{-1}$

$\Delta p = 20\ kg.m.s^{-1}$ South

- The **momentum** of the object is **decreasing** (with time).
- Therefore the **velocity** of the object is **decreasing**, and the net force is acting in the **opposite** direction of motion.
- In this example, South, therefore since the change in momentum is in the direction of the net force, the change in momentum is South.

1.5 Let North be positive

$p = -30\ kg.m.s^{-1}$

$p = 30\ kg.m.s^{-1}$ South

1.6 $F_{net}\Delta t = \Delta p$

Determine the change in momentum between 40s and 60s, this will be equal to the impulse acting on the object between 40s and 60s.

Let North be positive

$\Delta p = mv_f - mv_i$

$\Delta p = p_f - p_i$

$\Delta p = (-30) - (0)$

$\Delta p = -30\ kg.m.s^{-1}$

$\Delta p = 30\ kg.m.s^{-1}$ South

- The **momentum** of the object is **increasing** (with time).
- Therefore the **velocity** of the object is **increasing**, and the net force is acting **in the direction** of motion.
- In this example, South, therefore since the change in momentum is in the direction of the net force, the change in momentum is South.
- The impulse acting on the object is **equal** to the change in momentum and is in the direction of the net force or the change in momentum, namely South.

$F_{net}\Delta t = \Delta p$

$F_{net}\Delta t = 30\ kg.m.s^{-1}$ South

Worked example

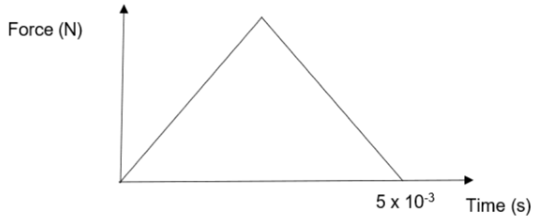
A+P

2. A golf ball of mass 0,045 kg is struck by a club and leaves the club with a velocity of 50 $m.s^{-1}$.

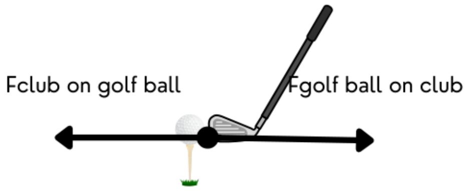
2.1 Calculate the magnitude of the change in momentum of the ball. (3)

2.2 Calculate the average net force applied to the ball if the ball is in contact with the club for 5×10^{-3} s. (5)

The force – time graph below represents the force acting on the golf ball.



2.3 Calculate the MAXIMUM force exerted by the club on the ball.



2.1

Let left be positive

$\Delta p = mv_f - mv_i$

$\Delta p = (0{,}045)(50) - (0{,}045)(0)$

$\Delta p = 2{,}25\ kg.m.s^{-1}$

OR

Let right be positive

$\Delta p = mv_f - mv_i$

$\Delta p = (0{,}045)(-50) - (0{,}045)(0)$

$\Delta p = -2{,}25\ kg.m.s^{-1}$

$\Delta p = 2{,}25\ kg.m.s^{-1}$

2.2

OPTION 1

Let left be positive

$F_{net} = \frac{\Delta p}{\Delta t}$

$F_{net} = \frac{(+2{,}25)}{(5\times10^{-3})}$

$F_{net} = 450$ N left

OR

OPTION 1

Let right be positive

$F_{net} = \frac{\Delta p}{\Delta t}$

$F_{net} = \frac{(-2{,}25)}{(5\times10^{-3})}$

$F_{net} = -450$ N

$F_{net} = 450$ N left

OPTION 2

Let left be positive

$F_{net}\Delta t = \Delta p$

$F_{net}(5 \times 10^{-3}) = (+2{,}25)$

$F_{net} = 450$ N left

OPTION 2

Let right be positive

$F_{net}\Delta t = \Delta p$

$F_{net}(5 \times 10^{-3}) = (-2{,}25)$

$F_{net} = -450$ N

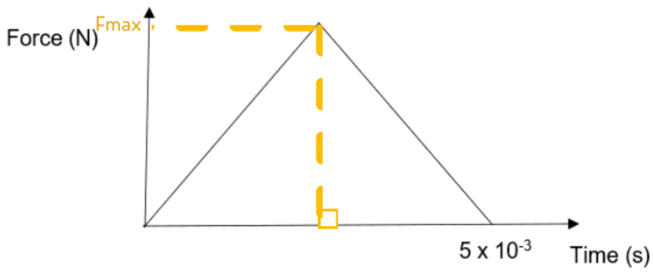
$F_{net} = 450$ N left

PRO-TIPS

Take note of the question.
If it is a VECTOR quantity and the question states "magnitude", only give the magnitude, do not include the direction.

2.3 Analyse the force - time graph:

- The force exerted by the club on the golf ball is not constant and varies from 0 N to a maximum and back to 0 N when the club loses contact with the golf ball.
- The area under the force - time graph represents the impulse. Impulse = change in momentum. The change in momentum of the golf ball was calculated in **question 2.1**
- Additional constructions can be added to the graph:
  - A perpendicular line drawn from the corner of the triangle to the one side of the triangle. This is the **perpendicular height** of the triangle.



2.3 **continued...**

From **question 2.1:**

$F_{net}\Delta t = \Delta p$

$F_{net}\Delta t = 2{,}25$ N.s (left)

Area under the force – time graph = Impulse

Area under the force – time graph = $\frac{1}{2} b \times \perp h$

Area under the force – time graph = $\frac{1}{2} \Delta t \times F_{max}$

$(2{,}25) = \frac{1}{2}(5 \times 10^{-3}) \times F_{max}$

$F_{max} = 900$ N left

REMINDER :QUESTION DIFFICULTY

C – COMPREHENSION AND RECALL QUESTIONS

These are common questions which include definitions and calculation questions that look similar to the questions covered in class. Approximately **50%** of Paper 1 (Physics) will include questions on this level.

A – ANALYSIS AND APPLICATION QUESTIONS

These are more complex questions which involves applying the knowledge and skills learned in this chapter. Approximately **40%** of Paper 1 (Physics) will include questions on this level.

P – PROBLEM- SOLVING QUESTIONS

These are questions that require critical thinking and being able to make connections between different representations of information and integrating different topics. They are not familiar questions, but are able to be solved through critical analysis. Approximately **10%** of Paper 1 (Physics) will include questions on this level.

THE PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

In Physics, an **isolated system** is often encountered. However, an isolated system is rare in real - life.

What is an isolated system (in Physics)?

Definition: An isolated system is a system on which the net or resultant external force is zero.

Only the internal forces e.g., the colliding forces between the objects is considered.

External forces change the **mechanical energy** of the system. Mechanical energy is the sum of the kinetic and gravitational potential energy of the object.

PRO-TIPS

The term 'isolated system' **ONLY** applies in Physics and 'closed system' **ONLY** applies in Chemistry. These terms cannot be switched around as they mean different things.

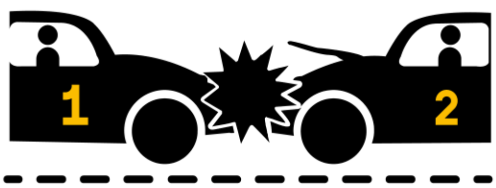
Examples of external forces:

- Frictional force (including air friction/ air resistance)
- Applied force
- Tension

In an isolated system the mechanical energy and linear momentum is conserved.

Principle of conservation of linear momentum: states that the total linear momentum of an isolated system remains constant (is conserved).

Below is a scenario where two objects, **1** and **2,** in an isolated system collide with each other, head on:



PRO-TIPS

Linear means 'in a straight line'

Only the internal forces, i.e. colliding forces are factored in. The impulse experienced by object 1 is equal in magnitude, but opposite in direction to the impulse experienced by object 2:

Let left be positive

$$\triangle p_1 = -\triangle p_2$$

$$m_1 v_{f1} - m_1 v_{i1} = -(m_2 v_{f2} - m_2 v_{i2})$$

Rearrange the formula: $$m_1 v_{i1} + m_2 v_{i2} = m_1 v_{f1} + m_2 v_{f2}$$

In summary: $$\Sigma p_i = \Sigma p_f$$

Sum of the momentums before the collision = Sum of the momentums after the collision

PRO-TIPS

The principle of conservation of linear momentum can be applied to collisions or 'explosions'.

There are three common scenarios of isolated systems encountered in the principle of conservation of linear momentum:

<table>
<tr><td>Scenario 1:
Objects are separated BEFORE and AFTER the collision</td><td>Scenario 2:
Objects are separated BEFORE and stick together AFTER the collision</td></tr>
<tr><td>Two separate objects collide and remain separated after the collision.
$\sum p_i = \sum p_f$
$m_1v_{i1} + m_2v_{i2} = m_1v_{f1} + m_2v_{f2}$</td><td>Two separate objects collide and stick together after the collision.
$\sum p_i = \sum p_f$
$m_1v_{i1} + m_2v_{i2} = (m_1 + m_2)\,v_f$
The combined objects have the same final velocity, after the collision.</td></tr>
<tr><td colspan="2">Scenario 3:
Objects are stuck together BEFORE the explosion and separate AFTER the explosion.</td></tr>
<tr><td colspan="2">This scenario is an example of an 'explosion'. For example, when a bullet is shot out of a gun or rifle. The two objects are "stuck together" before the explosion, and separate after the explosion.
$\sum p_i = \sum p_f$
$(m_1 + m_2)\,v_i = m_1v_{f1} + m_2v_{f2}$
The combined objects have the same initial velocity, before the collision.</td></tr>
</table>

PRO-TIPS

The above scenarios do not have to be learned, the original formula can be used for all scenarios involving an isolated system:

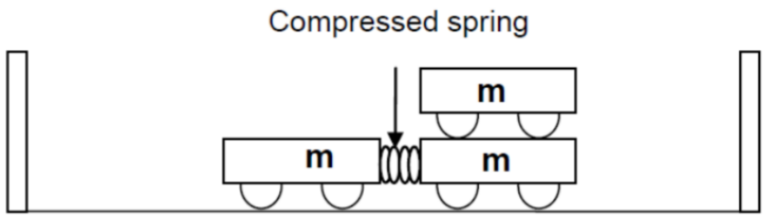
$$\sum p_i = \sum p_f$$
$$m_1v_{i1} + m_2v_{i2} = m_1v_{f1} + m_2v_{f2}$$

Worked examples

Multiple choice questions

C+A

Learners perform an experiment using identical trolleys, each of mass **m**.
The trolleys are arranged, as shown in the diagram below. They are initially at rest on a frictionless surface and are connected with a compressed, massless spring.



When the spring is released it falls vertically down and the single trolley moves with velocity **v** to the left. The magnitude of the velocity of the two trolleys moving to the right will be:

A **v**
B 2**v**
C 0,5**v**
D 0,25**v**

Answer: C
The trolleys are attached (stuck together) before the spring is released. The system is at rest, therefore the sum of the momentums before the collision is 0 $kg.m.s^{-1}$.

Let left be positive

$$\sum p_i = \sum p_f$$
$$(m_1 + m_2)v_i = m_1 v f_1 + m_2 v f_2$$
$$0 = mv + 2mv_{f2}$$
$$-m\mathbf{v} = 2mv_{f2}$$
$$v_{f2} = -\frac{v}{2}$$
$$v_{f2} = -0{,}5\mathbf{v}$$
$v_{f2} = 0{,}5\mathbf{v}$ (magnitude only)

PRO-TIPS

For multiple choice questions that do not involve values, follow the same process using symbols
OR
substitute in your own values and compare it to the options.

Worked example

C+A

1. A girl wearing skates stands stationary on ice. The mass of the girl is 60 kg. The girl throws a 200 g ball horizontally at a speed of 15 $m.s^{-1}$. Ignore any effects of friction.

Calculate the velocity of the girl after throwing the ball. (4)

Let the direction of motion of the ball be positive

$$\sum p_i = \sum p_f$$
$$(m_{girl} + m_{ball})\, v_i = m_{girl}v_{fgirl} + m_{ball}v_{fball}$$
$$(60 + 0,2)(0) = (60)v_{fgirl} + (0,2)(+15)$$
$$v_{fgirl} = -0,05 \text{ m.s}^{-1}$$
$v_{fgirl} = 0,05$ $m.s^{-1}$ in the opposite direction of motion of the ball.

1. This is an isolated system involving two bodies, therefore the momentum of the system is conserved and the principle of conservation of linear momentum can be used to calculate the final velocity of the girl.

PRO-TIPS

- In the principle of conservation of linear momentum formula, symbols 1 and 2 can be used to represent object or body 1 and 2.
- However, words to represent the objects (e.g., ball, girl, boy etc.) can be used instead.

Revision

In an isolated system, the Principle of Conservation of Mechanical energy can also be applied.

Definition: Principle of conservation of mechanical energy: states that the total mechanical energy in a isolated system remains constant.

$$E_M \text{ (at a point)} = E_M \text{ (at another point)}$$
$$(E_k + E_p)\text{at a point} = (E_k + E_p)\text{at another point}$$

Worked example

A+P

2. A boy with a mass of 45 kg jumps with a speed of 5 $m.s^{-1}$ onto a stationary skateboard, with a mass of 5 kg, resting on a horizontal surface of a skateboard ramp.

PRO-TIPS

Kinetic energy formula:

$E_k = \frac{1}{2}mv^2$

Gravitational potential energy formula:

$E_p = mgh$

2.1 Prove, by calculation, that the speed of the boy is 4,5 $m.s^{-1}$ immediately after he has jumped onto the skateboard. (4)

2.2 After jumping onto the skateboard, the boy moves up an ramp. At what vertical height above the ground will the boy on the skateboard rise to before coming to a stop. Ignore all effects of friction. (4)

2.1 Let towards the skateboard be positive

$$\sum p_i = \sum p_f$$

$$m_{boy}v_{iboy} + m_{skateboard}v_{iskateboard} = (m_{boy} + m_{skateboard})v_f$$

$$(45)(+5) + (5)(0) = (45 + 5)v_f$$

$$v_f = 4,5\ m.s^{-1}$$

2.2 **Take the bottom of the ramp as the reference point**

E_M(at the bottom of the ramp) = E_M(at the top of the ramp)

$(E_k + E_p)_{\text{at the bottom of the ramp}} = (E_k + E_p)_{\text{at the top of the ramp}}$

$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mv^2 + mgh$

$\frac{1}{2}(50)(4,5)^2 + (50)(9,8)(0) = \frac{1}{2}(50)(0)^2 + (50)(9,8)h$

$h = 1,03$ m

PRO-TIPS

When applying the principle of conservation of mechanical energy, choose a **reference point**, from which the height is measured relative to.

ELASTIC AND INELASTIC COLLISIONS

All collisions can be classified as either **ELASTIC COLLISIONS** or **INELASTIC COLLISIONS**, depending on whether the kinetic energy of the system is conserved or not when the object's collide.

Inelastic collisions

When objects collide, they sometimes **deform** (change shape), make a **sound** or release **light** energy or **heat** energy during a collision. This results in some of the **kinetic** energy of the system being converted into other forms of energy, and therefore the total kinetic energy of the system before the collision, is not the same as after the collision. These collisions are known as inelastic collisions.

Definition: **Inelastic collision**: Type of collision where the kinetic energy is not conserved

Most collisions that occur in real - life are inelastic collisions, for example, when a hard objects collide.

Sum of the kinetic energy before the collision $(\sum E_{kf}) : \frac{1}{2} m_1 v_{i1}^2 + \frac{1}{2} m_2 v_{i2}^2$

Sum of the kinetic energy after the collision $(\sum E_{kf}) = \frac{1}{2} m_1 v_{f1}^2 + \frac{1}{2} m_2 v_{f2}^2$

Conclusion for an inelastic collision:

$$\sum E_{ki} \neq \sum E_{kf}$$

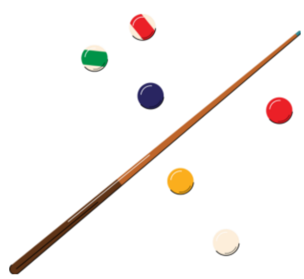
Elastic collisions

Definition: **Elastic collision**: Type of collision where the kinetic energy is conserved.

In an elastic collision, the sum of the kinetic energy before the collision = sum of the kinetic energy after the collision, such that the kinetic energy of the system is conserved:

$$\sum E_{ki} = \sum E_{kf}$$

Elastic collisions are very rare, as in real - life some of the kinetic energy of the system is usually converted into other forms of energy. One example of an elastic collision is the collisions between billiard balls when they are struck by the white ball.



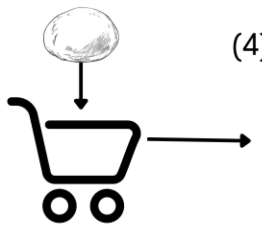
PRO-TIPS

In both elastic collisions and inelastic collisions, the **momentum** is **conserved**.

Worked example

C+A

1. A trolley of mass 5 kg is moving at 10 $m.s^{-1}$ right when a 2 kg piece of dough is dropped vertically onto the trolley and stays on it. The velocity of the trolley decreases to 8 $m.s^{-1}$. Determine whether the collision is an elastic or an inelastic collision. Ignore the effects of friction. (4)



1. **Tips when approaching elastic collision and inelastic collision questions:**

- Calculate the sum of the kinetic energy before the collision as a separate calculation from the sum of the kinetic energy after the collision.
- Only the magnitudes of the velocities are substituted into the kinetic energy formula, as it is a scalar quantity.

$\sum E_{ki} = \frac{1}{2} m_{trolley} v_{itrolley}^2 + \frac{1}{2} m_{dough} v_{idough}^2$

$\sum E_{ki} = \frac{1}{2}(5)(10)^2 + \frac{1}{2}(2)(0)^2$

$\sum E_{ki} = 250$ J

$\sum E_{kf} = \frac{1}{2} m_{trolley} v_{ftrolley}^2 + \frac{1}{2} m_{dough} v_{fdough}^2$

$\sum E_{kf} = \frac{1}{2}(5)(8)^2 + \frac{1}{2}(2)(8)^2$

$\sum E_{kf} = 224$ J

$\sum E_{kf} < \sum E_{ki}$

$\sum E_{ki} \neq \sum E_{kf}$ $\therefore$ Inelastic collision

PRO-TIPS

NOTE: The final kinetic energy is less than the initial kinetic energy of the system, because some of the kinetic energy of the system was converted into other forms of energy.

Worked example

C+A

2. A 0,3 kg cart, moving to the right on a frictionless linear air track at 4 $m.s^{-1}$ strikes a second cart of mass 0,5 kg, travelling in the opposite direction. The collision between the two carts is elastic. After the collision, the first cart is moving in the opposite direction at 4,75 $m.s^{-1}$ and the second cart moves at a velocity of 2,25 $m.s^{-1}$ right.
Calculate the velocity of the second cart before the collision, using two methods.

METHOD 1: ELASTIC COLLISION

$$\sum E_{ki} = \sum E_{kf}$$

$\frac{1}{2} m_1 v_{i1}^2 + \frac{1}{2} m_2 v_{i2}^2 = \frac{1}{2} m_1 v_{f1}^2 + \frac{1}{2} m_2 v_{f2}^2$

$\frac{1}{2}(0,3)(4)^2 + \frac{1}{2}(0,5)v_{i2}^2 = \frac{1}{2}(0,3)(4,75)^2 + \frac{1}{2}(0,5)(2,25)^2$

$v_{i2} = 3$ $m.s^{-1}$ left

- The type of collision is an elastic collision, therefore the sum of the kinetic energy before the collision is equal to the sum of the kinetic energy after the collision.
- This concept can be used to calculate the initial velocity of the second cart.

METHOD 2: PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

Let right be positive

$$\sum p_i = \sum p_f$$

$m_1 v_{i1} + m_2 v_{i2} = m_1 v_{f1} + m_2 v_{f2}$

$(0,3)(4) + (0,5)v_{i2} = (0,3)(-4,75) + (0,5)(2,25)$

$v_{i2} = -3$ $m.s^{-1}$

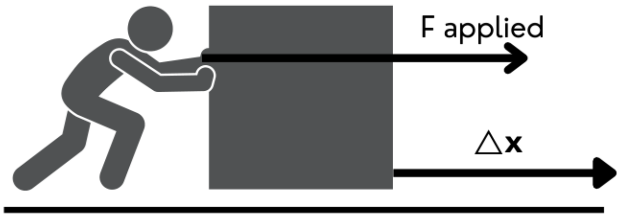
$v_{i2} = 3$ $m.s^{-1}$ left

- The system is an isolated system, where two objects collide, and only the internal colliding forces are factored in.
- Therefore, the principle of conservation of linear momentum can also be used to calculate the initial velocity of the second cart.

WORK, ENERGY AND POWER

The word ‘work’ is a common term used in every day language.
For example: “Grade 12 is a lot of work” or“I need to finish my Physics work”.
However, in Physics, the term ‘work’ has a definite meaning.

The example below explains the meaning of **work done** in Physics:



When a force, **F**, is exerted onto the object and the object undergoes a displacement, $\triangle$**x**, we say that **work is done by the force (W)** and when work is done, **energy is transferred** to or **energy is transferred from** the object. In this example, kinetic energy is transferred to the object.

Formula to calculate the work done by a force:

$$W = F\triangle x\cos\theta$$

PRO-TIPS

It is the forces that do work - forces transfer energy **to** or **from** a system.

<u>What do these variables mean and what are the SI units?</u>

W = Work done (by a force on object) in Joules (J)

F = MAGNITUDE of the force (acting on an object) in Newtons (N)

$\triangle$x= MAGNITUDE of the displacement (of the object) in metres (m)

NOTE: $\triangle$y can be used for an object being displaced vertically.

θ= angle between the force and the displacement.

Definition:

Work done: the work done on an object by a constant force F as F$\triangle$x cos θ , where F is the magnitude of the force, $\triangle$x the magnitude of the displacement and θ the angle between the force and the displacement.

PRO-TIPS

In the work done formula, Force and displacement are **VECTOR** quantities, however, only the **MAGNITUDE** is substituted into the work done formula. cos θ sorts out the direction and determines whether the work done is positive, negative or zero

POSITIVE, NEGATIVE AND ZERO WORK DONE

PRO-TIPS

Did you know?
Energy is defined as the ability to do work. Therefore there is a relationship between work done and energy.

Forces acting on an object (system) do the work, and therefore either:

- Transfer energy **TO** or **ADD** energy to the system (object)

OR

- Transfer energy **FROM** the system or **REMOVE** energy from the system(object)

OR

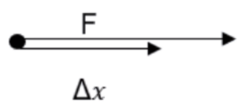
- Neither add nor remove energy from the system (i.e., do no work/ no work done).

Since work done is associated with energy, it is a SCALAR QUANTITY, therefore it has MAGNITUDE ONLY. However, the work done by a force can either be **POSITIVE, NEGATIVE** or **ZERO**, depending on whether it transfers energy TO (+) or FROM (-) the system (object) or NEITHER (0)!

COMMON WORK DONE SCENARIOS

<table>
<tr><th>SCENARIO 1</th><th>SCENARIO 2</th></tr>
<tr>
<td>

F and Δx are in the same direction
i.e. $\theta = 0°$



$W = F\Delta x\cos\theta$
$W = F\Delta x\cos 0°$ ($\cos 0° = 1$)
$W = +F\Delta x$

- Work done is **positive** and has a maximum value.
- Energy is transferred to the system (object). The object gains energy.

</td>
<td>

F and Δx are in opposite directions
i.e. $\theta = 180°$

$W = F\,\Delta x\cos\theta$
$W = F\Delta x\cos 180°$ ($\cos 180° = -1$)
$W = -F\Delta x$

- Work done is **negative** and has a minimum value.
- Energy is transferred from the system (object). The object converts some of the energy it has into other forms of energy.

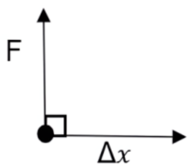
</td>
</tr>
</table>

Friction is a common example of a force that does **negative** work and **converts** some of the energy of the system into **heat**, **light** or **sound** energy!

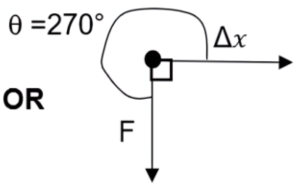
Common work done scenarios continued…

SCENARIO 3

F and Δx are perpendicular to one another
(θ = 90° or θ =270°)



OR



W = FΔxcosθ
W= FΔxcos90° (cos 90° = 0)
W = 0 J

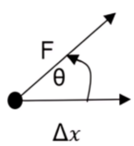
W = FΔxcosθ
W= FΔxcos90° (cos 270° = 0)
W = 0 J

- cos90° = 0 therefore the work done is 0 J
- No work is done by the force; therefore, no energy is transferred to or from the object by the force.

The normal force and gravitational force (weight) are common examples of forces that do no work, if the object is on a **horizontal surface**.

SCENARIO 4 (FORCES AT AN ANGLE)

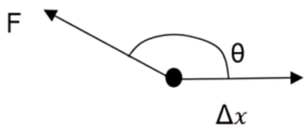
Angle between F and Δx lies between $0° < \theta < 90°$



- The horizontal component of the force is in the direction of the displacement.
- Work done is positive.
- Energy is transferred to the system (object). The object gains energy.

OR

Angle between F and Δx lies between $90° < \theta < 180°$



- The horizontal component of the force is in the **opposite** direction of the displacement.
- Work done is negative.
- Energy is transferred from (or removed from) the system (object).

Worked examples

C + A

Multiple choice questions

Which ONE of the following angles between the force and the displacement will result in the SMALLEST amount of energy being transferred to an object?

A 0°
B 10°
C 80°
D 180°

Answer: C

Energy transferred to the object means that **positive work** is done. Therefore, the angle between the force and the displacement must lie between 0° and 90° (but excluding 90°). For the smallest amount of energy to be transferred to the object, the angle between the force and the displacement must be the largest value closest to 90°, since a greater angle between the force and the displacement results in less energy being transferred to the object.

An object on a smooth horizontal surface is initially at rest. A constant force **F** is applied to the object. After **t** seconds, the displacement of the object is $\triangle$**x** and the work done on the object is **W.**
Which ONE of the following represents the work done on the object after 2**t** seconds?

A **W**
B 2**W**
C 3**W**
D 4**W**

Answer: D

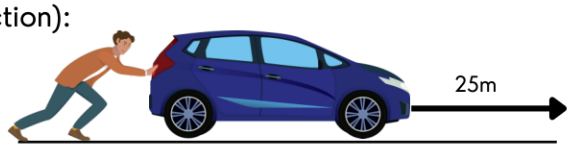
NOTE: The time that the constant net force acts on the object is doubled, therefore the velocity of the object increases, and the displacement of the object also increases, increasing the work done, the key goal of this question is to determine by what **factor** the work done increases.
There are two parts to solving this question:

1. After 2**t** seconds (double the time) what is the displacement of the object? Since force **F** is constant, the acceleration of the object is constant, therefore using an equation of motion that has vi, a and Δt in it, can be used to determine the effect on the displacement $\triangle$x:
From the equation of motion: $\Delta x = v_i \Delta t + \frac{1}{2}a\Delta t^2$, $v_i = 0$ m.s^{-1} , therefore $v_i \Delta t = 0$, acceleration, a, remains constant, therefore $\triangle x \propto \triangle t^2$:
$\triangle x \propto (2)^2$ $\therefore$ $4\triangle x$. The displacement of the object has increased by a factor of 4.

2. $W = F\Delta x\cos\theta$
$W \propto \Delta x$, since F and θ, the angle between the force and the displacement, remains constant $\therefore$ 4**W**

Worked example (C)

1. Peter tries to impress Bonnie with his new car, but the engine dies in the middle of the intersection. While Bonnie steers in a straight line, Steve pushes the car 25 m. If he pushes the car in the direction of motion with a constant force of 200 N (ignore the effects of friction):

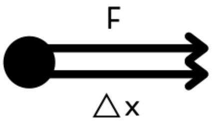


1.1 Calculate how much work Peter does on the car. (3)

Peter now exerts a force at an angle of 40° anti- clockwise to the direction of motion of the car, calculate:

1.2 The amount of work done by Peter. (3)

1.1 Draw a vector diagram showing the angle between the force and the displacement:



Note that the force and the displacement are in the SAME direction, therefore $\theta = 0°$

PRO-TIPS

In work, energy and power always draw a **vector diagram** assist you with determining the angle between the force and the displacement.

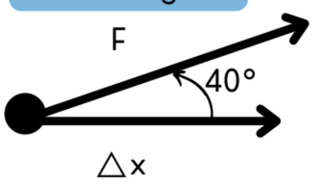
$$W = F\Delta x\cos\theta$$
$$W = (200)(25)\cos 0°$$
$$W = +5000\ \text{J}$$

NOTE: The work done by Peter on the car is positive work done, therefore, energy, in this example **kinetic energy** is added to the car. This makes sense because the car is **moving** as Peter pushes it with a force!

PRO-TIPS

Kinetic energy is the energy that an object has due to its motion.

Vector diagram:



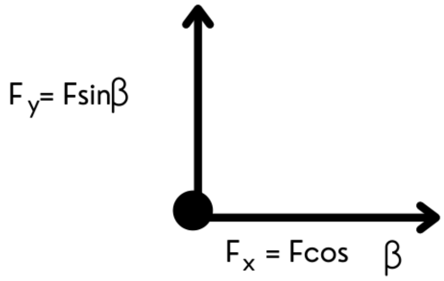
$$W = F\Delta x\cos\theta$$
$$W = (200)(25)\cos 40°$$
$$W = +3830{,}22\ \text{J}$$

NOTE: The work done by Peter on the car is still positive work done, however, less kinetic energy has been transferred to the car, because the angle between the force Peter exerts and the displacement of the car increased.

OPTION 2 (a less common, alternative method)

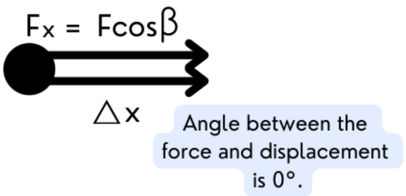
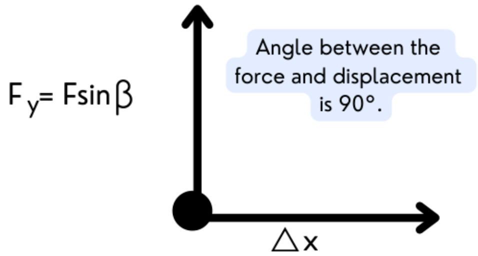
A force at an angle can be resolved into its horizontal or x - component and vertical or y - component.

The free - body diagram below represents force, **F**, resolved into its components:



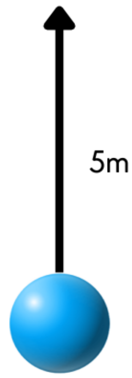
> **PRO-TIPS**
>
> Instead of using θ to represent the angle of the components, an alternative Greek letter e.g., β is used to avoid confusion.

The work done by each component of the force can then be calculated and added at the end to find the total work done by the force:

X - COMPONENT	Y- COMPONENT
 $W = F\Delta x\cos\theta$ $W = F\cos\beta\Delta x\cos\theta$ $W = (200)\cos40°(25)\cos0°$ $W = +3830,22$ J	 $W = F\Delta x\cos\theta$ $W = F\sin\beta\Delta x\cos\theta$ $W = (200)\sin40°(25)\cos90°$ $W = 0$ J
Total work done by the force: $W = +3830,22 + 0$ $W = +3830,22$ J	NOTE: No work is done by the y or vertical component of the force!

Worked example

2. A 200 g ball is thrown vertically upwards and reaches a height of 5 m. Ignore the effects of friction.

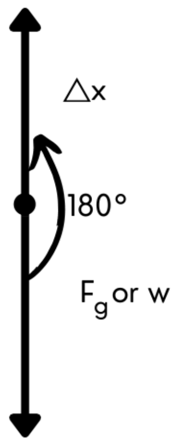


2.1 Calculate the work done by the gravitational force on the ball. (3)

2.2 Write down the energy conversion taking place. (2)

2.1 **NOTE:** The ball is in free - fall as it moves vertically upwards. This is because the only force acting on the ball is the gravitational force (or the force of gravity).

$$W = F\Delta x\cos\theta$$
$$W = F_g\Delta x\cos\theta$$
$$W = mg\Delta x\cos\theta$$
$$W = (0,2)(9,8)(5)\cos 180°$$
$$W = -9,8\ J$$



PRO-TIPS

NOTE:
- The gravitational force or weight always acts vertically downwards.
- To calculate the gravitational force or weight: $F_g = mg$ or $w = mg$
- SI units for mass: kg
- 1000 g = 1 kg

NOTE:
- The work done by the gravitational force or the weight is negative work done.
- This is because energy (more specifically kinetic energy in this example) is being "**removed**" from the ball, that is, it is being converted into gravitational potential energy.
- Work done is a scalar quantity, therefore, the negative answer for work done does **NOT** indicate a negative direction.
- The answer **must** therefore remain **negative**.

Data

m = 200 ÷1000
m = 0,2 kg

g = 9,8 $m.s^{-2}$

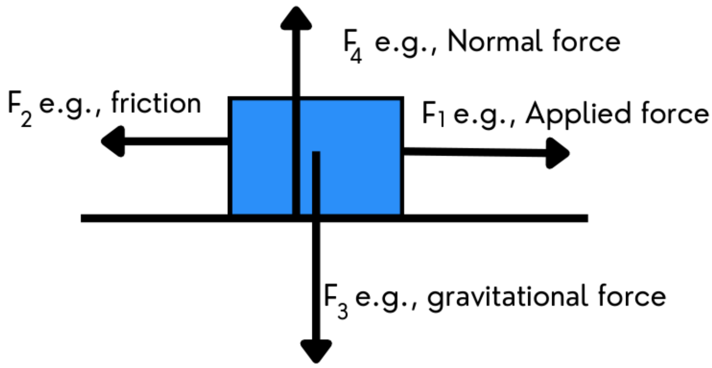
Δx = 5m

θ = 180°

NET WORK DONE

Recall from Mechanics the meaning of the word '**net**'. In Physics, the word net means **total** or **resultant**.

Therefore, in work, energy and power, the **net work done** is the total/ resultant work done by all the forces acting on the object; this is because there is usually more than one force acting on an object at any given time.



There are two methods to calculate the net work done by the forces:

METHOD 1: CALCULATING THE WORK DONE BY EACH FORCE	METHOD 2: CALCULATING THE NET WORK DONE BY CALCULATING THE NET FORCE
• Calculate the work done by each force. • The total (or net) work done is the algebraic sum of the work done by each force. • $W_{net} = W_1 + W_2 + \ldots$ Where $W_1 = F\Delta x\cos\theta$; $W_2 = F\Delta x\cos\theta$ **Note:** The symbols of the actual forces can be used and is recommended.	**$W_{net} = F_{net}\Delta x\cos\theta$** • Calculate the resultant or net force (Fnet) acting on the object. • Calculate the work done by using the equation $W_{net} = F_{net}\Delta x\cos\theta$ To calculate the net force acting on an object, it is the **vector** sum of the forces acting on the object. Remember to state your sign convention (positive direction). **NOTE**: Only the **magnitude** of the net force is substituted into the net work done formula.

PRO-TIPS

NOTE:

The net work done on an object can be positive, negative or zero.

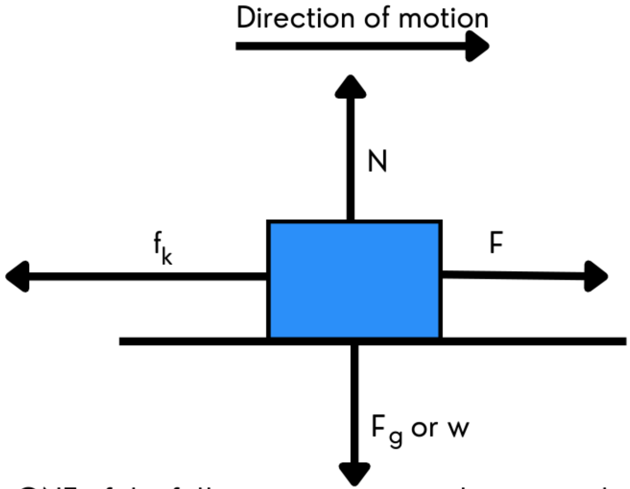
- **Positive net work done** on a system will Increase the energy of the system.
- **Negative net work done** on a system will decrease the energy of the system.
- **Zero net work done** on a system means that the energy of the system remains constant or the same.

Worked examples

Multiple choice question

C + A

The diagram below represents the forces acting on a box..



1.1 Which ONE of the following represents the net work done on the box?

A Positive
B Negative
C Zero
D Positive then zero

Answer: B

The direction of motion of the box is to the **right**. However, the direction of the net force is to the **left**, since the kinetic frictional force is greater than the applied force, F. (Note that the length of the arrow representing friction is longer than the length of the arrow representing the applied force, F).

From the formula, $W_{net} = F_{net}\Delta x\cos\theta$, $\theta = 180°$, therefore:

$W_{net} = F_{net}\Delta x\cos 180°$

$W_{net} = -F_{net}\Delta x$

Negative work is done on the box, and energy (kinetic energy) is transferred from the box (or removed from the box), causing the box to slow down, and its velocity to decrease uniformly.

NOTE:

- Did you notice that the vertical forces (Normal force and gravitational force) acting on the box are in **equilibrium**?
- Therefore, the net/ resultant vertical force is **zero**, and the net work done by the vertical forces is also zero.
- Calculating the work done by these individual forces: Since the angle between the **force** and the **displacement** is 90°, cos 90° = 0

Worked examples

Multiple choice question

A + P

Vuyo and Kalin run up a hill. Vuyo has a mass **m** and Kalin has a mass 2**m**.
Vuyo travels the same distance as Kalin, in half the time.
Which ONE of the following represents the net work done by Kalin and Vuyo?

	Net work done by Kalin	**Net work done by Vuyo**
A	$2\mathbf{W}_{\mathbf{net}}$	$\mathbf{W}_{\mathbf{net}}$
B	$\mathbf{W}_{\mathbf{net}}$	$2\mathbf{W}_{\mathbf{net}}$
C	$\mathbf{W}_{\mathbf{net}}$	$4\mathbf{W}_{\mathbf{net}}$
D	$4\mathbf{W}_{\mathbf{net}}$	$\mathbf{W}_{\mathbf{net}}$

Answer: B

In this complex question, there are many variables to consider. To calculate the net work done, the formula **Wnet = Fnet∆xcosθ** is used.

The following variables for Vuyo and Kalin in the same:

- Same displacement, Δx
- Same angle, θ, between the force and the displacement, namely θ = 0° therefore cos0° = 1

Therefore **Wnet** ∝ **Fnet**

Which variables affect the net force Vuyo and Kalin will experience:

1. Vuyo and Kalin do not have the same mass, this affects the net force, since Fnet = ma.
2. Vuyo and Kalin have the same initial velocity (we can assume they both started from rest), but not necessarily the same final velocity and hence not necessarily the same change in velocity).
3. The time taken is different for the same displacement.

Before comparing how mass affects net force, determine the relationship between time and acceleration, using an equation of motion that has a, **∆**t; as well as constant variables vi and **∆**x: $\Delta x = v_i\Delta t + \frac{1}{2}a\Delta t^2$, $v_i = 0\ m.s^{-1}$, therefore $v_i\Delta t = 0$,
Δx, displacement remains constant (taken as 1), therefore: $a \propto \frac{1}{\Delta t^2}$

Since Kalin completes the displacement in time, **t**, $a \propto \frac{1}{\Delta t^2}$
Vuyo completes the displacement in $\frac{1}{2}$**t:**

$$a \propto \frac{1}{\left(\frac{1}{2}\right)^2}$$

$$\therefore 4\mathbf{a}$$

The acceleration of Vuyo is 4 times the acceleration of Kalin. **HOWEVER**, it must not be forgotten that Kalin is DOUBLE the mass of Vuyo:

Vuyo	**Kalin**
$F_{net} = ma$	$F_{net} = ma$
$F_{net} \propto a$	$F_{net} \propto m$
$F_{net} \propto 4$	$F_{net} \propto 2$
$4\ F_{net}$	$2\ F_{net}$

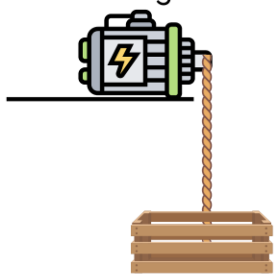
Conclusion: The net force acting on Vuyo is **DOUBLE** the net force acting on Kalin

Since $W_{net} \propto F_{net}$
The net work done on Vuyo
$W_{net} \propto 2$
$\therefore 2W_{net}$

The net work done on Vuyo is **DOUBLE** the net work done on Kalin.

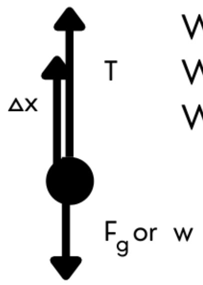
Worked example

1. An electric motor is used to raise a crate of mass 60 kg, through a vertical height of 5 m. The force of the motor is 700 N. Ignore the effects of air resistance and friction.



1.1 Calculate the work done by the rope on the crate. (3)
1.2 Calculate the work done by the gravitational force. (3)
1.3 Calculate the net work done on the crate. (3)

1.1 Draw a vector diagram representing the forces acting on the crate and the displacement.



$W_T = F\Delta x\cos\theta$
$W_T = (700)(5)\cos 0°$
$W_T = +3500$ J

1.1 In this question, the work done by the rope (on the crate) is due to the force of the motor, pulling the rope, and hence the crate, upwards. Therefore, the tension force in the rope is equal in magnitude to the force of the motor. The displacement of the crate is 5m.

NOTE:
- Assume the mass of the rope is negligible.
- The gravitational force can be calculated separately.
- The angle between the gravitational force (weight) and the displacement is 180°, since they are in opposite directions

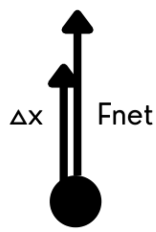
1.2 $W_{Fg} = F\Delta x\cos\theta$
$W_{Fg} = mg\Delta x\cos\theta$
$W_{Fg} = (60)(9,8)(5)\cos 180°$
$W_{Fg} = -2940$ J

OPTION 1
$W_{net} = W_T + W_{Fg}$
$W_{net} = (+3500) + (-2940)$
$W_{net} = +560$ J

OR

OPTION 2
Let upwards be positive
Fnet = T + (-mg)
Fnet = (700) - (60 x 9,8)
Fnet = 112 N upwards

$W_{net} = F_{net}\Delta x\cos\theta$
$W_{net} = (112)(5)\cos 0°$
$W_{net} = +560$ J



WORK - ENERGY THEOREM

Recall to date that:

- If the net **work** done on an object is **positive**, energy is **added** to the system.
- If the net **work** done on an object is **negative**, energy is **removed** to the system.
- If the net **work** done on an object is **zero**. energy is **neither** added nor removed from the system.

The energy added to or removed from the system is **kinetic energy**, which affects the motion of the object.

Definition:
The **work - energy theorem**: The net work done on an object is equal to the change in the object's kinetic energy,

In symbols: $\boxed{W_{net} = \triangle E_k}$ Formula on the data sheet

What do these variables mean and what are the SI units?
Wnet = net work done on an object in Joules (J).
$\triangle E_k$ = change in kinetic energy of the object in Joules (J).

This formula can be further expanded:

$$W_{net} = \triangle E_k$$

$$W_{net} = E_{kf} - E_{ki}$$

$$W_{net} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

PRO-TIPS

Remember to always write the original formula from the data sheet, before expanding the formula further.

Any of these expanded formulae or a combination of these expanded formulae can be used to do calculations (depending on the information given).

Conclusions from the work - energy theorem:

- If Wnet is positive then, Ekf > Eki because vf > vi, since the object is **speeding up**.
- If Wnet is negative then, Ekf < Eki because vf < vi since the object is **slowing down**.
- If Wnet is **ZERO** then, Ekf = Eki because vf = vi since the object is **moving** at a **constant** speed or a constant velocity.

Good to know:
Derivation of the work - energy theorem

Consider an object, mass m, moving a displacement Δx, and a constant force, F_{net} is acting on the object, changing the velocity from v_i to v_f, for this the following is true:

$W_{net} = F_{net}\Delta x\cos\theta$
$F_{net} = ma$

$W_{net} = ma\Delta x\cos\theta$ Assume the force and the displacement are in the same direction $\theta = 0°$

$W_{net} = ma\Delta x$ -------------1

$v_f^2 = v_i^2 + 2a\Delta x$

$a\Delta x = \frac{1}{2}(v_f^2 - v_i^2)$ -----------------2

substitute 2 into 1: $W_{net} = \frac{1}{2}m(v_f^2 - v_i^2)$

$W_{net} = \Delta E_k$ ----------------- work- energy theorem

Worked examples

Multiple choice question

C + A

Ball A and ball B of equal mass are dropped from different heights.
Ball A's final velocity is **v** and ball B's final velocity is 4**v**.
The final kinetic energy of ball A is **K.**

Which ONE of the following represents the final kinetic energy of ball B?

A **K**
B 2**K**
C 4**K**
D 16**K**

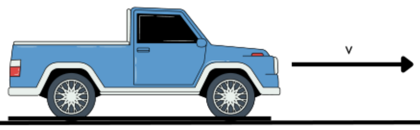
Answer: D
The final velocity of ball B is four times greater than the final velocity of ball A.
From the kinetic energy formula: $E_k = \frac{1}{2}mv^2$, the relationship between E_k and v:

$E_k \propto v^2$
$E_k \propto (4)^2$
$E_k \propto 16$
$\therefore 16E_k$
$\therefore 16$**K**

Worked example

C + A

A car of mass 800 kg is travelling in a straight line at an initial velocity of 120 $km.h^{-1}$.
The force of the engine is 20 000 N and the co - efficient of kinetic friction between the wheels of the car and the road is 0,3. Calculate the final speed of the car after it has travelled 40 m, using energy principles. (6)



"Using energy principles" is common wording seen in questions. It implies that only one of the following methods can be used to solve the question:

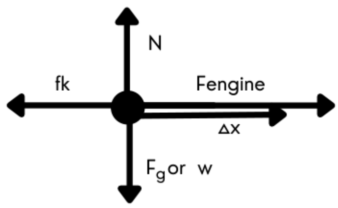
1. **The work - energy theorem** or/and
2. The **Principle of conservation of mechanical energy** (Remember this can only be applied in an isolated system where the net or resultant external force is zero!)

In this question, because it is not an isolated system (due to friction), the work - energy theorem ($W_{net}=\triangle E_k$) can be used to determine the final speed of the car.
Firstly, since the initial speed of the car is given and the final speed of the car is being determined in this question, the work - energy theorem formula can be expanded to:

vi = 120 ÷ 3,6
vi = 33,33 $m.s^{-1}$

$$\mathbf{W_{net} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2}$$

Draw a vector diagram showing all the forces acting on the car, and the displacement:



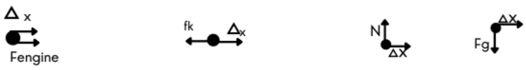
Before calculating the net work done, the magnitude of the kinetic frictional force must be determined;

$f_k = \mu_k N$
$f_k = \mu_k mg$
$f_k = (0,3)(800)(9,8)$
$f_k = 2352$ N

OPTION 1

OR

Wnet = ΔE_k
Wnet = $\frac{1}{2} mv_f^2 - \frac{1}{2}mv_i^2$



$W_{Fengine} + W_{fk} + W_N + W_{Fg} = \frac{1}{2} mv_f^2 - \frac{1}{2}mv_i^2$

$F\Delta x\cos\theta + F\Delta x\cos\theta + F\Delta x\cos\theta + F\Delta x\cos\theta = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2$

$(20000)(40)\cos 0° + (2352)(40)\cos 180° + (7840)(40)\cos 90° + (7840)(40)\cos 270° = \frac{1}{2}(800)v_f^2 - \frac{1}{2}(800)(33,33)^2$

$v_f^2 = 2875,68...$

$\sqrt{vf^2} = \sqrt{2875,68...}$

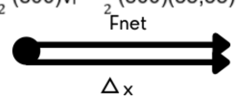
$v_f = 53,63\ m.s^{-1}$

OPTION 2

Let right be positive

$F_{net} = F_{engine} + (-f_k)$
$F_{net} = (20000) + (-2352)$
$F_{net} = 17648$ N right

$W_{net} = \Delta E_k$
$F_{net}\Delta x\cos\theta = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2$
$(17648)(40)\cos 0° = \frac{1}{2}(800)v_f^2 - \frac{1}{2}(800)(33,33)^2$
$v_f^2 = 2875,68...$
$\sqrt{vf^2} = \sqrt{2875,68...}$
$v_f = 53,63\ m.s^{-1}$



PRO-TIPS

Did you notice all the vector diagrams drawn for each calculation?

WORK – ENERGY THEOREM CONTINUED: CONSERVATIVE AND NON – CONSERVATIVE FORCES

In the context of work and energy, the word "conservative" gets its meaning from the conservation of **mechanical energy**.

Mechanical energy is the sum of the kinetic energy and the gravitational potential energy of an object.

In symbols:
$$E_M = E_k + E_p$$
$$E_M = \frac{1}{2}mv^2 + mgh$$

What is a conservative force?

Logically, a conservative force is a force that conserves the mechanical energy of the system and does not convert mechanical energy into other forms of energy such as heat, light or sound energy.

A common example of a conservative force is the **gravitational force** or **weight**.

To explain this, a scenario is shown below:

A ball of mass 1 kg is thrown vertically upwards. It reaches its maximum height of 1 m and free - falls vertically downwards to the same point as its starting point in motion. If we ignore friction, then the only force acting on the ball during its motion is the gravitational force.

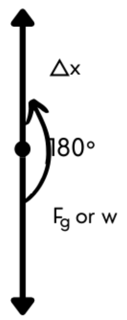
Work done by the gravitational force during the balls upwards motion:

$W = F\Delta x\cos\theta$

$W = mg\Delta x\cos\theta$

$W = (1)(9,8)(1)\cos180°$

$W = -9,8$ J



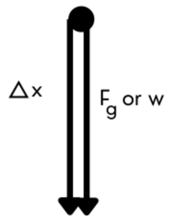
Work done by the gravitational force during the balls downwards motion:

$W = F\Delta x\cos\theta$

$W = mg\Delta x\cos\theta$

$W = (1)(9,8)(1)\cos0°$

$W = +9,8$ J



The net work done for the motion of the ball in a **closed path** can be determined.
A closed path is a path where the start and end point is at the same point.

W_{net} = -9,8 + (+9,8)
W_{net} = 0 J (for motion in a closed path)

Due to the net work done in a closed path being zero, this means that when the ball moved vertically upwards and reached its **highest** point, all of its **kinetic** energy was **converted** into **gravitational potential** energy and as the ball moved vertically **downwards** to the same point in motion as its started, all of its **gravitational potential energy** was converted into **kinetic** energy. This means that at any point during its motion upwards or downwards, the **sum** of the kinetic energy and gravitational potential energy is equal, and therefore conserved:

E_M (at a point) = E_M(at any other point)
$E_{k1} + E_{p1} = E_{k2} + E_{p2}$

Think about it: Would the net work done on the ball still be zero if it was **thrown higher**? YES! And if it was thrown lower? Absolutely. Therefore, the work done on the ball does **not** depend on the path taken. This can be used to formally define a conservative force.

PRO-TIPS

Stating and applying the principle of conservation of mechanical energy is still tested in Grade 12.
Can you remember what the principle of conservation of mechanical energy states?
The principal of conservation of mechanical energy states that the total mechanical energy in an isolated system remains constant.

Definition: Conservative force: Force for which the work done in moving an object between two points is independent of the path taken.

Other examples of conservative forces: **Electrostatic force** (between charges) and the **elastic force in a spring.**

Non - conservative forces

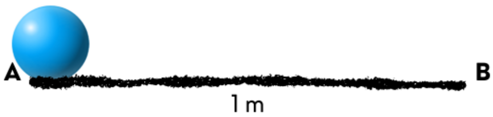
What are non - conservative forces?

Logically, a non - conservative force is force that **does not** conserve the **mechanical** energy of the system, and converts the mechanical energy of the system into other forms of energy such as heat, light or sound energy.

A common example of a non - conservative force is **friction** (frictional force).

To explain this, a scenario is shown below:

A ball of mass 1 kg is rolled along a rough surface. It starts at point **A** and ends at point **B**, where it covers a distance of 1 m.
The ball then moves from point **A** to point **B**, forming a closed path. A frictional force of 1 N acts on the ball during its motion in a closed path.



Work done by the frictional force during the balls motion from point A to B:

$W = F\Delta x\cos\theta$
$W = f\Delta x\cos\theta$
$W = (1)(1)\cos 180°$
$W = -1$ J

Work done by the frictional force during the balls motion from point B to A:

$W = F\Delta x\cos\theta$
$W = f\Delta x\cos\theta$
$W = (1)(1)\cos 180°$
$W = -1$ J

Note :
that same amount of negative work is done as the ball moves from point A to point B, as when it moves from point B, back to A in a closed path. The net work done by the frictional force for the motion in a closed path can be calculated:

$W_{net} = -1 + (-1)$
$W_{net} = -2$ J **(for motion in a closed path)**

Note:
that the net work done by the frictional force for motion in a closed path is NOT zero! 2 J of negative work has been done. What does this mean? 2 J of the mechanical energy of the system has been converted into other forms of energy (that is not mechanical energy), such as heat energy. The total energy of the system is still conserved, but the **mechanical energy** is not conserved, therefore, friction is a non - conservative force.

Think about it: Would the net work done by friction change if the length of the path was longer? **YES**, it would **INCREASE** the amount of negative work done by friction, since the displacement over which the frictional force acts, increases. And if the path length was shorter? It would **DECREASE** the amount of negative work done. Therefore, the work done on the ball **does** depend on the path taken. This can be used to formally define a non - conservative force.

Definition: Non - conservative force: Force for which the work done in moving an object between two points is dependents on the path taken.

Other examples of non - conservative forces: Applied force, tension (in a rope, string or chord), air resistance (or air friction), normal force.

Calculating the work done by non - conservative forces

You learned that non - conservative forces change the mechanical energy of the system. This can be represented in an equation as:

$$W_{nc} = \triangle E_M$$

$$\boxed{W_{nc} = \triangle E_k + \triangle E_p}$$

Formula on the data sheet.
This equation is the **second version of the work - energy theorem.**

What do these variables means and what are the SI units?

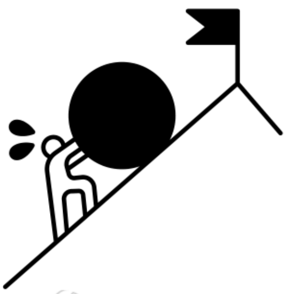
W_{nc} = net (total) work done by all the non - conservative forces acting on an object in Joules (J).
$\triangle E_k$ = change in kinetic energy in Joules (J).
$\triangle E_p$ = change in gravitational potential energy in Joules (J).

The above equation is second version of the work - energy theorem. In words:
The work done by all non - conservative forces equals a change in the total mechanical energy of the system.

If no non - conservative forces are acting on an object then $W_{nc} = 0$ J and the equation becomes:

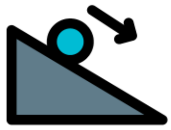
$$W_{nc} = \Delta E_k + \Delta E_p$$
$$0 = E_{kf} - E_{ki} + E_{pf} - E_{pi}$$
$$E_{ki} + E_{pi} = E_{kf} + E_{pf}$$

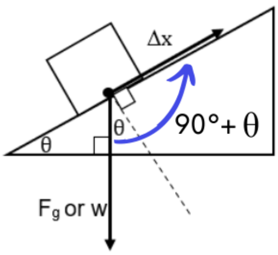
Note: that the mechanical energy of the system is conserved when non - conservative forces are not present.

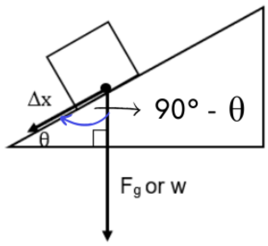


INCLINED PLANES: CALCULATING THE WORK DONE BY THE GRAVITATIONAL FORCE

- When an object is on an inclined plane, there are **two common methods** to calculate the work done by the gravitational force, or weight.
- Remember that the gravitational force is a conservative force (that is, it does **not** change the mechanical energy of the system), however, it still does **work** by **converting** one form of mechanical energy (e.g., kinetic energy) into another form of mechanical energy (e.g., gravitational potential energy) and vice versa.



<table>
<tr><th colspan="2">METHOD 1 (RECOMMENDED METHOD):
CALCULATING THE WORK DONE BY WEIGHT WHEN THE ANGLE OF THE INCLINE IS KNOWN OR CAN BE CALCULATED</th></tr>
<tr><td>Object is moving up the inclined plane</td><td>Object is moving down the inclined plane</td></tr>
<tr><td colspan="2">Key goal in method 1: Using the angle of the incline to find the angle between the gravitational force or weight and the displacement</td></tr>
<tr><td>

$W_{Fg} = F\Delta x\cos\theta$
$W_{Fg} = mg\Delta x\cos(90° + \theta)$</td><td>

$W_{Fg} = F\Delta x\cos\theta$
$W_{Fg} = mg\Delta x\cos(90° - \theta)$</td></tr>
</table>

PRO-TIPS

Trigonometry can be used to determine the angle of the incline if the lengths of TWO sides of the incline, which is a right - angled triangle, is known Remember the trigonometric ratios: **SOH. CAH, TOA**

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

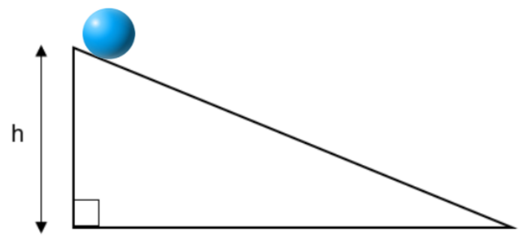
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

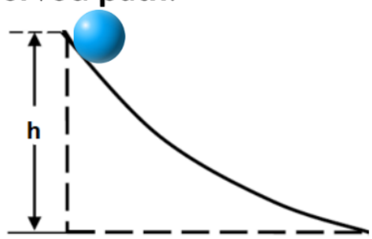
PRO-TIPS

The work done by the gravitational force or weight can also be determined using the **components of the gravitational force or the components of weight**. However, this method is less commonly used.

METHOD 2: CALCULATING THE WORK DONE BY THE GRAVITATIONAL FORCE OR WEIGHT USING THE VERTICAL HEIGHT OF THE INCLINE OR CURVED PATH.

NOTE: This method is commonly used to calculate the work done by the gravitational force if the angle of the incline is not known or the path is a **curved path**.





The formula used is NOT on the data sheet, and therefore must be derived if used, using other formulae on the data sheet:

From the work – energy theorem: $W_{net} = \Delta E_k$

$W_{\text{conservative forces}} + W_{\text{non – conservative forces}} = \Delta E_k$

$W_c + W_{nc} = \Delta E_k$

From the second version of the work – energy theorem ($W_{nc} = \Delta E_k + \Delta E_p$):

$W_c + W_{nc} = \Delta E_k$

$W_c + \Delta E_k + \Delta E_p = \Delta E_k$

$\mathbf{W_c = -\Delta E_p}$

$W_c = -(mgh_f – mgh_i)$

$W_c = -mg\,(h_f – h_i)$

PRO-TIPS

- It is possible to use **ANY** of these methods to calculate the work done by the gravitational force or the weight.
- However, **method 1**, using the work done formula is much simpler and is the most used and recommended method.

Worked examples

Multiple choice question

C

1. Which ONE of the following forces is an example of a non - conservative force?
   A Tension
   B Gravitational force
   C Electrostatic force
   D Elastic force in a spring

PRO-TIPS

- Being able to **identify** which forces are conservative forces and non - conservative forces is a **common** question.
- Know a few examples of both conservative and non - conservative forces.

Answer: A

The **gravitational** force, **electrostatic** force and **elastic** force in a spring are all examples of **conservative** forces. **Tension** is the **ONLY** non - conservative force that depends on the path taken, and **DOES NOT** conserve the mechanical energy of the system.

Multiple choice questions

C + A

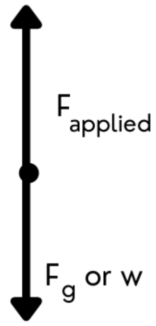
2. A girl lifts a package vertically upwards by applying a constant force, **F**, to it of magnitude greater than the weight of the package. The work done by force, **F**, on the package equals the gain in ... of the package. Ignore the effects of friction.

A gravitational potential energy
B gravitational potential energy plus the gain in kinetic energy
C kinetic energy
D gravitational potential energy minus the gain the kinetic energy

Answer: B

- In this question, we are dealing with two types of energy - **kinetic** energy and **gravitational** potential energy.
- To determine the packages kinetic energy the **type of motion** of the object must be determined i.e., is the velocity of the object changing or not, and if so, is it increasing or decreasing?
- Since the package is moving upwards, its **height above** the reference point increases, therefore, its **gravitational** potential energy must **increase**. **HOWEVER**, it needs to be determined if the kinetic energy of the package is changing:

1. Determine the forces acting on the ball as the girl lifts it vertically upwards, and draw a free - body diagram:



PRO-TIPS

From Newton's second law of motion:
$\mathbf{F_{net} = ma}$

The direction of the acceleration of the object is in the same direction as the net force.

2. Since the applied force is greater than the gravitational force or weight of the package, there is an **upwards net or resultant force** acting on the package, therefore the package will accelerate vertically upwards.
Since the direction of the motion and displacement of the package is also upwards, in the same direction as the net force and the acceleration, the velocity of the package will increase, therefore the kinetic energy of the package will increase.

3. The package is also moving vertically upwards and its height above the reference point increases, therefore the potential energy of the package also increases.

In summary, the applied force, **F**, is a non - conservative force which changes the mechanical energy of the system, and in this example **increases** both the kinetic energy and the gravitational potential energy of the package.

3. An object moves in a straight line on a ROUGH horizontal surface. If the net work done on the object is zero, then…

A the object has zero kinetic energy.
B the object moves at constant speed.
C the object moves at constant acceleration.
D there is no frictional force acting on the object

Answer: B

- The object is **MOVING** on a **ROUGH** surface, therefore, there is friction acting on the object, and the object has a certain velocity because it is in motion, therefore, it has kinetic energy.
- If the net work done is ZERO, from the work - energy theorem:

$W_{net} = \Delta E_k$
$W_{net} = E_{kf} - E_{ki}$

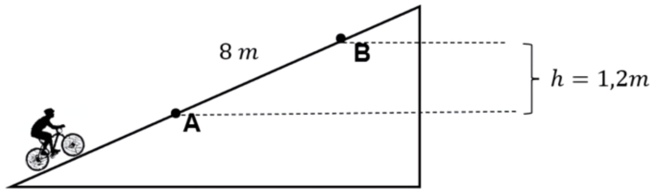
For W_{net} = 0 J, then $E_{kf} = E_{ki}$

Therefore $v_f = v_i$ and the object is moving at a constant speed/velocity and the forces acting on the object are in equilibrium.

Worked example

C + A

1. A cyclist is peddling up a slope and when she reaches point **A**, her kinetic energy is equal to 2750 J. She stops pedaling at point **A** and free – wheels further up the slope. The distance between **A** and **B** is 8 m and point **B** is 1,2 m higher than point **A**. While free-wheeling up the ramp she experiences a frictional force of 18 N. The total mass of the cyclist and her bicycle is 55 kg.



1.1 State the work - energy theorem in words. (2)
1.2 Calculate the velocity of the cyclist at point **A**. (3)
1.3 Calculate the cyclist's kinetic energy at point **B,** using energy principles. (5)

1.1 The work - energy theorem states that the net work done on an object is equal to the change in the object's kinetic energy.

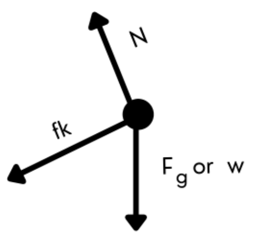
1.2 The cyclist's kinetic energy at point **A** is known (2750 J). Using the kinetic energy formula, the velocity of the cyclist at point **A** can be determined.
Remember that velocity is a vector quantity, therefore the direction must be included in the answer.

$$E_{kA} = \frac{1}{2}mv^2$$
$$E_{kA} = \frac{1}{2}mv^2$$
$$(2750) = \frac{1}{2}(55)v^2$$
$$v^2 = 100$$
$$\sqrt{v^2} = \sqrt{100}$$
$v = 10\ m.s^{-1}$ up the slope / up the incline

1.3 To determine the cyclist's kinetic energy at point B, the velocity at point **B** needs to first be determined. Since energy principles must be used, only one of the two versions of the work - energy theorem can be used.

What cannot be used to calculate the velocity at point **B**?
1. Equations of motion (this is not energy principles).
2. The Principle of conservation of mechanical energy since there is friction, a non - conservative force, acting on the bicycle.

Vector diagram showing the forces acting on the cyclist and the displacement from point **A** to point **B**:

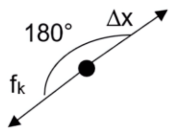


Remember that the cyclist is free - wheeling from point **A** to point **B**, therefore no force is being applied by the cyclist.

Option 1: Using the second version of the work - energy theorem by calculating the work done by the non - conservative forces:

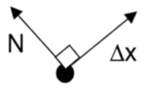
Identify the non – conservative forces acting on the cyclist:

1. **Frictional force**



Angle between the frictional force and the displacement = 180°

2. **Normal force**



Angle between the normal force and the displacement = 90°
cos90° = 0
No work done by the normal force (0 J)

Take point **A** as the reference point.

$W_{nc} = \Delta E_k + \Delta E_p$

$W_{nc} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgh_f - mgh_i$

$W_f + W_N = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgh_f - mgh_i$

$F\Delta x\cos\theta + F\Delta x\cos\theta = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgh_f - mgh_i$

$(18)(8)\cos180° + 0 = \frac{1}{2}(55)v_f^2 - \frac{1}{2}(55)(10)^2 + (55)(9,8)(1,2) - (55)(9,8)(0)$

$1959,20 = 27,5v_f^2$

$v_f^2 = 71,2436...$

$\sqrt{vf^2} = \sqrt{71,2436...}$

$v_f = 8,44 \text{ m.s}^{-1}$

$E_{kB} = \frac{1}{2}mv^2$

$E_{kB} = \frac{1}{2}(55)(8,44)^2$

$E_{kB} = 1959,20 \text{ J}$

PRO-TIPS

When calculating the gravitational potential energy of an object, always choose a reference point (if not stated) or use the reference point given.

OR

Option 2: Using the work - energy theorem by calculating the net work done by all the forces (conservative and non - conservative forces).

Work done by the gravitational force

Method 1

Take point A as the reference point

From the work – energy theorem: $W_{net} = \Delta E_k$

$W_c + W_{nc} = \Delta E_k$

From the second version of the work – energy theorem ($W_{nc} = \Delta E_k + \Delta E_p$):

$W_c + W_{nc} = \Delta E_k$

$W_c + \Delta E_k + \Delta E_p = \Delta E_k$

$\boldsymbol{W_c = -\Delta E_p}$

$W_c = -(mgh_f - mgh_i)$

$W_c = -mg\,(h_f - h_i)$

$W_c = -(55)(9,8)(1,2 - 0)$

$W_c = -646,80\ J$

PRO-TIPS

When doing separate calculations, do not round off until the **FINAL** answer.

Method 2

Calculate the angle of the incline:

$\sin\theta = \frac{O}{H}$

$\sin\theta = \frac{(1,2)}{(8)}$

$\theta = 8,626...°$

Angle of the incline = 8,626...°

Angle between the gravitational force (or weight) and the displacement = 90°+ 8,626...°
= 98,626...°

$W_{Fg} = F\Delta x\cos\theta$

$W_{Fg} = mg\Delta x\cos\theta$

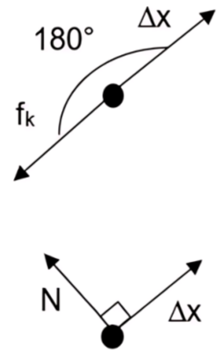
$W_{Fg} = (55)(9,8)(8)\cos 98,626...°$

$W_{Fg} = -646,80\ J$

$W_{net} = \Delta E_k$

$W_{Fg} + W_f + W_N = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

Vector diagrams



$W_{Fg} + F\Delta x\cos\theta + F\Delta x\cos\theta = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$(-646,80) + (18)(8)\cos 180° + 0 = \frac{1}{2}(55)v_f^2 - \frac{1}{2}(55)(10)^2$

$1959,20 = 27,5v_f^2$

$v_f^2 = 71,2436...$

$\sqrt{vf^2} = \sqrt{71,2436...}$

$v_f = 8,44\ m.s^{-1}$

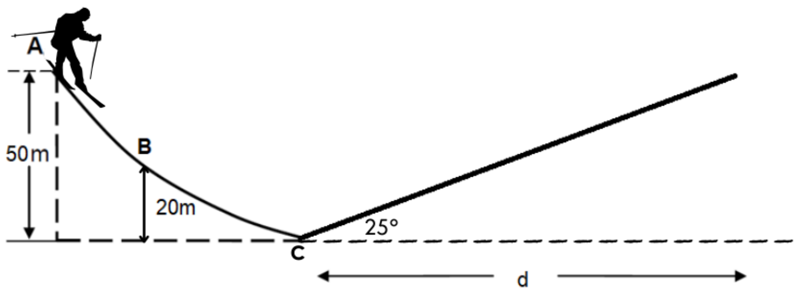
$E_{kB} = \frac{1}{2}mv^2$

$E_{kB} = \frac{1}{2}(55)(8,44)^2$

$E_{kB} = 1959,20\ J$

Worked example

2. A skier with skies of total mass 70 kg start from rest at point **A**. The down - slope is curved and the up - slope is an inclined plane. The curved down slope is frictionless, but the inclined plane is rough. Take the ground as the reference point.



2.1 Describe the energy changes that take place as the skier moves from point **A** to point **C**. (2)

2.2 Calculate the potential energy of the skier at point **B**. (3)

2.3 Calculate the speed of the skier at point **C**. (4)

The skier experiences a frictional force of 150 N as he moves up the inclined plane.

2.4 Calculate the maximum horizontal distance, d, that the skier reaches, from point **C.** (7)

NOTE:
At point **A**, the initial velocity of the skier is zero, therefore the kinetic energy is zero. However, the skier has gravitational potential energy at point **A**, as the skier is the maximum height above the reference point (ground) at point **A**. The system is an isolated system as it is a frictionless surface, therefore only the gravitational force (a conservative force) does work as the skier moves from point **A** to point **C**.

2.1 Gravitational potential energy is converted into kinetic energy.

2.2 The height above the reference point, h, is known at point **B**. (h = 20 m)

$E_p = mgh$

$E_p = (70)(9{,}8)(20)$

$E_p = 13720$ J

2.3 **OPTION 1:** Using the principle of conservation of mechanical energy.
The system is an isolated system, therefore the mechanical energy of the system is conserved.

$E_M(A) = E_M(C)$

$(E_k + E_p)_A = (E_k + E_p)_C$

$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mv^2 + mgh$

$\frac{1}{2}(70)(0)^2 + (70)(9{,}8)(50) = \frac{1}{2}(70)v^2 + (70)(9{,}8)(0)$

$v^2 = 980$

$\sqrt{v^2} = \sqrt{980}$

$v = 31{,}30\ m.s^{-1}$

Speed at point C = $31{,}30\ m.s^{-1}$

OR

OPTION 2: Using the work - energy theorem.
Due to it being a curved surface, the formula to calculate the work done by the gravitational force must be derived.

Work done by the gravitational force or weight

From the work – energy theorem: $W_{net} = \Delta E_k$

$W_c + W_{nc} = \Delta E_k$

From the second version of the work – energy theorem ($W_{nc} = \Delta E_k + \Delta E_p$):

$W_c + W_{nc} = \Delta E_k$

$W_c + \Delta E_k + \Delta E_p = \Delta E_k$

$\mathbf{W_c = -\Delta E_p}$

$W_c = -(mgh_f – mgh_i)$

$W_c = -mg\ (h_f – h_i)$

$W_c = -(70)(9,8)(0 – 50)$

$W_c = 34300$ J

$W_{net} = \Delta E_k$

$W_{Fg} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$34300= \frac{1}{2}(70)v_f^2 - \frac{1}{2}(70)(0)^2$

$34300 = 35v_f^2$

$v_f^2 = 980$

$\sqrt{vf^2} = \sqrt{980}$

$v_f = 31,30$ m.s^{-1}

Speed at point C = 31,30 m.s^{-1}

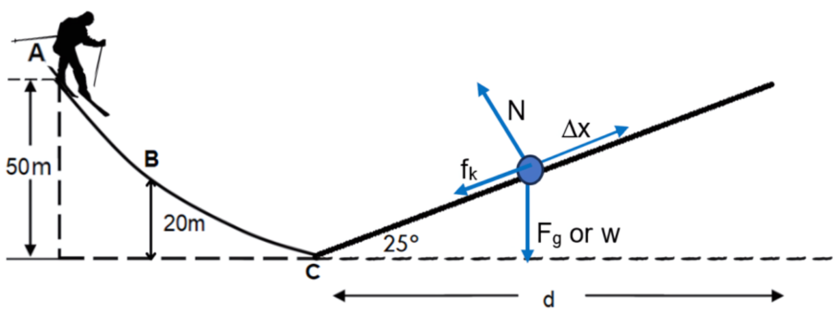
2.4

NOTE:

- To determine the maximum horizontal distance from **C**, the maximum displacement from point **C** needs to first be determined - this is at a **point on the rough incline where the final velocity is zero**. The velocity at point **C** was calculated in Question 2.3 as 31,30 m.s^{-1}
- The work - energy theorem $W_{net} = \Delta E_k$, can be used to calculate the unknown maximum displacement.
  The second version of the work - energy theorem can be used to calculate the maximum displacement, however, this requires some additional mathematical application.

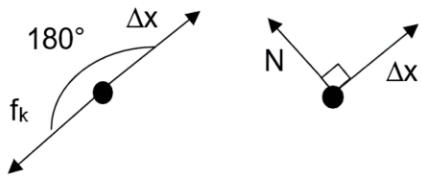
OPTION 1: Using the work - energy theorem

Identify the forces acting on the skier and the direction of the displacement as he moves up the rough inclined plane:



Angle between the gravitational force (or weight) and the displacement = 90°+ 25°
= 115°

Vector diagrams



$W_{net} = \Delta E_k$

$W_{F_g} + W_f + W_N = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

$F\Delta x\cos\theta + F\Delta x\cos\theta + F\Delta x\cos\theta = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

$mg\Delta x\cos\theta + f\Delta x\cos\theta + 0 = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

$(70)(9{,}8)\Delta x\cos 115° + (150)\Delta x\cos 180° + 0 = \frac{1}{2}(70)(0)^2 - \frac{1}{2}(70)(31{,}30)^2$

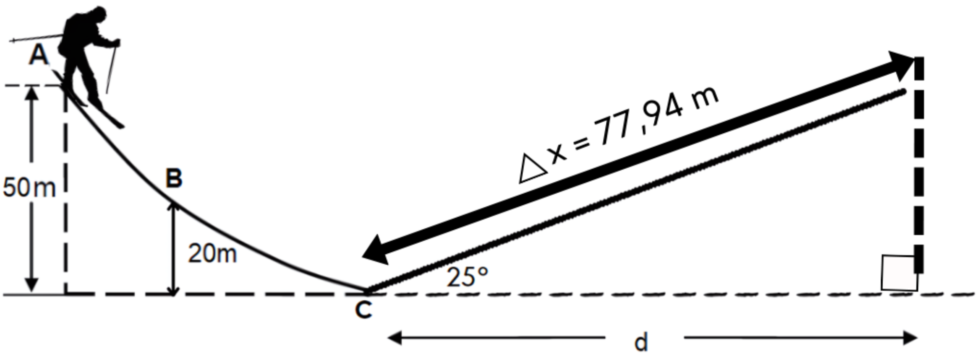
$-439{,}9161...\Delta x = -34289{,}15$

$\Delta x = 77{,}94$ m

∴Maximum displacement = 77, 94 m

Use of trigonometry to determine the maximum horizontal distance,d, reached:

Treat the inclined plane as a right - angled triangle:



- The angle of the incline is known (25°)
- The hypotenuse is known (77,94 m)
- Relative to the angle of the incline, the length of the adjacent side needs to be determined, this is the maximum horizontal distance reached.
- cosθ is the trigonometric ratio used to calculate the maximum horizontal distance, d, reached.

$\cos\theta = \frac{A}{H}$

$\cos 25° = \frac{d}{(77{,}94)}$

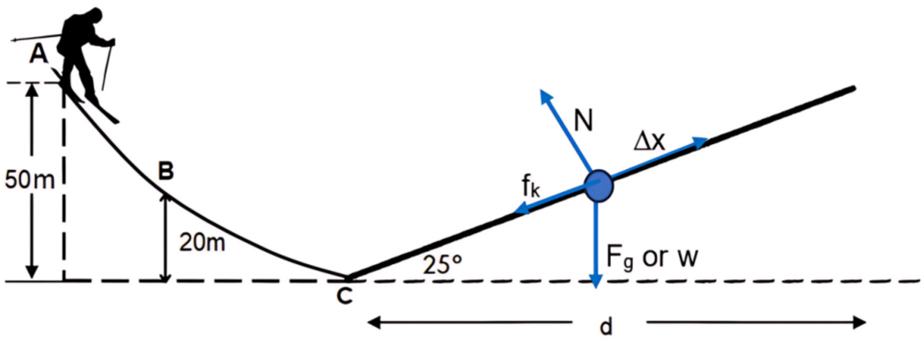
$d = (\cos 25°)(77{,}94)$

$d = 70{,}64$ m

OR

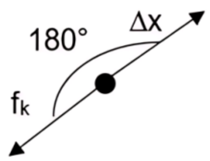
OPTION 2: Using the second version of the work - energy theorem.
NOTE: This method is more mathematical and requires the application of simultaneous equations.

$$W_{nc} = \Delta E_k + \Delta E_p$$



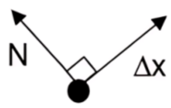
Identify the non – conservative forces acting on the cyclist:

1. **Frictional force**



Angle between the frictional force and the displacement = 180°

2. **Normal force**



Angle between the normal force and the displacement = 90°
cos90° = 0
No work done by the normal force (0 J)

Take point **C** as the reference point.

$W_{nc} = \Delta E_k + \Delta E_p$

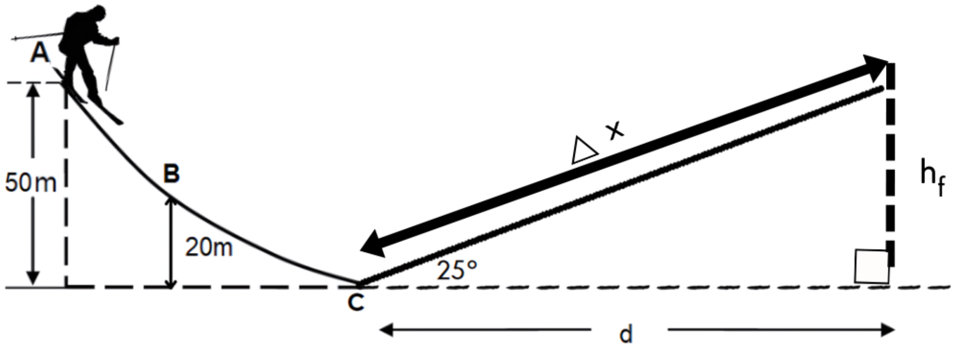
$W_{nc} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgh_f - mgh_i$

$W_f + W_N = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgh_f - mgh_i$

$F\Delta x\cos\theta + F\Delta x\cos\theta = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgh_f - mgh_i$

$(150)\mathbf{\Delta x}\cos180° + 0 = \frac{1}{2}(70)(0)^2 - \frac{1}{2}(70)(31{,}30)^2 + (55)(9{,}8)\mathbf{h_f} - (55)(9{,}8)(0)$ ------ 1

Getting h_f (final height) in terms of Δx:



$\sin 25° = \frac{O}{H}$

$\sin 25° = \frac{h_f}{\Delta x}$

$h_f = \sin 25° \Delta x$ ---------------- 2

Substitute equation 2 into 1:

$(150)\mathbf{\Delta x}\cos 180° + 0 = \frac{1}{2}(70)(0)^2 - \frac{1}{2}(70)(31,30)^2 + (55)(9,8)\mathbf{h_f} - (55)(9,8)(0)$

$(150)\mathbf{\Delta x}\cos 180° + 0 = \frac{1}{2}(70)(0)^2 - \frac{1}{2}(70)(31,30)^2 + (70)(9,8)\mathbf{\sin 25° \Delta x} - (70)(9,8)(0)$

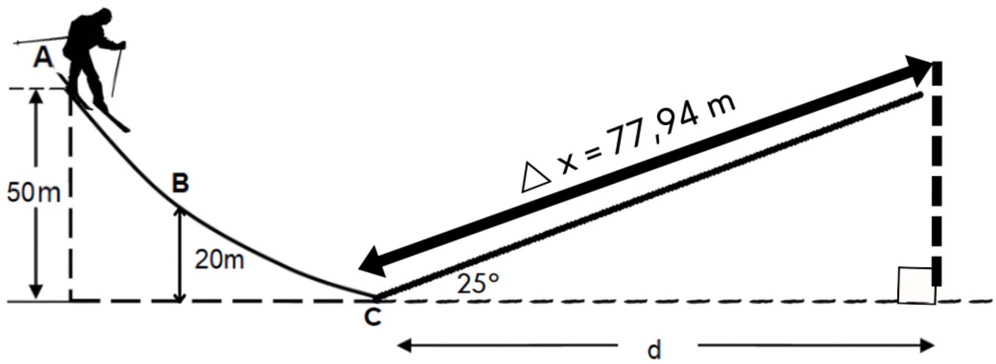
$-439,9161276\Delta x = -34289,15$

$\Delta x = 77,94$ m

∴Maximum displacement = 77, 94 m

Use of trigonometry to determine the maximum horizontal distance,d, reached:

Treat the inclined plane as a right - angled triangle:



- The angle of the incline is known (25°)
- The hypotenuse is known (77,94 m)
- Relative to the angle of the incline, the length of the adjacent side needs to be determined, this is the maximum horizontal distance reached.
- $\cos\theta$ is the trigonometric ratio used to calculate the maximum horizontal distance, d, reached.

$\cos\theta = \frac{A}{H}$

$\cos 25° = \frac{d}{(77,94)}$

$d = (\cos 25°)(77,94)$

$d = 70,64$ m

AVERAGE POWER

It is not only important to know how much work is done, but also the rate at which work is done or energy transferred, that is, the amount of work done or energy transferred per unit of time, i.e., per second.

Did you know?
The units of power, Watts, was named after James Watt, a scientist that discovered steam engines in trains.

The symbol for power: **P**
SI units for power: **Watts (W)**
NOTE: 1000 W = 1 kW (1 kilowatt)

Definition: Power: Power is defined as the rate at which work is done.

From the definition, the formula for power in symbols:

$$\boxed{P = \frac{W}{\Delta t}}$$

PRO-TIPS

Power is a **scalar quantity**. Therefore, it has magnitude only and is not associated with direction.

What do these variables mean and what are the SI units?
P = Power (average power) is measure in Watts (W)
W = Work done (energy transferred) in Joules (J)
Δt= time taken to do the work in seconds (s)

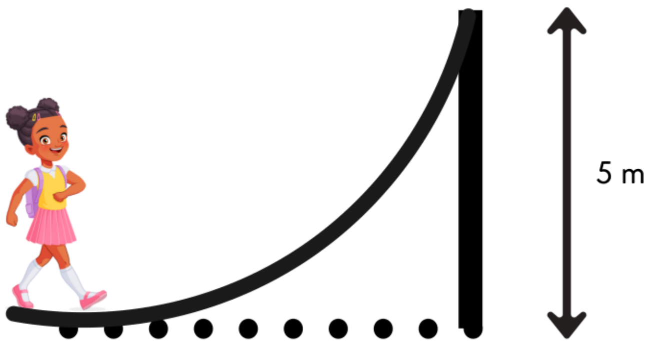
NOTE: 1 watt is 1 Joule per second.

$$1\ W = \frac{1J}{1s} = 1\ J.s^{-1}$$

Worked example

C + A

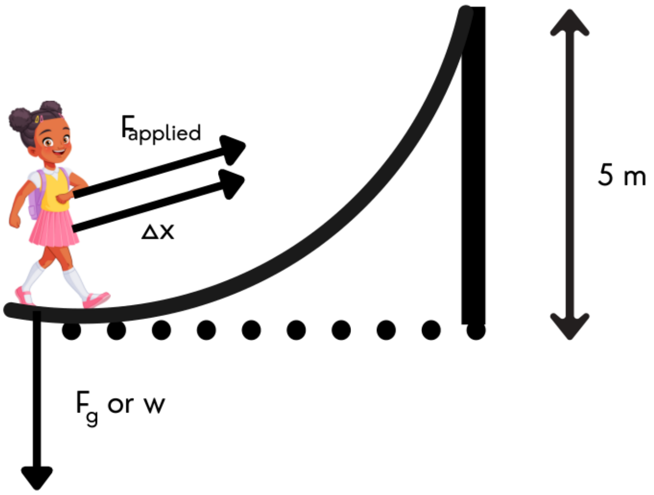
1. Sarah investigates the amount of work done in walking from rest up a 5 m high frictionless curved bridge.
   Sarah has a mass of 50 kg. It takes her 7 seconds to reach the top of the bridge. Her speed at the top of the bridge is 4 m.s^{-1}.



1.1 Calculate the work done by Sarah when she climbs to the top of the bridge. (4)
1.2 Calculate Sarah's power output. (3)

1.1 **NOTE:** Sarah exerts a constant applied force to get herself to move up the bridge.

Identify the **constant** forces acting on Sarah and her displacement as she moves up the bridge:



The work done by the applied force Sarah exerts will change the mechanical energy of the system.
The applied force Sarah exerts is a non - conservative force.

Take the bottom of the bridge as the reference point.

$W_{nc} = \Delta E_k + \Delta E_p$

$W_{nc} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgh_f - mgh_i$

$W_F = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgh_f - mgh_i$

$W_F = \frac{1}{2}(50)(4)^2 - \frac{1}{2}(50)(0)^2 + (50)(9,8)(5) - (50)(9,8)(0)$

$W_F = 2850 \text{ J}$

Sarah's work done to walk up the bridge was calculated in Question 2.1 (2850 J) and the time taken to do that amount of work (7s). This can be used to calculate her power output, that is, the amount of work done or energy transferred per second.

$P = \frac{W}{\Delta t}$

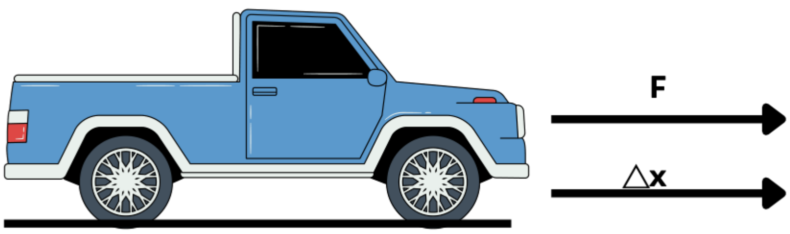
$P = \frac{(2850)}{(7)}$

$P = 407,14 \text{ W}$

AVERAGE POWER CONTINUED

There is a **second formula** that can be used to calculate the **average power** of an object.

Consider a situation where a constant force, **F**, is acting on an object, causing the object to move in a straight line. Assume that the force causes the object to undergo a displacement Δx during a time interval Δt.



For this situation the following is true:
Work done by force, **F**, on object: $W = F\Delta x\cos\theta$

The average power of the force can be calculated:

PRO-TIPS

Average velocity can also be calculated using the formula:

$$v_{ave} = \frac{v_i + v_f}{2}$$

$$P_{ave} = \frac{W}{\Delta t}$$

$$P_{ave} = \frac{F\Delta x\cos\theta}{\Delta t}$$

but, $v_{ave} = \frac{\Delta x}{\Delta t}$

From grade 10 mechanics, average velocity = $\frac{\text{displacement}}{\text{time}}$

$$\mathbf{P_{ave} = Fv_{ave}\cos\theta}$$

P_{ave} = average power in Watts (W)
v_{ave} = average velocity of the object in $m.s^{-1}$
F = Force resulting in the power in Newtons (N)
θ = angle between the force and the velocity (motion) in degrees (°)

For object moving at a constant velocity, the force resulting in the power is in the same direction as the velocity (motion), therefore $\theta = 0°$, $\cos 0° = 1$, the average power formula is simplified to:

$$\mathbf{P_{ave} = Fv_{ave}}$$

Formula on the data sheet

Worked examples

Multiple choice question

C + A

1. A force, **F**, acts on a car moving at a constant velocity **v** for time **t** resulting in a power **P**.

Which ONE of the following changes will increase the power to 4**P**?

A $\frac{1}{4}\mathbf{t}$

B 4**t**

C 2**F**

D 8**F**

Answer: A

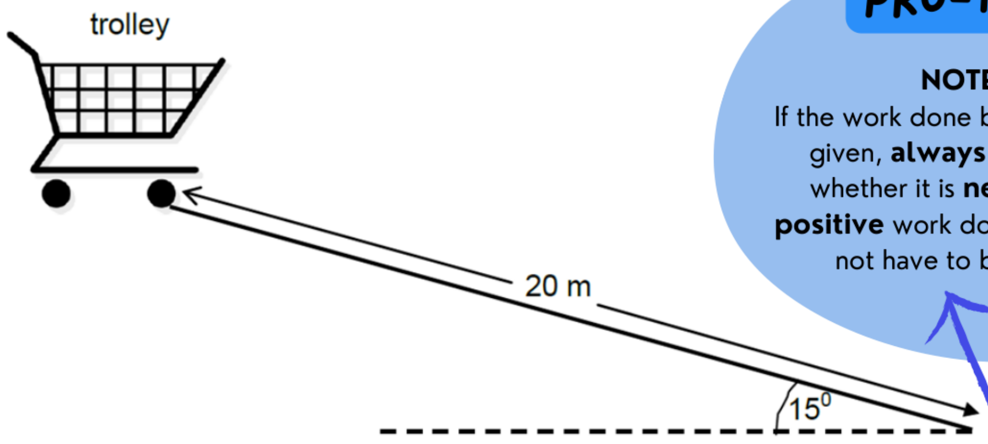
There are two ways to calculate average power. Let's discuss all the options below:

1. $P_{ave} = \frac{W}{\Delta t}$	**2. $P_{ave} = Fv_{ave}$**
What is the relationship between average power (P_{ave}) and time (Δt)? $P_{ave} \propto \frac{1}{\Delta t}$ Average power is inversely proportional to time taken, provided the work done remains constant. **OPTION A:** If the time is decreased by a factor of 4 (i.e., $\frac{1}{4}\mathbf{t}$ as in option A): $P_{ave} \propto \frac{1}{\Delta t}$ $P_{ave} \propto \frac{1}{(\frac{1}{4})}$ ∴4**P** **OPTION B:** If the time is increased by a factor of 4 (I.e., 4**t** as in option B): $P_{ave} \propto \frac{1}{\Delta t}$ $P_{ave} \propto \frac{1}{(4)}$ ∴ $\frac{1}{4}$**P**	What is the relationship between average power (P_{ave}) and force? $P_{ave} \propto F$ Average power is directly proportional to force, provided the average velocity remains constant. **OPTION C:** If the force is increased by a factor of 2 or doubled (i.e., 2**F** as in option C): $P_{ave} \propto F$ $P_{ave} \propto (2)$ ∴ 2**P** **OPTION D:** If the force is increased by a factor of 8 (i.e., 8**F** as in option D): $P_{ave} \propto F$ $P_{ave} \propto (8)$ ∴ 8**P**

Worked example

C + A

1. A shopper with her trolley approaches an incline, 20 m long, that makes an angle of 15° with the horizontal as shown below.



PRO-TIPS

NOTE:
If the work done by the force is given, **always** determine whether it is **negative** or **positive** work done - this does not have to be given!

The combined mass of the trolley and groceries is 30 kg. The work done by the constant frictional force that acts on the trolley as it moves down the incline is 700 J. The shopper applies a pulling force, **F**, on the trolley so that it moves down the incline at a CONSTANT SPEED of 1 $m.s^{-1}$.

1.1 Write down two non – conservative forces acting on the trolley as it moves down the incline. (2)

1.2 Draw a labelled free – body diagram showing all the forces acting on the trolley while it moves down the incline. (4)

1.3 Calculate the average power output of the shopper as she takes the trolley down the incline. (8)

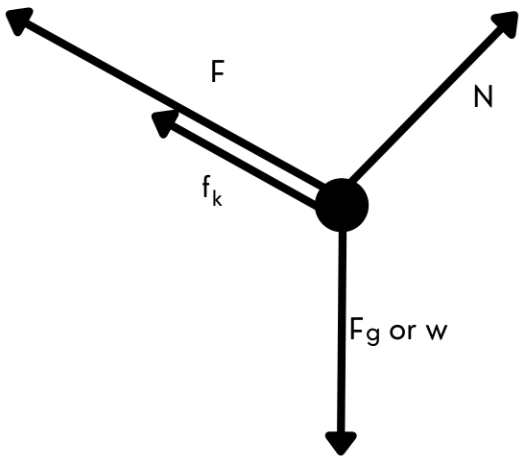
1.1 Recall that non - conservative forces are forces for which the work done in moving an object between two points depends on the path taken.

The pulling force and the frictional force (or friction).

1.2 **Notes regarding free - body diagrams:**

- A free - body diagram is a **diagram that represents the forces acting on an object**.
- In a free - body diagram the **dot represents the object**.
- Arrows representing the forces are always be drawn **AWAY** from the dot and must be drawn in **proportion and at the correct angles, as they act on the object.**
- Always label the forces and draw a **key** naming the forces when drawing a free - body diagram.
- **<u>NEW RULE IMPLEMENTED</u>: Forces must be represented as is, the components of the force (e.g., the components of the gravitational force or a force at an angle) will no longer be accepted.**

Free - body diagram showing the forces acting on the trolley



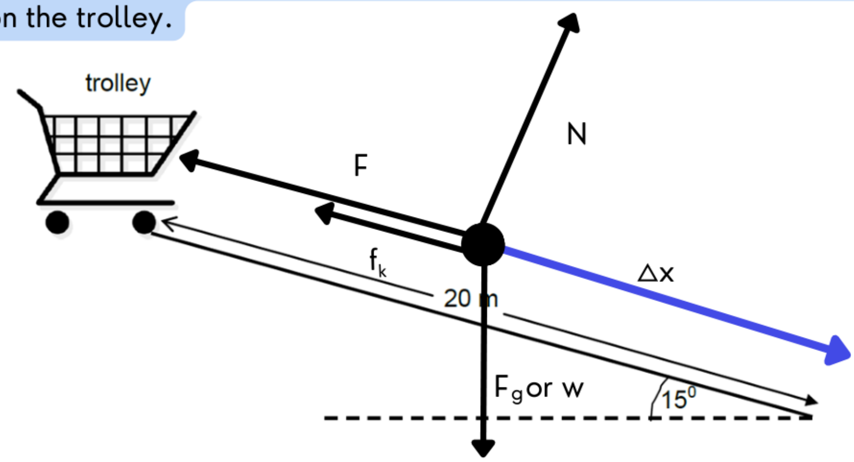
Key
F = Pulling force
N = normal force
f_k = kinetic frictional force
F_g or w = gravitational force or weight

PRO-TIPS

Draw all diagrams and graphs in **pencil**, incase of any mistakes or errors. Make sure to use a ruler as well!

1.3

To calculate the average power output of the shopper, by the pulling force of the shopper, the formula $P_{ave} = FV_{ave}$ will be used, as the average velocity of the trolley is known.
The trolley is moving at a constant velocity, therefore $v_f = v_i$ and the average velocity is 1 $m.s^{-1}$.
The force, **F**, exerted by the shopper on the trolley needs to be determined. Since the trolley is moving at a constant velocity, $Ek_f = Ek_i$ and the net work done is zero, working backwards with the work - energy theorem can be used to determine the force, **F**, exerted by the shopper on the trolley.



$W_{net} = \Delta E_k$
$W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$
$W_F + W_{Fg} + W_f + W_N = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$
$F\Delta x\cos\theta + F\Delta x\cos\theta + F\Delta x\cos\theta + F\Delta x\cos\theta = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$
$F(20)\cos180° + (30)(9{,}8)(20)\cos60° + (-700) + 0 = \frac{1}{2}(30)(1)^2 - \frac{1}{2}(30)(1)^2$
$-20F = -2240$
$F = 112$ N

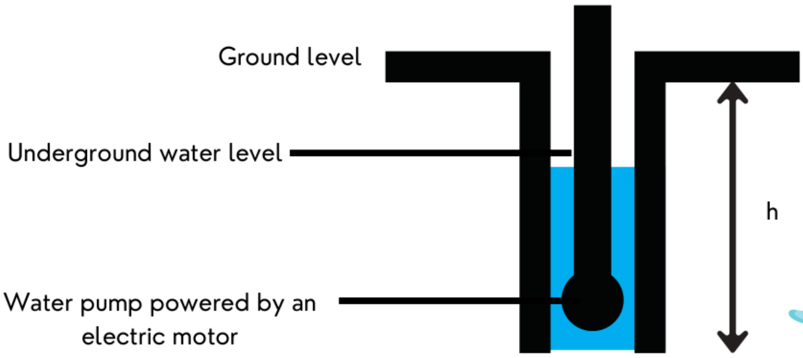
The work done by friction was given as 700 J. However, remember that friction does **NEGATIVE** work, as the angle between the frictional force and the displacement is 180°, and cos180°= -1

$P_{ave} = Fv_{ave}$
$P_{ave} = (112)(1)$
$P_{ave} = 112$ W

POWER OF AN ELECTRIC MOTOR

Electric motors are commonly used. One common application of an electric motor is to pump water out of a borehole. A borehole is a narrow hole made into the ground to extract underground water.
An electric motor is used to pump the water up to ground level.

A simplified sketch of a borehole is represented below:



Water has a density of 1 $g.cm^{-3}$, therefore 1 cm^3 of water (in volume) is equivalent to 1 g of water (in mass).

Worked example

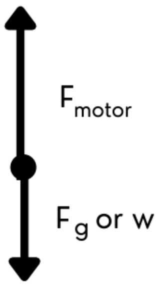
C + A

Calculate the minimum power required by an electric motor to pump water at a constant rate of 120 litres per minute from a depth of 10 m.

NOTE: 1 litre = 1000 cm^3 = 1000 ml

Mass of water pumped (in kg)
Therefore 120 litres = 120 000 cm^3 = 120 000 g = 120 kg

Forces acting on the water:



PRO-TIPS

Volume conversion chart
1 cm^3 = 1 ml

1 litre = 1000 cm^3 = 1000 ml

PRO-TIPS

Mass conversion chart
1000 g = 1 kg
g to kg ÷ 1000

The force of the motor is an example of a non - conservative force. Therefore:

Take the lowest point of the water as the reference point.

$W_{nc} = \Delta E_k + \Delta E_p$
$W_{nc} = 0 + \Delta E_p$
$W_{Fmotor} = mgh_f - mgh_i$
$W_{Fmotor} = (120)(9,8)(10) - (120)(9,8)(0)$
$W_{Fmotor} = 11760 \text{ J}$

The change in kinetic energy is zero because the water is being pumped at a constant speed.

1 minute = 60 seconds

$P = \frac{W}{\Delta t}$

$P = \frac{(11760)}{(60)}$

$P = 196 \text{ W}$

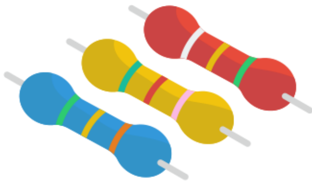
DID YOU KNOW?
Horsepower is a term commonly used to describe the power of an engine.
One horsepower is equivalent to approximately 746 W

REMINDER :QUESTION DIFFICULTY

C — COMPREHENSION AND RECALL QUESTIONS
These are common questions which include definitions and calculation questions that look similar to the questions covered in class.
Approximately **50%** of Paper 1 (Physics) will include questions on this level.

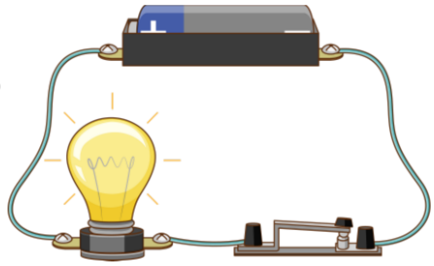
A — ANALYSIS AND APPLICATION QUESTIONS
These are more complex questions which involves applying the knowledge and skills learned in this chapter.
Approximately **40%** of Paper 1 (Physics) will include questions on this level.

P — PROBLEM- SOLVING QUESTIONS
These are questions that require critical thinking and being able to make connections between different representations of information and integrating different topics. They are not familiar questions, but are able to be solved through critical analysis. Approximately **10%** of Paper 1 (Physics) will include questions on this level.



ELECTRIC CIRCUITS

REVISION FROM GRADE 10 AND GRADE 11:



What is necessary for current to flow in a circuit?

1. A **cell or a battery** or a power supply.
This supplies the electrons with energy to move.

2. A **closed circuit**.
A circuit where there are no "breaks" from the positive to the negative terminal of the battery.

Potential difference (voltage) (V)

To simply understand potential difference, it can be described as the energy supplied by the cell / battery/power supply to the charge (electrons), that causes charge to flow in the circuit.

Definition: **Potential difference** (V): The potential difference across the ends of a conductor or between two points is the work done or energy transferred per unit charge flowing through it.

This can be expressed as an equation:

$$V = \frac{W}{q}$$

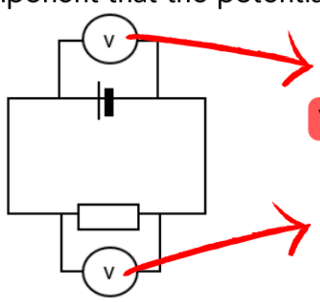
$$W = Vq$$

Both forms of the formulae are on the data sheet

What do these variables mean and what are the SI units?

W = work done or energy transferred in joules (J)
Q = charge flowing between two points in coulombs (C)
V = potential difference/ voltage in volts (V).

How is potential difference measured?
Potential difference is measured using a high resistance **voltmeter**, that is connected in **parallel** to the component that the potential difference is measured across.



Voltmeters connected in parallel to components

Electric current (I)

Definition: Electric current: The amount of charge that flows past a point per second. **OR** The rate of flow of charge.

PRO-TIPS

The **direction** of conventional current is from the **positive** terminal of the battery (or cell) to the **negative** terminal of the battery (or cell)

This can be expressed as an equation:

$$I = \frac{q}{\Delta t}$$

$$q = I\Delta t$$

Formula on the data sheet

What do these variables mean and what are the SI units?

q = Amount of charge flowing past a point in coulombs (C)
$\triangle$t = time taken for the charge to flow past a point in seconds (s)
I = current in amperes (A).

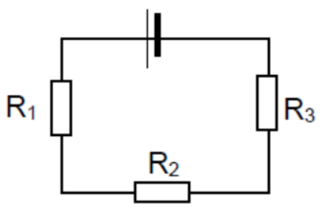
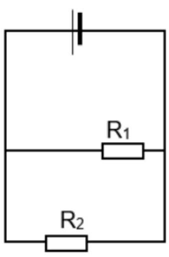
Current is measured using a very low resistance **ammeter**, which is always connected in series.

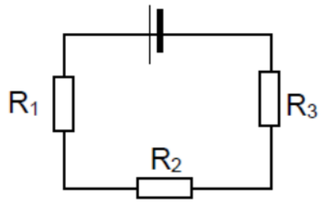
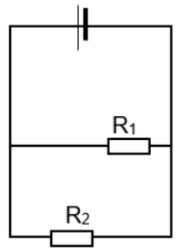
Resistance (R)

Resistance is the opposition to the flow of charge. Resistors are necessary in the external circuit to control the flow of electric charge.

The SI unit for resistance is Ohm. (Ω)

Resistors in the external circuit can be connected in **series** or in **parallel.**
The table below compares the differences between resistors in series and resistors in parallel:

RESISTORS IN SERIES	RESISTORS IN PARALLEL
	
Resistors connected in series are connected along the **same** conducting pathway. There is only one path through which current can flow. **The current through each resistor in series in the same.** $I_1 = I_2 = I_3$	Resistors connected in parallel are connected along **different** or **separate** conducting pathways. There are multiple pathways through which current can flow. **Resistors in parallel divide the current in a fixed ratio, according to the resistances.** $I_{total} = I_1 + I_2$

<table>
<tr><th>RESISTORS IN SERIES</th><th>RESISTORS IN PARALLEL</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>Resistors in series are potential difference (voltage) dividers. Resistors in series divide the voltage in a fixed ratio.

$V_{total} = V_1 + V_2 + V_3$</td><td>The potential difference across resistors in parallel is the same.

$V_1 = V_2 = V_{total}$</td></tr>
<tr><td>To calculate the total resistance for resistors in series, the equation below is used:

$R_s = R_1 + R_2 + \ldots$

Resistors in series increase the total resistance in the circuit.
Adding resistors in series increases the total resistance.
Removing resistors in series decreases the total resistance.
</td><td>To calculate the total resistance for resistors connected in parallel, the equation below is used:

$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$

Resistors in parallel decreases the total resistance in the circuit.
The total resistance for the parallel combination is always less than the smallest resistor in parallel.
Adding resistors in parallel decreases the total resistance.
Removing resistors in parallel increases the total resistance.
</td></tr>
</table>

PRO-TIPS

To **analyse** the circuit and to determine whether resistors are connected in **series** or **parallel**, follow the **direction of current** flow or "walk" the circuit.

Ohm's law

Ohm's law was developed by Georg Simon Ohm. It is an equation that describes the relationship between potential difference, resistance and current.

Definition: **Ohm's law**: Ohm's law states that the potential difference across a conductor is directly proportional to the current in the conductor at constant temperature.

PRO-TIPS

Ohm's law is a very common definition asked.
NOTE:
By keeping the temperature constant, the resistance in the circuit is kept constant.

Ohm's law represented using symbols (mathematically):

$$\mathbf{V = IR}$$

$$R = \frac{V}{I}$$

Formula on the data sheet

Relationship between variables:

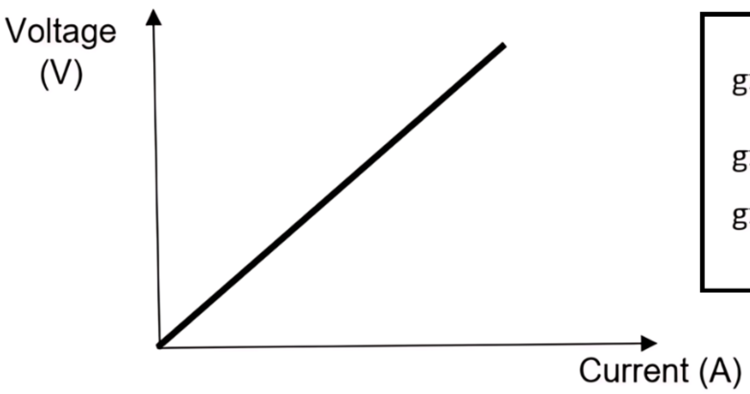
$V \propto I$ provided **R** remains constant.

This relationship between potential difference and current can be represented in a straight - line graph:

PRO-TIPS

A directly proportional relationship between two variables is always represented by a **straight - line graph** that **starts at the origin.**

Graph showing the relationship between voltage (V) and current (A)



$$\text{gradient} = \frac{\Delta y}{\Delta x}$$
$$\text{gradient} = \frac{V}{I}$$
$$\text{gradient} = R \text{ (resistance)}$$

$y = mx + c$

$V = RI + 0$

$\mathbf{V = IR}$

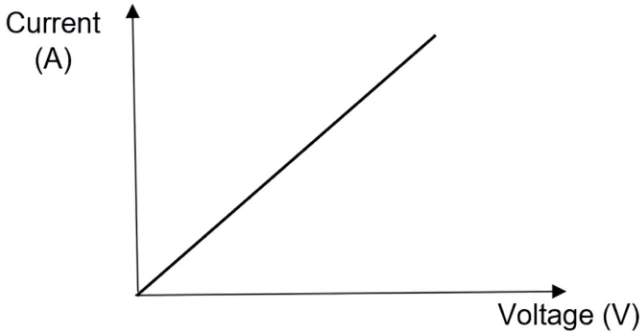
$R = \frac{V}{I}$

Equation of a straight - line graph

Ohm's law

OR

Graph showing the relationship between voltage (V) and current (A)



$$gradient = \frac{\Delta y}{\Delta x}$$
$$gradient = \frac{I}{V}$$
$$gradient = \frac{1}{R}$$

NOTE: gradient = inverse of resistance)
To find the resistance, invert/ flip the gradient.

y = mx + c
$I = \frac{1}{R}.V + 0$
IR = V
V = IR
R = $\frac{V}{I}$

Ohm's law

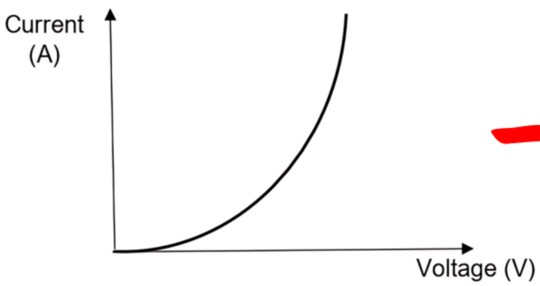
Ohm<u>ic conductors</u>

The above graphs represent the relationship between current and voltage in an Ohmic conductor.
An Ohmic conductor is a conductor that **obeys** Ohm's law.
Examples of Ohmic conductors: Resistors, copper and silver wire, incandescent light bulbs (only at higher temperatures).

Non-ohmic conductors

Non - ohmic conductors are conductors that do **not obey** Ohm's law and deviate from Ohm's law.
In a non - ohmic conductor, the relationship between current and voltage is not a directly proportional relationship therefore, the graph is **NOT** a straight - line graph that starts at the origin.
Example of potential difference versus current graph for a non - ohmic conductor:

Graph showing the relationship between voltage (V) and current (A)



Graph is non - linear

Examples of non - ohmic conductors: diodes, thermistors, light bulb filaments at high temperatures deviate from Ohm's law

Ohm's law relationship between variables (continued)...

- **V** $\propto$ **R** provided **I** remains constant.
- $I \propto \frac{1}{R}$ provided **V** remains constant

Inversely proportional relationship

Power (P)

When electrons flow through a circuit, they do work, that is, they transfer energy to the components e.g., resistors, light bulbs etc. in the circuit. It is useful to know how much work is done in a given time period, this is referred to as power (**P**).

Definition: Power: Power is defined as the rate at which work is done.

From the definition, the formula for power in symbols:

$$\boxed{P = \frac{W}{\Delta t}}$$

What do these variables mean and what are the SI units?

P = Power (average power) is measure in Watts (W).**NOTE:** 1000 W = 1 kW (1 kilowatt)

W = Work done (energy transferred) in Joules (J)

Δt= time taken to do the work in seconds (s)

NOTE: 1 watt is equivalent to 1 Joule per second.

$1\ W = \frac{1J}{1s} = 1\ J.s^{-1}$

In electricity, there are other formulae that can be used to calculate power:

$$\mathbf{P = VI}$$

$$\mathbf{P = \frac{V^2}{R}}$$

$$\mathbf{P = I^2R}$$

PRO-TIPS

You can go straight to these work done equations if the question asks:
"Calculate the amount of energy transferred..."

Work done or energy transferred (W)

Since $P = \frac{W}{\Delta t}$, W = P Δt. Using this the work done or energy transferred formulae can be derived by substituting the various power formulae into W = P Δt.

$$\mathbf{W = Vq}$$

This equation is from the potential difference formula

$$\mathbf{W = VI\Delta t}$$

$$\mathbf{W = \frac{V^2\Delta t}{R}}$$

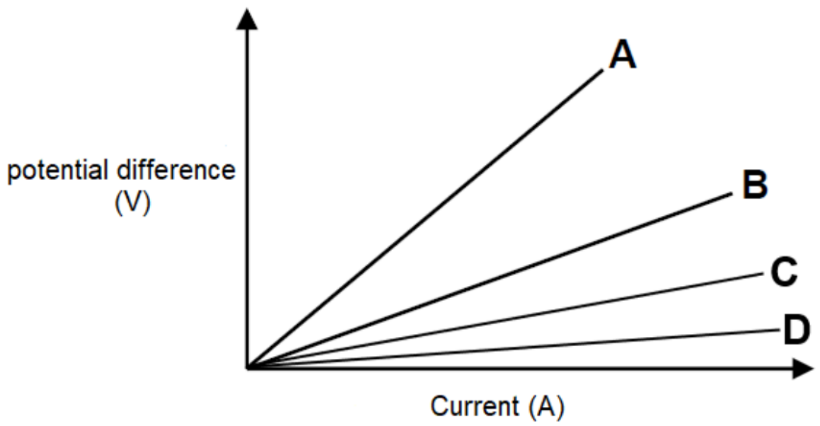
$$\mathbf{W = I^2R\Delta t}$$

Worked examples

Multiple choice questions

C + A

1. The potential difference is measured across resistors **A**, **B**, **C** and **D** by varying the current. The graph below represents the results:



Which ONE of the following resistors has the lowest resistance?

A Resistor **A**
B Resistor **B**
C Resistor **C**
D Resistor **D**

Answer: D

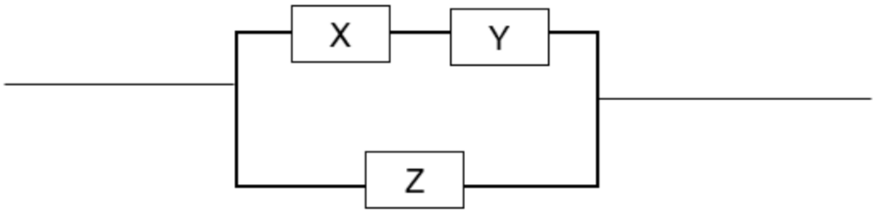
1. When given a graph, always analyse the graph first by determining the physical quantity represented by the gradient, area under the graph etc.

   What physical quantity is represented by the gradient of this graph?

$$\text{gradient} = \frac{\Delta y}{\Delta x}$$
$$\text{gradient} = \frac{V}{I}$$
$$\text{gradient} = R\ (\text{resistance})$$

The gradient of the graph represents the resistance of the resistor, therefore the **steeper/higher** the gradient, the **higher** the **resistance** of the resistor, and the **smaller**/more gradual the gradient, the **lower** the **resistance**. Therefore, resistor **D** has the most gradual/ smallest gradient and therefore the lowest resistance.

2. Three identical resistors are connected as shown below:



How do the potential differences across the individual resistors compare?

A $V_X = V_Y \neq V_Z$

B $V_X = V_Y = V_Z$

C $V_X = 2V_Z$

D $V_X = \frac{1}{4}V_Z$

Answer: A

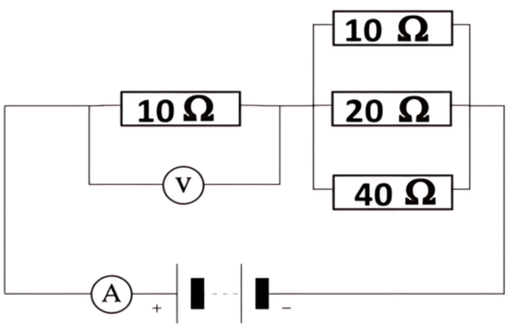
- Resistor **X** and **Y** are connected in series with each other, and together they are in parallel with resistor **Z**.
- Resistors in series divide the potential difference (or voltage).
- Since resistor **X** and **Y** have the same resistance, the potential difference is divided equally, therefore the potential difference across resistor **X** equals the potential difference across resistor **Y**.
- The potential difference across resistors in parallel are the same.
- Therefore, the **sum** of the potential differences across resistor **X** and **Y** equals the potential difference across resistor **Z**.
- **NOTE: For option C and D**: The potential difference across resistor **X** is **HALF** the potential difference across resistor **Z**. Therefore, these options are not correct.

Worked example

C + A

NOTE: This revision example still deals with circuits without the internal resistance of the battery being factored in.

1. In the circuit shown below, the reading on the ammeter is 3,2 A.



PRO-TIPS

Circuit questions involve the application of Ohm's law:

$$R = \frac{V}{I}$$

Calculate:

1.1 The reading on the voltmeter. (3)

1.2 The total resistance in the circuit. (4)

1.3 The potential difference across the 40 Ω resistor. (4)

1.4 The current in the 20 Ω resistor. (3)

1.1 **Start off by "walking" the circuit or following the direction of current flow to draw conclusions:**

- The ammeter is positioned such that it reads the TOTAL current in the circuit. Therefore the total current in the circuit is 3,2 A.
- The voltmeter is connected in parallel with the 10 Ω resistor in series and all the current will flow through the 10 Ω resistor.
- Using Ohm's law, the potential difference across it can be determined:

$$\mathbf{R} = \frac{\mathbf{V}}{\mathbf{I}}$$

$$(\mathbf{10}) = \frac{\mathbf{V}}{(\mathbf{3{,}2})}$$

V = 32 V

1.2

When calculating the total resistance in the external circuit, always start by calculating the total resistance of the parallel combination, followed by the resistors in series.

The total external resistance can then be calculated using the formula:

$R_{total} = R_s + R_p$

There are three resistors connected in parallel:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_p} = \frac{1}{(10)} + \frac{1}{(20)} + \frac{1}{(40)}$$

$$\frac{1}{R_p} = \frac{7}{40}$$

$R_p = \frac{40}{7}$

$R_p = 5{,}71\ \Omega$

$R_{total} = R_s + R_p$

$R_{total} = (10) + (5{,}71)$

$R_{total} = 15{,}71\ \Omega$

NOTE:

- Do not forget to invert the fraction when solving for R_p
- Do not leave the answer as a fraction. All answers must be in decimal form rounded off to a minimum of two decimal places.

1.3 **OPTION 1**

NOTE: The 40 Ω resistor is connected in parallel to the 10 Ω and 20 Ω resistor. By **calculating** the potential difference across the **parallel** combination, that will equal the potential difference across each of the resistors in parallel.

To determine the potential difference across the parallel combination, information regarding the parallel combination is needed:

$R_p = 5{,}71\ \Omega$

$I_p = 3{,}2$ A ⟶ This is the sum of the currents through all of the branches, which is the **TOTAL** current in the circuit.

1.3 **OPTION 1**

$$\mathbf{R} = \frac{\mathbf{V}}{\mathbf{I}}$$

$$(\mathbf{5,71}) = \frac{\mathbf{V}}{\mathbf{(3{,}2)}}$$

V = 18,27 V

$V_{40\Omega} = V_p = 18{,}27$ V

OR

1.3 **OPTION 2**

NOTE: The 40 Ω resistor is connected in parallel to the 10 Ω and 20 Ω resistor. The voltage across the 10 Ω resistor in series was calculated in question 1.1 as 32 V. By calculating the total voltage of the battery, and subtracting the voltage across the 10 Ω in series, the remaining voltage is the voltage across the parallel combination.

To calculate the total voltage of the battery, the total resistance and total current in the circuit is required, this information is known.

$R_{total} = 15{,}71\ \Omega$

$I_{total} = 3{,}2$ A

$$\mathbf{R} = \frac{\mathbf{V}}{\mathbf{I}}$$

$$(\mathbf{15,71}) = \frac{\mathbf{V}}{\mathbf{(3{,}2)}}$$

V = 50,27 V

$V_{40\Omega} = V_p = V_{total} - V_{10\Omega}$

$V_{40\Omega} = V_p = (50{,}27) - (32)$

$V_{40\Omega} = V_p = 18{,}27$ V

1.4

NOTE: The 20 Ω resistor is connected in parallel with the 40 Ω and 10 Ω resistor. The potential difference across the 20 Ω resistor in parallel is equal to the potential difference across the parallel combination, which was calculated in Question 1.3.

To calculate the current through the 20 Ω resistor:

R = 20 Ω

V = 18,27V

$$\mathbf{R} = \frac{\mathbf{V}}{\mathbf{I}}$$

$$(\mathbf{20}) = \frac{\mathbf{(18{,}27)}}{\mathbf{I}}$$

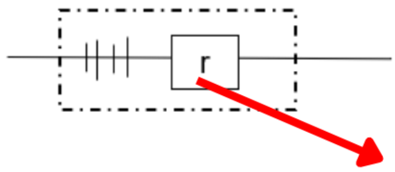
I = 0,91 A

Current through 20 Ω resistor = 0,91 A

INTERNAL RESISTANCE OF A BATTERY OR CELL

In grade 10 and 11, all electric circuit problems were simplified by **ignoring** the **internal resistance** of the battery or cell.
In reality, all batteries and cells have some resistance due to the materials that make up the battery.
This is called internal resistance. The symbol for internal resistance is r.

The diagram below represents a battery with internal resistance:



Internal resistance (r)

Emf (ε) of a battery or cell

Batteries often have a voltage value written on them, for example, fresh standard AA size batteries have a voltage of 1,5 V. This is the **emf** of the battery.

Definition: emf: The maximum energy provided by a battery per unit charge passing through it.

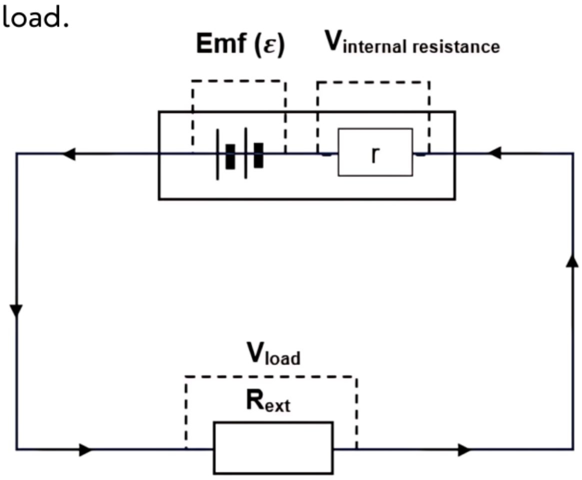
The emf of the battery is the maximum potential difference or voltage that the battery can supply. It is the terminal potential difference measured across the terminals of the battery when the circuit is open and there is no current flowing.

Emf (ε) of a battery or cell , internal resistance (r) and voltage across the external circuit (V_{load})

When a circuit is closed and charge flows through the battery, due to the internal resistance of the battery, some of the **electrical potential energy** of the battery converted into **heat energy.**

This electrical potential energy is no longer available to the external circuit, and is referred to as "lost" volts, $V_{\text{"lost"}}$ or $V_{\text{internal resistance}}$. The 'volts' are not **really** lost, it is just converted into heat energy and is therefore not available to the external circuit.
As a result of this, **the potential difference/voltage available to the external circuit decreases and is less than the emf of the battery.**

In the diagram below, a battery of emf is connected in a closed circuit to a external resistor with resistance, R, called the load.



Recall that resistors in series divide the voltage.
The internal resistance of the battery (r) is treated as another resistor in series with the resistors in the external circuit (R_{ext}). Therefore, the voltage across the external circuit (V_{load}) [this is the voltage available to the external circuit] plus the voltage across the internal resistance ($V_{internal\ resistance}$) is equal to the battery's emf (ε)

This can be represented as an equation, in symbols:

$$\text{emf } (\varepsilon) = V_{load} + V_{internal\ resistance}$$
$$\text{emf } (\varepsilon) = IR_{ext} + Ir$$

$$\text{emf } (\varepsilon) = I\,(R + r)$$

Formula on the data sheet

PRO-TIPS

From Ohm's law:
V = IR

What do these variables mean and what are the SI units?

emf (ε) = emf of the battery i.e. total potential difference or voltage of the battery in volts (V).

V_{load} = terminal potential difference; potential difference or voltage across the external circuit or load in volts (V).

$V_{internal\ resistance}$ = potential difference or voltage across the internal resistance of the cell, i.e., the "lost" volts that are not available to the external circuit in volts (V).

I = **Total** current in the circuit in amperes (A).

R_{ext} = **Total** resistance in the **external** circuit in ohms (Ω)

r = internal resistance of the battery or cell in ohms (Ω)

NOTE: The above equation can be separated into the following equations, which can be used as separate equations when doing calculations - it is just applying Ohm's law!

$V_{load} = IR_{ext}$ & $V_{internal\ resistance} = Ir$

Important notes regarding circuits with internal resistance:

- The **emf** of the battery and the **internal resistance** of the battery always **remain constant** (because the battery is not changed) regardless of any changes made to the resistance in the external circuit.
- When the switch is closed and current flows through the circuit and battery, the voltage across the battery **DROPS** or **DECREASES**, this is due to internal resistance resulting in "lost" volts or
  ($V_{\text{internal resistance}}$)
  Another way to calculate $V_{\text{internal resistance}}$:

$$\text{emf } (\varepsilon) = V_{\text{load}} + V_{\text{internal resistance}}$$

$$V_{\text{internal resistance}} = \text{emf } (\varepsilon) - V_{\text{load}}$$

- The voltage that the voltmeter connected parallel to the battery reads is the voltage available to the external circuit (V_{load}). This will be **LOWER** than the emf of the battery when internal resistance is factored in:

$$V_{\text{load}} < \varepsilon \text{ (emf)}$$

Another way to calculate V_{load}:

$$\text{emf } (\varepsilon) = V_{\text{load}} + V_{\text{internal resistance}}$$

$$V_{\text{load}} = \text{emf } (\varepsilon) - V_{\text{internal resistance}}$$

NOTE: If the circuit is an open circuit and no current flowing, the voltmeter connected parallel to the battery/across the battery will read the emf of the battery.

- It is very helpful to compile a list of all the important electricity formulae that have been discussed in this chapter and to write a list of these formulae for all electricity questions to help you identify which formula to use.If you forget a formula - do not panic - the main formulae are on the data sheet to reference
  and can be expanded from there.

A comprehensive list of all the electricity formulae:

Ohm's law – including circuits with internal resistance

$\text{emf } (\varepsilon) = I(R + r)$

Start with this formula and expand from there!

$\text{emf } (\varepsilon) = IR_{\text{ext}} + Ir$

$\text{emf } (\varepsilon) = V_{\text{load}} + V_{\text{internal resistance}}$

$V_{\text{load}} = \text{emf } (\varepsilon) - V_{\text{internal resistance}}$ **OR** $IR_{\text{ext}} = \text{emf } (\varepsilon) - Ir$

$V_{\text{internal resistance}} = \text{emf } (\varepsilon) - V_{\text{load}}$ **OR** $Ir = \text{emf } (\varepsilon) - IR_{\text{ext}}$

$V_{\text{load}} = IR_{\text{ext}}$

$V_{\text{internal resistance}} = Ir$

Separated formulae

$R = \frac{V}{I}$

Power

$P = \frac{W}{\Delta t}$

$P = VI$

$P = \frac{V^2}{R}$

$P = I^2R$

Energy

$W = Vq$

$W = VI\Delta t$

$W = \frac{V^2\Delta t}{R}$

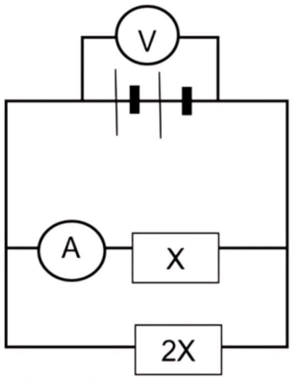
$W = I^2R\Delta t$

Worked examples

Multiple choice questions

C + A

1. In the circuit represented below, a battery of emf **E** is connected to two resistors in parallel. The total current in the circuit is **I**.
   The internal resistance of the battery cannot be ignored.



PRO-TIPS

The internal resistance of the battery only plays a role if current is flowing in the circuit. Therefore internal resistance is defined as:
The resistance within a cell or battery when current is flowing.

Which ONE of the following combinations is CORRECT?

	Ammeter reading	Voltmeter reading
A	$\frac{1}{3}\mathbf{I}$	Less than **E**
B	$\frac{2}{3}\mathbf{I}$	Less than **E**
C	$\frac{1}{3}\mathbf{I}$	Equal to **E**
D	$\frac{2}{3}\mathbf{I}$	Equal to **E**

Answer: B

<u>Voltmeter reading</u>

The internal resistance of the battery is factored in, therefore since the circuit is a closed circuit, the reading on the voltmeter across the battery decreases or drops and reads V_{load}, which is less than the emf.
This leaves option A and B.

<u>Ammeter reading</u>

The resistors **X** and 2**X** are connected in parallel, therefore, the potential difference across them is the same and equal to V_{load}. However, the resistors in parallel divide the total current, **I,** in a fixed ratio according to their resistances. Remember that resistor **X** has a lower resistance than resistor 2**X**, therefore more current will flow through it.

I is divided in a ratio 1 : 2

$I_X = \frac{2}{3} \times I$

$I_X = \frac{2}{3} I$

A ratio is a fraction; when working with ratios apply the following formula:

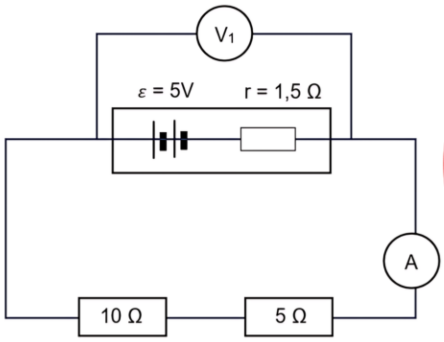
$$\frac{part}{whole} \times total$$

Worked example

1. A circuit is set up as shown in the diagram. The battery has an emf of 5 V and an internal resistance of 1,5 Ω.

PRO-TIPS

Did you remember to write down your formula sheet and to "walk" the circuit or follow the direction of current flow?



PRO-TIPS

Note whether the question is asking you to calculate the **total external resistance** or the **total resistance** in the circuit. If the question asks "total resistance" it is also factoring in the internal resistance of the battery.

Calculate:

1.1	The total resistance in the circuit.	(2)
1.2	The reading on the ammeter.	(3)
1.3	The reading on the voltmeter.	(3)
1.4	The energy dissipated by the 10 Ω resistor in 2 minutes.	(3)

1.1 This circuit has two resistors in series, however, the internal resistance of the battery also acts as a resistor in series, and is therefore factored in when calculating the **TOTAL** resistance in the circuit.

$R_{total} = R_{ext} + r$
$R_{total} = R_s + r$
$R_{total} = (10) + (5) + (1,5)$
$R_{total} = 16,5\ \Omega$

1.2 The ammeter is positioned such that it reads the **total current** in the circuit. It must be noted that this circuit has internal resistance, and this must be factored into the calculation as it affects the total current in the circuit.

emf $(\varepsilon) = I\,(R + r)$
$(5) = I\,(15 + 1,5)$
$I = \frac{5}{16,5}$
$I = 0,30$ A

1.3 This question is **indirectly** asking you to calculate V_{load}.
The voltmeter is connected parallel to the battery, therefore it reads the potential difference or voltage across the battery. Due to the circuit being a **closed** circuit, internal resistance plays a role, resulting in "lost" volts, therefore, the potential difference across the battery **decreases** or drops and is **less than** the **emf**.

1.4

The energy dissipated is the work done by the 10Ω resistor. The time given is in minutes, this must be converted to seconds by multiplying it by 60:

$\Delta t = 2 \times 60$

$\Delta t = 120$ s

The resistance of the 10Ω resistor is known, and because the resistor is in series, the current flowing through it is the total current, which was calculated as 0,30 A in Question 1.2.

$W = I^2R\Delta t$

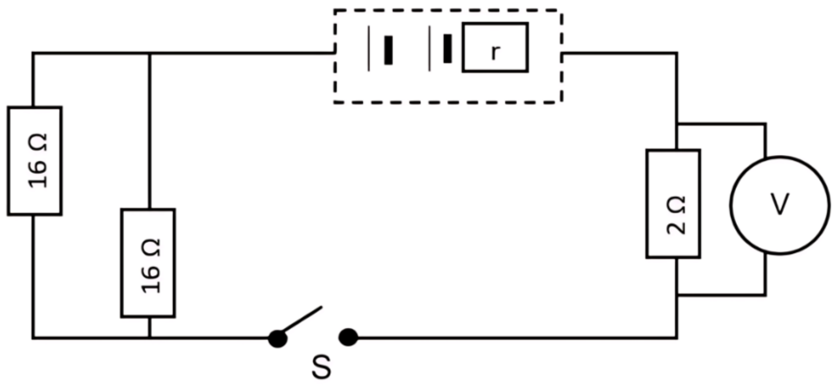
$W = (0,30)^2(10)(120)$

$W = 108$ J

Worked example

C + A

2. A circuit with an unknown internal resistance, r, is shown below.
   The maximum energy provided by the battery per coulomb of charge passing through it is 30 $J.C^{-1}$.



2.1 Provide one word for the underlined words above. (1)

When switch S is closed, the potential difference across the battery **decreases** by 4 V.

2.2 Calculate the current flowing through the 2Ω resistor. (4)
2.3 Calculate the reading on the voltmeter. (3)
2.4 Calculate the internal resistance of the battery. (3)

2.1 Emf of the battery.

NOTE: The maximum energy provided by the battery per unit (or per coulomb) of charge passing through it is defined as the emf of the battery. Therefore, the emf of the battery is 30 V.

2.2 The decrease in the potential difference across the battery is due to the internal resistance of the battery resulting in "lost" volts or $V_{\text{internal resistance}}$

Due to the potential difference across the battery decreasing by 4 V, the following can be concluded:

- $V_{\text{internal resistance}} = 4V$
- $V_{load} = 30 - 4$
  $V_{load} = 26$ V

OPTION 1

The 2Ω resistor is connected in series, therefore the total current flows through it. The question is indirectly asking "calculate the **total** current in the circuit".

There are many unknowns in this question, therefore, it is not as simple as substituting it into the emf $(\varepsilon) = I(R + r)$ formula.

However, this formula can be separated, and the formula $V_{load} = IR_{ext}$ can be used to calculate the total current in the circuit, as V_{load} was deduced as 26V, and the total external resistance can be calculated:

There are two resistors in parallel (16Ω resistors) and one resistor in series (2Ω resistor) in the external circuit:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$\frac{1}{R_p} = \frac{1}{(16)} + \frac{1}{(16)}$$
$$\frac{1}{R_p} = \frac{2}{16}$$
$$R_p = \frac{16}{2}$$
$$R_p = 8\ \Omega$$

$$R_{ext} = R_s + R_p$$
$$R_{ext} = (2) + (8)$$
$$R_{ext} = 10\ \Omega$$

$$V_{load} = 30 - 4$$
$$V_{load} = 26V$$

$$V_{load} = IR_{ext}$$
$$(26) = I(10)$$
$$I = 2{,}60\ A$$

∴ Current through 2Ω resistor = 2,60 A.

PRO-TIPS

Did you remember to "walk the circuit" or follow the direction of current flow to determine which resistors are in series and which resistors are in parallel?

OR

OPTION 2

The 2Ω resistor is connected in series, therefore the total current flows through it. The question is indirectly asking "calculate the total current in the circuit".

There are many unknowns in this question, therefore, it is not as simple as substituting it into the emf $(\varepsilon) = I\,(R + r)$ formula.

This formula can, however, be adjusted and expanded to suit the information that you have. When the formula is expanded, it becomes:

emf $(\varepsilon) = I\,(R + r)$

emf $(\varepsilon) = IR_{ext} + Ir$

The following is known:

emf (ε) = 30 V

$V_{internal\ resistance}$ = 4V, therefore the entire '**Ir**' in the formula can be replaced with 4V.

The total external resistance can be calculated:

There are two resistors in parallel (16Ω resistors) and one resistor in series (2Ω resistor) in the external circuit:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$\frac{1}{R_p} = \frac{1}{(16)} + \frac{1}{(16)}$$
$$\frac{1}{R_p} = \frac{2}{16}$$
$$R_p = \frac{16}{2}$$
$$R_p = 8\ \Omega$$

$$R_{ext} = R_s + R_p$$
$$R_{ext} = (2) + (8)$$
$$R_{ext} = 10\ \Omega$$

$$V_{load} = 30 - 4$$
$$V_{load} = 26V$$

$$V_{load} = IR_{ext}$$
$$(26) = I(10)$$
$$I = 2{,}60\ A$$

∴ Current through 2Ω resistor = 2,60 A.

2.3 The voltmeter reads the potential difference or voltage across the 2Ω resistor. Since the current through the resistor was calculated in Question 2.2 as 2,60 A and the resistance is 2Ω, calculating the potential difference is a simple Ohm's law calculation:

$$\mathbf{R} = \frac{\mathbf{V}}{\mathbf{I}}$$

$$(\mathbf{2}) = \frac{\mathbf{V}}{(\mathbf{2{,}60})}$$

$$V = 5{,}20\ V$$

2.4 OPTION 1

There is sufficient information to calculate the internal resistance, r, of the battery using the formula: emf $(\varepsilon) = I\,(R + r)$

emf = 30 V
I = 2,60 A
R_{ext} = 10 Ω

$$\text{emf }(\varepsilon) = I\,(R + r)$$
$$(30) = 2{,}6(10 + r)$$
$$\frac{30}{2{,}6} = 10 + r$$
$$r = 11{,}538\ldots - 10$$
$$r = 1{,}54\ \Omega$$

OR

$$\text{emf }(\varepsilon) = I\,(R + r)$$
$$(30) = 2{,}6(10 + r)$$
$$30 = 26 + 2{,}6r$$
$$30 - 26 = 2{,}6r$$
$$4 = 2{,}6r$$
$$r = \frac{4}{2{,}6}$$
$$r = 1{,}54\ \Omega$$

Distributive law

OR

PRO-TIPS

NOTE:
There are two different ways to solve for **r** mathematically using this formula.
If using the distributive law, remember to apply it correctly.

2.4 OPTION 2

The amount of "lost" volts ($V_{internal\ resistance}$) is known (4V) and the **total** current flowing through the circuit is 2,60 A, therefore the formula $V_{internal\ resistance} = Ir$ can be used to determine the internal resistance of the battery.

$$V_{internal\ resistance} = Ir$$
$$(4) = (2{,}6)r$$
$$r = \frac{4}{2{,}6}$$
$$r = 1{,}54\ \Omega$$

EFFECT OF INCREASING AND DECREASING THE EXTERNAL RESISTANCE IN THE CIRCUIT

It is important to be able to explain what effect increasing or decreasing the external resistance has on:

- The voltage available to the external circuit (V_{load})
- The amount of "lost" volts ($V_{internal\ resistance}$)

Effect of increasing the external resistance on V_{load}

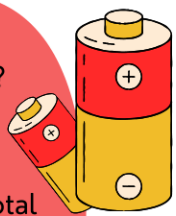
<u>How can the **external** resistance in the circuit be increased</u>?

1. By adding resistors in series.
2. By removing resistors in parallel.

- By increasing the external resistance in the circuit, the total resistance increases, therefore the total current in the circuit decreases, since the emf of the battery remains constant.

- From the formula emf $(\varepsilon) = IR_{ext} + Ir$, IR_{ext} = emf $(\varepsilon) - Ir$, therefore if the emf and the internal resistance of the battery remains constant and the total current decreases, this results in less "lost" volts or $V_{internal\ resistance}$ decreases, therefore the voltage available to the external circuit (V_{load}) increases.

Effect of decreasing the external resistance on V_{load}



<u>How can the **external** resistance in the circuit be decreased</u>?

1. By removing resistors in series.
2. By adding resistors in parallel.

- By decreasing the external resistance in the circuit, the total resistance decreases, therefore the total current in the circuit increases, since the emf of the battery remains constant.

- From the formula emf $(\varepsilon) = IR_{ext} + Ir$, IR_{ext} = emf $(\varepsilon) - Ir$, therefore if the emf and the internal resistance of the battery remains constant and the total current increases, this results in more "lost" volts or $V_{internal\ resistance}$ increases, therefore the voltage available to the external circuit (V_{load}) decreases.

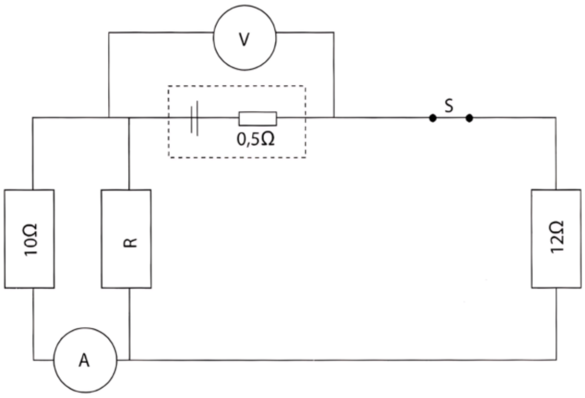
PRO-TIPS

These types of explanation questions are being asked in almost every exam - it is important to know **HOW** to explain the answer.

Worked example

A + P

1. In the circuit represented below, the resistance of **R** is unknown. When the switch, S, is open, the reading on V is 45 V. The battery has an internal resistance of 0,5 Ω. When switch S is closed, the reading on V is 43,5 V.



1.1 State Ohm's law in words. (2)

1.2 Calculate:
1.2.1 The reading on ammeter A. Show all calculations. (7)
1.2.2 The resistance of **R**. (4)

1.3 Resistor **R** burns out. How will this affect the reading on voltmeter V? Write down only INCREASES, DECREASES or REMAINS THE SAME. Explain the answer. (4)

1.1 Ohm's law states that the potential difference across a conductor is directly proportional to the current in the conductor at constant temperature.

1.2 **NOTE**: Analyze the circuit and remember to "walk" the circuit or follow the direction of current flow.
Some conclusions can be drawn about this circuit:

- When switch S is open, the circuit is an open circuit, therefore no current flows in the circuit and the voltmeter reads the emf of the battery. Emf = 45 V.
- When switch S is closed, the circuit is a closed circuit, therefore current flows in the circuit and through the battery the voltage across the battery drops due to "lost" volts or $V_{\text{internal resistance}}$

  The voltmeter reads V_{load}. V_{load}= 43,5 V. The "lost" volts are able to be calculated:
- $V_{\text{internal resistance}} = 45 - 43,5$

  $V_{\text{internal resistance}} = 1,5$ V
- The 10 Ω resistor and resistor **R** are in parallel with each other, and the 12 Ω resistor is in series, therefore the total current in the circuit flows through it.

1.2.1 • The reading on the ammeter is the the current through the 10Ω resistor which is in parallel with resistor **R**. Remember that resistors in parallel divide the current, but the potential difference across each resistor in parallel is the same.

What information regarding the parallel combination can be determined?

- The resistance of resistor **R** is unknown, however, the potential difference across the parallel combination can still be determined if the total current is determined and the potential difference across the 12Ω resistor in series is determined, the current through the 10Ω resistor can then be determined since its resistance and the potential difference across it is known.

OR

- Can the resistance of the parallel combination be determined to calculate the current through the 10Ω resistor? Absolutely! However, the total current must first be determined and then the total external resistance and thereafter the resistance of the parallel combination and the potential difference across it, and finally the current through the 10Ω resistor.

OPTION 1

Total current in the circuit

The internal resistance of the battery and the amount of "lost" volts is known. This can be used to calculate the total current in the circuit:

$V_{\text{internal resistance}} = \text{emf } (\varepsilon) - V_{\text{load}}$
$V_{\text{internal resistance}} = (45) - (43{,}5)$
$V_{\text{internal resistance}} = 1{,}5 \text{ V}$

$V_{\text{internal resistance}} = \text{Ir}$
$(1{,}5) = \text{I}(0{,}5)$
$\text{I} = \frac{1{,}5}{0{,}5}$
$\text{I} = 3\text{A}$

OR

$\text{emf } (\varepsilon) = \text{I }(\text{R} + \text{r})$
$\text{emf } (\varepsilon) = \text{IR}_{\text{ext}} + \text{Ir}$
$\text{emf } (\varepsilon) = V_{\text{load}} + \text{Ir}$
$(45) = (43{,}5) + \text{I}(0{,}5)$
$45 - 43{,}5 = 0{,}5\text{I}$
$\text{I} = 3\text{A}$

Potential difference or voltage across the 12Ω resistor in series

$\text{R} = \frac{\text{V}}{\text{I}}$
$(12) = \frac{\text{V}}{(3)}$
$\text{V} = 36 \text{ V}$

Potential difference or voltage across the parallel combination and therefore across the 10 Ω resistor.

$V_p = V_{\text{load}} - V_{12\Omega}$
$V_p = V_{\text{load}} - V_{12\Omega}$
$V_p = (43{,}5) - (36)$
$V_p = 7{,}5 \text{ V}$

Remember that there is only 43,5 V available to the external circuit - not all the emf of the battery (45 V) is available to the external circuit because of internal resistance.

$\therefore V_{10\Omega} = V_p = 7{,}5 \text{ V}$

Current through the 10 Ω resistor in parallel:

$R = \frac{V}{I}$

$(10) = \frac{(7,5)}{I}$

$10I = 7,5$

$I = \frac{7,5}{10}$

$I = 0,75$ A

∴ Reading on the ammeter = 0,75 A

OR

OPTION 2

Total current in the circuit

The internal resistance of the battery and the amount of "lost" volts is known. This can be used to calculate the total current in the circuit:

$V_{\text{internal resistance}} = \text{emf } (\varepsilon) - V_{\text{load}}$

$V_{\text{internal resistance}} = (45) - (43,5)$

$V_{\text{internal resistance}} = 1,5$ V

$V_{\text{internal resistance}} = Ir$

$(1,5) = I(0,5)$

$I = \frac{1,5}{0,5}$

$I = 3$A

OR

$\text{emf } (\varepsilon) = I(R + r)$

$\text{emf } (\varepsilon) = IR_{\text{ext}} + Ir$

$\text{emf } (\varepsilon) = V_{\text{load}} + Ir$

$(45) = (43,5) + I(0,5)$

$45 - 43,5 = 0,5I$

$I = 3$A

Total external resistance and resistance of the parallel combination

$V_{\text{load}} = IR_{\text{ext}}$

$(43,5) = (3)R_{\text{ext}}$

$R_{\text{ext}} = \frac{43,5}{3}$

$R_{\text{ext}} = 14,5\ \Omega$

$R_{\text{ext}} = R_s + R_p$

$(14,5) = (12) + R_p$

$R_p = 14,5 - 12$

$R_p = 2,5\ \Omega$

Potential difference across the 10Ω resistor and the parallel combination

$R = \frac{V}{I}$

$(2,5) = \frac{V}{(3)}$

$V = 7,5$ V

$\therefore V_p = V_{10\Omega} = 7,5$ V

Current through the 10 Ω resistor in parallel:

$R = \frac{V}{I}$

$(10) = \frac{(7,5)}{I}$

$10I = 7,5$

$I = \frac{7,5}{10}$

$I = 0,75$ A

∴ Reading on the ammeter = 0,75 A

1.2.2 To determine the resistance of resistor **R** the potential difference across resistor **R** and the current flowing through resistor **R** must first be determined. The resistance of resistor **R** can be determined using a simple Ohm's law calculation.

Resistor **R** is connected in parallel with the 10Ω resistor, therefore the potential difference across these resistors are the same as the potential difference across the parallel combination:

$V_R = V_{10\,\Omega} = V_P = 7,5$ V

Resistors in parallel divide the current in a fixed ratio, therefore, the since the total current and the current through the 10 Ω resistor in parallel is known, the current through resistor **R** in parallel can be determined:

$I_R = I_{total} - I_{10\Omega}$

$I_R = (3) - (0,75)$

$I_R = 2,25$ A

Using Ohm's law, the resistor of resistor **R** can be determined:

$R = \frac{V}{I}$

$R = \frac{(7,5)}{(2,25)}$

$R = 3,33\ \Omega$

∴ Resistance of resistor **R** = 3,33 Ω

PRO-TIPS

When writing explanations represent the answer in **bullet point form**. It is easier for the marker to read and mark, and it ensures that you have covered all the needed points.

1.3 **Increases.**

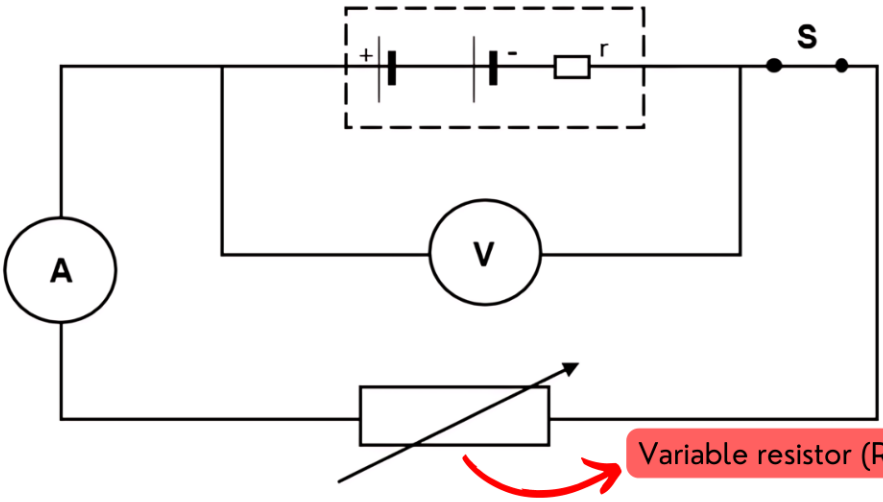
- When resistor **R** burns out, it creates a short circuit and no current flows through resistor **R**. Therefore, the 10 Ω resistor is now in series with the 12Ω resistor and the total external resistance increases, therefore, the total resistance increases and the total current in the circuit decreases, since the emf of the battery remains constant.

- From the formula , $\text{emf}\ (\varepsilon) = IR_{ext} + Ir$, $IR_{ext} = \text{emf}\ (\varepsilon) - Ir$ therefore if the emf and the internal resistance of the battery remains constant and the total current decreases, this results in less "lost" volts or $V_{\text{internal resistance}}$ decreases, therefore the voltage available to the external circuit (V_{load}) increases.

ELECTRIC CIRCUITS WITH INTERNAL RESISTANCE: COMMON GRAPHS

An experiment can be set up as shown in the diagram below to determine the emf and the internal resistance of a battery.

- A voltmeter is connected in parallel across a battery with an internal resistance, **r**.
- In the circuit a variable resistor (rheostat) is used. The function of the variable resistor is to vary the resistance in the circuit, therefore also varying the current in the circuit.



Variable resistor (Rheostat).

The data obtained from the experiment is displayed below:

READING ON AMMETER(A)	READING ON VOLTMETER (V)
0	1,5
0,25	1,3
0,5	1,1
0,75	0,85

Graph showing the relationship between the voltage (V) and current (A) for a circuit with internal resistance:

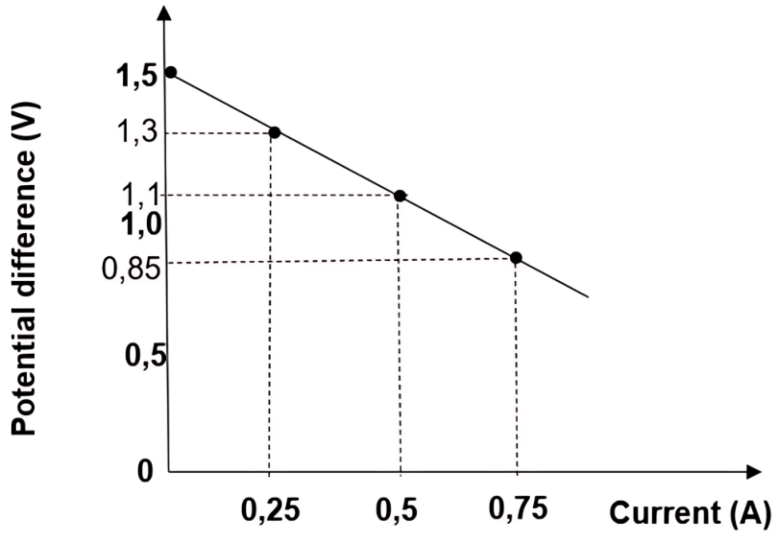


Figure 1

Interpreting the graph:

- **NOTE:** This is a potential difference versus current graph.
  The external resistance in the circuit is changed (or varied) using a variable resistor and the potential difference across the battery (V_{load}) is being measured.
- From the shape of the graph, it can be concluded that as the current INCREASES, the potential difference or voltage available to the external circuit (V_{load}) decreases.

Let's think why this makes sense:

As the current in the circuit increases, more current flows through the battery.

From the formula:

emf $(\varepsilon) = \mathrm{I}\,(\mathrm{R} + \mathrm{r})$

emf $(\varepsilon) = \mathrm{IR_{ext}} + \mathrm{Ir}$
$\mathrm{IR_{ext}}$ = emf $(\varepsilon) - \mathrm{Ir}$

The internal resistance (r) of the battery remains constant, and the current (I) increases, then **Ir** increases (this is the "lost volts"), and the number of "lost volts" (or $V_{internal\ resistance}$) increases.
If the number of lost volts increases, but the emf of the battery (ε) remains constant then the voltage available to the external circuit (V_{load}) decreases.

From the above graph in figure 1, the following can be concluded:

1. <u>Gradient of the graph</u>

gradient = $\frac{\Delta y}{\Delta x}$

gradient = $\frac{V}{I}$

gradient = **- r** (internal resistance)

- The gradient of the graph represents the **internal resistance.**
  The current in the circuit is being varied by changing the external resistance using the rheostat. However, the gradient of the graph is constant, therefore it represents the internal resistance, which remains constant.
- **NOTE:** the gradient is negative, however, internal resistance is a scalar quantity. To determine the internal resistance simply **use the positive answer**.

2. <u>Equation of a straight – line graph</u>

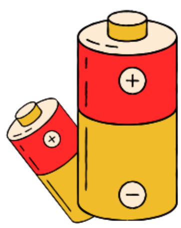
- The graph is a straight- line graph with a **negative** gradient (**Note:** the graph slopes to left). The equation for the standard form of a straight - line graph is:

  $$\mathbf{y = mx + c}$$

  Where:
  m = gradient of the graph. In this graph this is the internal resistance.
  c = y – intercept (where the graph cuts the y – axis).
  This is the emf of the battery, since the current at this point is zero.
  x = physical quantity on the x – axis. In this graph that is the current, $\mathbf{I}$.
  y = physical quantity on the y – axis. This is V_{load} or IR_{ext}.

The variables from the graph can be substituted into this formula y = mx + c:

$$y = mx + c$$
$$\therefore V_{load} = -rI + \varepsilon$$
$$IR_{ext} = -rI + \varepsilon$$
$$IR_{ext} = \varepsilon - Ir$$



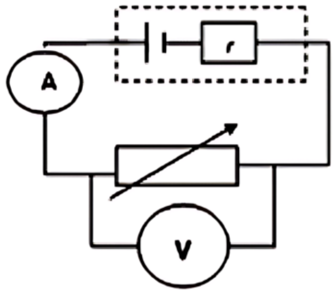
Make emf (ε) the subject of the formula:

$$\varepsilon = IR_{ext} + Ir$$
$$\text{emf}\,(\varepsilon) = I(R + r)$$

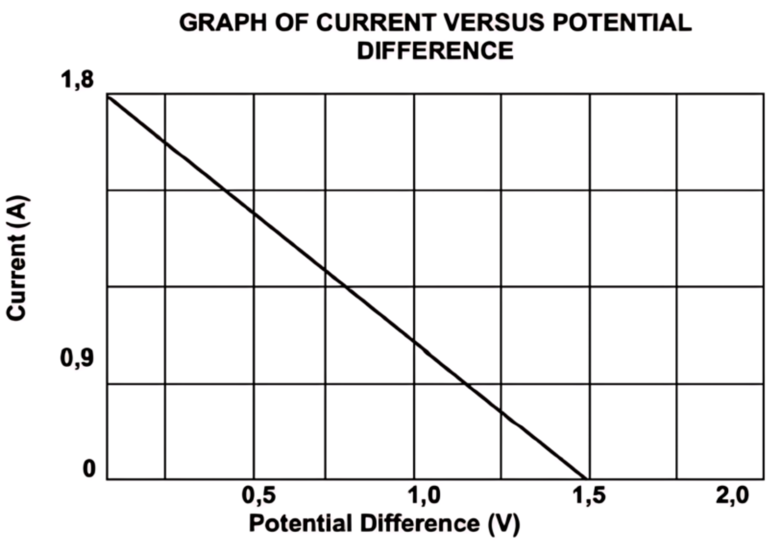
Worked example

C + A

1. Learners conduct an experiment as shown in the diagram below:



The results obtained are shown in the graph below.



Use the graph to determine the following:

1.1 The emf of the battery. (1)

1.2 The internal resistance of the battery - without using the equation emf (ε)= I(R+r). (4)

The resistance of the rheostat is now increased.

1.3 How will this change the voltmeter reading? Write down only INCREASES, DECREASES or REMAINS THE SAME. Explain the answer. (4)

1.1 **NOTE:** In this graph, the potential difference is on the x - axis. The emf of the battery is recorded when the current in the circuit is 0 A. Read from the graph:

Emf = 1,5 V

1.2 **Remember:** The internal resistance of the battery can be determined by calculating the gradient of the graph:

What physical quantity does the gradient of the graph represent?

gradient = $\frac{\Delta y}{\Delta x}$

gradient = $\frac{I}{V}$

gradient = $\frac{1}{r}$ (**inverse** of the internal resistance)

Gradient calculation
(Points chosen to calculate the gradient: Start and end point).

gradient = $\frac{\Delta y}{\Delta x}$

gradient = $\frac{(0)-(1,8)}{(1,5)-(0)}$

gradient = -1,2

Determine the internal resistance:

gradient = $\frac{1}{r}$

-1,2 = $\frac{1}{r}$

r = -0,83 Ω

∴ r = 0,83 Ω

NOTE: The voltmeter is connected parallel to the battery and reads V_{load} when the circuit is closed.

1.3 **Increases.**

- When the resistance of the rheostat is increased, the total external resistance increases, therefore, the total resistance increases and the total current in the circuit decreases, since the emf of the battery remains constant.

- From the formula , emf $(\varepsilon) = IR_{ext} + Ir$, $IR_{ext} = \text{emf}\ (\varepsilon) - Ir$ therefore if the emf and the internal resistance of the battery remains constant and the total current decreases, this results in less "lost" volts or $V_{internal\ resistance}$ decreases, therefore the voltage available to the external circuit (V_{load}) increases.

ELECTRODYNAMICS: MOTORS AND GENERATORS

Electrodynamics is the study that deals with the interaction between electricity, magnetism and movement (or motion).

Michael Faraday

In grade 11 you learned that an electric current (movement of charges) through a conductor induces a magnetic field. Michael Faraday used this concept to discover that a CHANGING magnetic field induces an electric current (and an emf).
The process is called **electromagnetic induction**, which is discussed later in this chapter.

Electrical machines: Generators and motors

In this chapter we will study two types of electrical machines, namely generators and motors.

GENERATORS

What is a generator?

A generator is a device that converts **mechanical energy (through rotation)** into **electrical energy (an induced current).**

The energy conversion in a generator.

A generator converts mechanical energy into electrical energy.

Types of generators.

There are TWO types of generators:

1. AC (alternating current) generator (also called an alternator)
2. DC (direct current) generator (also called a dynamo)

PRO-TIPS

The focus sections in electrical machines include:

- What is a generator or motor?
- Types of generators and motors (AC vs DC).
- It's simplified structure and the functions of the various components.
- Principles on which a generator and motor operate.
- Energy conversions in a generator and motor
- Advantages and disadvantages of AC and DC.

How does a generator work?
(On what principle does it operate?)

Generators operate on the principle of **electromagnetic induction,** developed by Michael Faraday. According to Michael Faraday if a conductor (e.g., a coil) **rotates or moves** through a magnetic field, this **changing** magnetic field results in a changing magnetic flux, causing the electrons in the conducting wire to move, inducing a current and an and emf (potential difference or voltage).

PRO-TIPS

Recall from grade 11 that **Magnetic flux** is the number of magnetic field lines passing through the surface area (of a coil); and acts perpendicular to the surface area.
Simply put, it is the "flow" of the magnetic field.

Definition: **Electromagnetic induction**: When a conductor is moved in a magnetic field, a current is induced across the conductor.

1. Alternating current (AC) generator (alternator)

What is alternating current? 

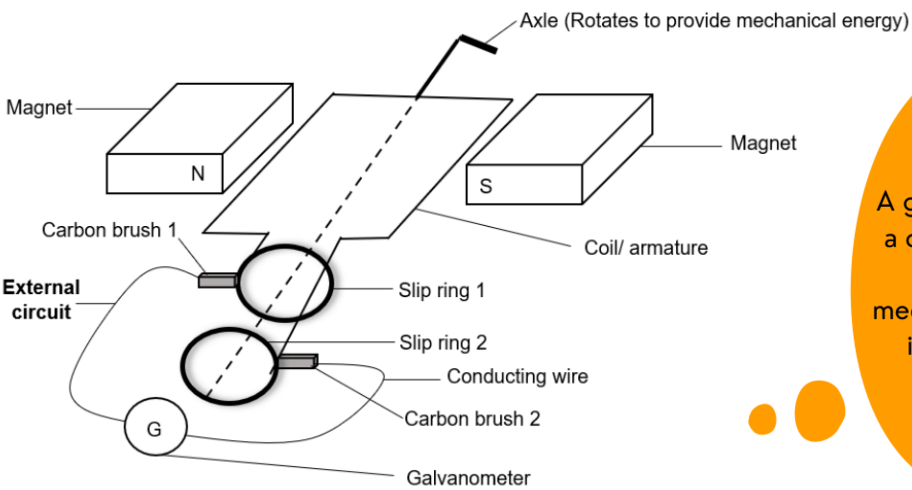
Alternating current is current that changes direction periodically. (i.e., at regular intervals or cycles).

In an AC generator, a coil is rotated through a magnetic field. The magnetic flux passing through the surface area of the coil is constantly changing. According to Faraday's law (of electromagnetic induction), an emf is induced in the coil, which in turn induces a current in the coil or circuit.

PRO-TIPS

Recall from grade 11 that **Faraday's law (of electromagnetic induction)** states that the induced emf in any closed circuit is equal to the rate of change of magnetic flux through the circuit.

Simplified structure of an AC generator



PRO-TIPS

NOTE:
A generator does **not** have a cell or battery or power supply as it converts mechanical energy (motion) into electrical energy.

Components and the functions of the components of an AC generator

- **Magnets:** Generates a magnetic field.
  Direction of the magnetic field: Magnetic field lines run away from north towards south (i.e., From North to South).

- **Coil/ armature:** Made of conducting wire (e.g., copper wire) winded into a coil or loops.

- **Axle:** Ensures that the coil and the slip rings turn and function as a unit. When the axle is rotated it provides mechanical energy to the system.

- **Slip rings:** The AC generator consists of two slip rings, referred to as slip ring **1** and slip ring **2**.
  The slip rings are made of copper bands around insulated rings.
  The slip rings are connected to the ends of the coil/ armature with one end of the coil (e.g. left side of the coil) connected to one slip ring (e.g. slip ring 1) and the other end of the coil (e.g. the right side of the coil) connected to the other slip ring (e.g. slip ring 2).
  The slip rings rotate with the coil and they help in the rotation of the coil.

Functions of the slip rings: Main function of the slip rings

1. **Provides electrical contact with the carbon brushes and ensures that the direction of the current in the external circuit is the same as the direction of the current in the coil.**
2. Assists with the rotation of the coil by preventing the conducting wire from entangling and damaging.

Each slip ring is in contact with (but not connected to) its own carbon brush.
When the coil rotates, the slip rings rotate with the coil and the rings just slip past the brushes, therefore they are called slip rings.
A slip ring can be used in an AC generator that requires a continuous rotation while transmitting power.

- **Carbon brushes:** The AC generator consist of two carbon brushes, namely

carbon brush **1** and carbon brush **2**. Each carbon brush makes contact with its own slip ring; however, it is not connected to the slip ring.

Functions of the carbon brushes:

1. Conducts the current induced in the coil to the external circuit.
2. Provides electrical contact.

- **Galvanometer:** A device used for detecting and measuring small amounts of current and voltage (potential difference).
  NOTE: The galvanometer can be replaced with a light bulb, ammeter or voltmeter.

Determining the direction of the induced current in the coil and in the external circuit of a generator or the direction the coil will rotate

When a coil is mechanically rotated through the magnetic field, it induces a current in the coil and in the external circuit. The galvanometer registers a reading.
Consider the AC generator shown below in **figure 1**, with the coil rotating clockwise.

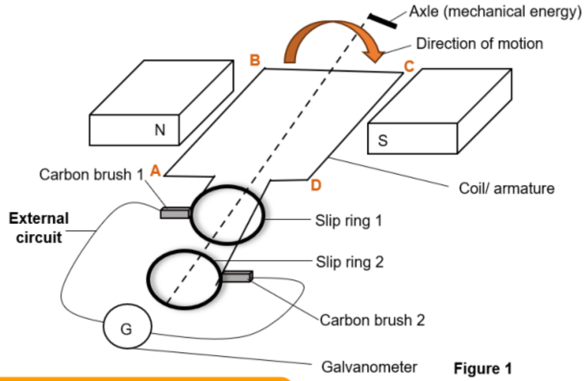


Figure 1

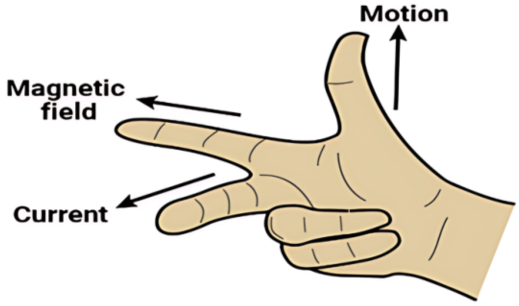
Fleming's Right Hand Rule for generators

Fleming's Right Hand Rule can be used to determine the:

- direction in which the induced current will flow in the coil and therefore the external circuit.
  OR
- the direction that the coil will rotate (clockwise or anticlockwise)
  OR
- the direction of the magnetic field (and therefore the polarity of the magnets)…in a **generator**.

How to use Fleming's Right Hand Rule (only applied to generators):

Hold your thumb, first finger (index finger) and second finger (middle finger) at **RIGHT ANGLES (90°)** to each other as seen below.



- **Thumb** points in the direction of **MOTION (Force).** **F**
- **First finger** points in the direction of the **magnetic FIELD, B,** from N to S. **B**
- **Second finger** points in the direction of the **CURRENT (induced current).** **I**

Apply Fleming's right hand rule to a AC generator, shown in **figure 1** below with the **coil rotating clockwise.**

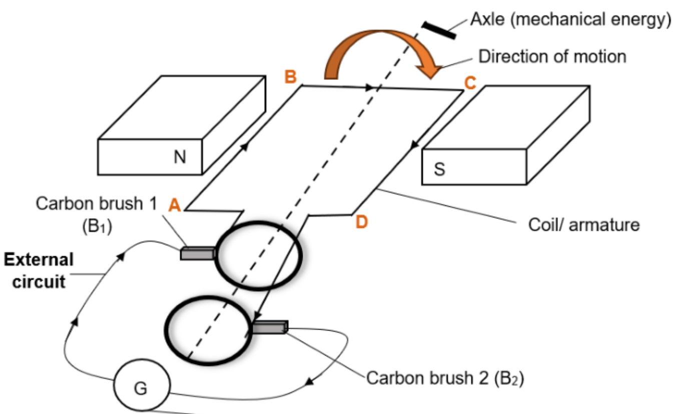


Figure 1

PRO-TIPS

The acronym '**FBI**' can be used to help you remember what each finger in Fleming's Right Hand Rule represents. Remember from grade 11 that the symbol for magnetic field is **B**.

In **figure 1** above, the coil is rotating in a clockwise direction, this means that side **AB** of the coil moves up and side **CD** of the coil moves down.

Using Fleming's right – hand rule, the induced current in the coil will flow from **A** to **B** to **C** to **D** (**ABCD**) then to the external circuit, from brush $\mathbf{B}_1$ to $\mathbf{B}_2$.

In **figure 2** below, the coil continues to rotate clockwise and completes half a revolution (180°) . Remember that the slip rings rotate with the coil. Side **AB** is now moving down and side **CD** is now moving up.
Applying Faraday's Right Hand Rule, the current in the coil flows from **D** to **C** to **B** to **A** (**DCBA**) and from **brush B_1 to B_2** in the external circuit.

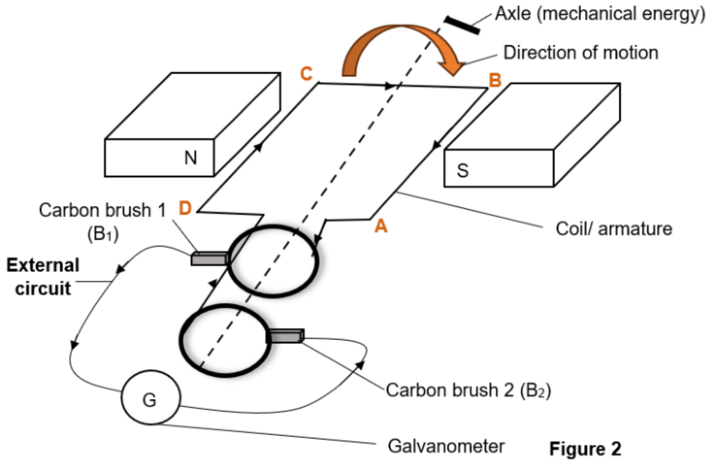


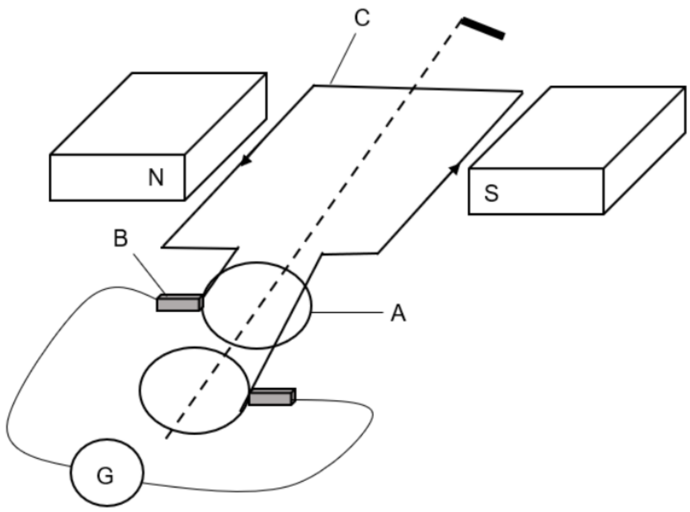
Figure 2

NOTE: When comparing the direction of current flow in figure 1 and figure 2, **the current changed direction in both the coil and the external circuit, since it is alternating current.**

Worked example

C + A

1. Below represents a simplified diagram of an AC generator:



1.1 Identify component **A, B** and **C**. (3)
1.2 Write down ONE function of component **A**. (1)
1.3 Write down ONE function of component **B**. (1)
1.4 Determine whether the coil is rotating CLOCKWISE or ANTI- CLOCKWISE. (2)

1.1 **NOTE:** The labels on the diagrams given must be learned, including the functions.

A - Slip ring
B - Carbon brush
C - Coil or armature

1.2 **NOTE:** The slip ring has two functions, however, the function listed below is the **main and most accepted function** in the exams.

Slip rings provide electrical contact with the carbon brushes and ensures that the direction of the current in the external circuit is the same as the direction of the current in the coil.

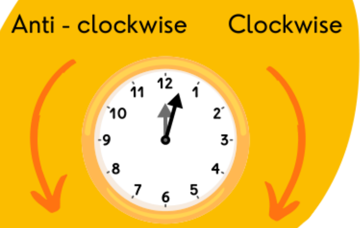
1.3 **NOTE: Any** of the functions of the carbon brushes listed below are accepted.

Conducts the current induced in the coil to the external circuit.

OR

Provides electrical contact.

PRO-TIPS



1.4 Anti - clockwise

This is an AC generator. Therefore, Faraday's Right Hand Rule can be used to determine the direction in which the coil is rotating.
Remember to use your RIGHT HAND.

Consider the **left - hand side of the coil** (closest to the North end of the magnet):

- The current is flowing **TOWARDS** you or **OUT OF THE PAGE**. Using Fleming's Right Hand Rule to determine the direction of motion:

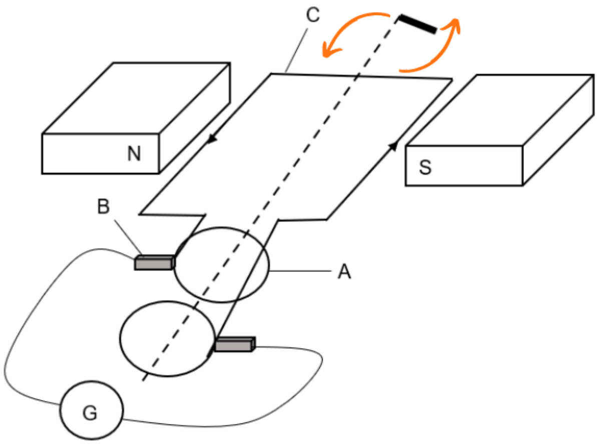
1. Point your first finger in the direction of the magnetic field, namely **NORTH** to **SOUTH**
2. Point your second finger **TOWARDS YOU**
3. Your thumb should point **DOWN/ DOWNWARDS**, this is the direction of the force and therefore the motion of the left - hand side of the coil.

Repeat the above, looking at the **right - hand side of the coil** (closest to the South end of the magnet):

- The current is flowing **AWAY FROM** you or **INTO THE PAGE**. Use Fleming's Right Hand Rule to determine the direction of motion:

1. Point your first finger in the direction of the magnetic field, namely **NORTH** to **SOUTH**
2. Point your second finger **AWAY FROM** you
3. Your thumb should point **UP/ UPWARDS**, this is the direction of the force and therefore the motion of the left - hand side of the coil.

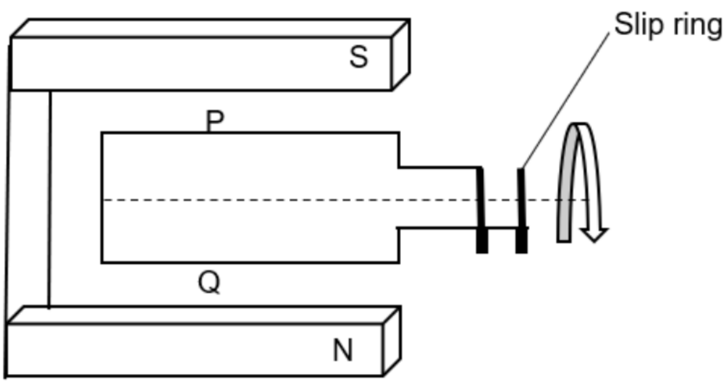
The diagram below shows how the coil rotates anti - clockwise:



Worked example

C + A

2. Below shows a simplified diagram of a generator, rotating anti-clockwise.



2.1 Give a reason why the above generator is an AC generator? (1)

2.2 In which direction will the current flow? Write down From **P** to **Q** or From **Q** to **P**. (2)

2.1 The generator has **slip rings,** therefore it is an AC generator.

One of the main ways to identify the **type** of generator, is to look at whether it has slip rings, which makes it an AC generator OR split ring commutators, which makes it a DC generator. We will learn about DC generators soon.

2.2 From **P** to **Q**

NOTE: The set of the magnets is not the usual horizontal set up, but rather a vertical set up.
The coil is rotating anti - clockwise (see arrow for guidance), therefore side **P** is moving TOWARDS YOU and side **Q** is moving **AWAY FROM YOU**.

This is an AC generator. Therefore, Faraday's Right Hand Rule can be used to determine the direction in which the coil is rotating. Remember to use your RIGHT HAND.

Consider side **P** of the coil (closest to the South end of the magnet):
Using your RIGHT HAND:

1. Point your **first finger** in the direction of the magnetic field, namely NORTH to SOUTH (upwards)
2. Point your **thumb TOWARDS YOU**, in the direction of the motion.
3. Your second finger should point **LEFT,** this is the direction in which the current is flowing.

Repeat the above, looking at side **Q** (closest to the North end of the magnet):
Using your RIGHT HAND:

1. Point your **first finger** in the direction of the magnetic field, namely NORTH to SOUTH (upwards)
2. Point your **thumb AWAY FROM YOU**, in the direction of the motion.
3. Your second finger should point **RIGHT,** this is the direction in which the current is flowing.

Therefore, the current flows from **P** to **Q**.

How does the AC generator work?

When the coil rotates within a magnetic field, this results in a change in magnetic flux. According to Faraday's law, the magnitude of the induced emf is **proportional to the rate of change of the magnetic flux.**

An AC generator is represented in **figure 3** below.

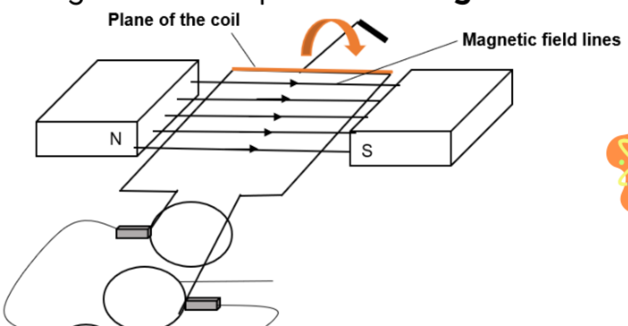


Figure 3

NOTE from figure 3: the magnetic field lines, the plane of the coil and the rotation of the coil, clockwise.

From Faraday's law of electromagnetic induction, it can be deduced that:

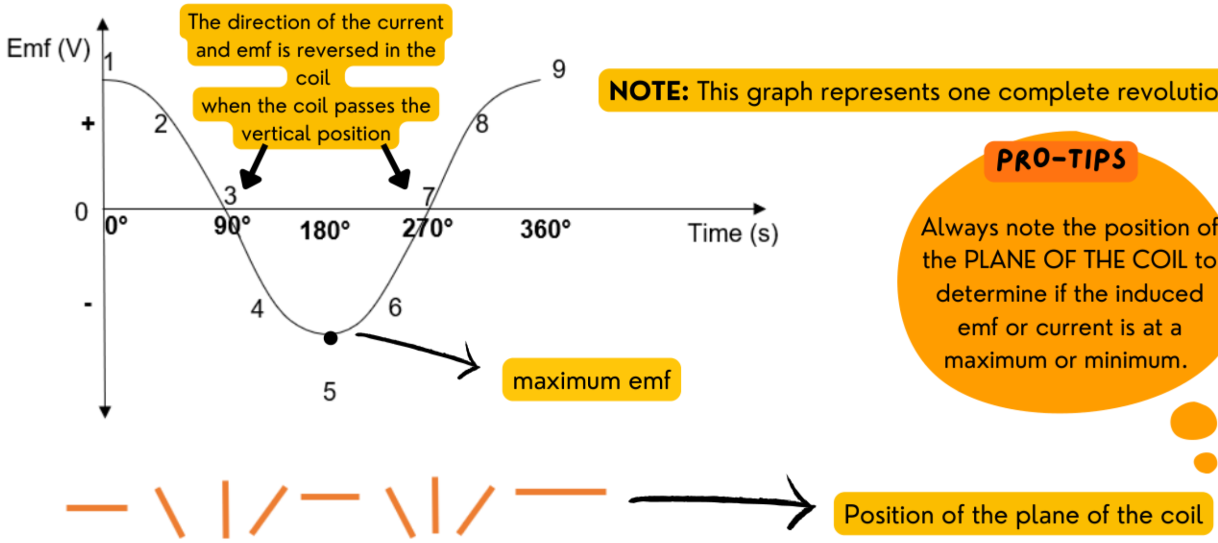
- If the coil cuts the maximum number of magnetic field lines per second (or per unit time), resulting in the greatest rate of change in magnetic flux therefore the maximum current and maximum emf will be induced in the circuit.
  This is when the plane of the coil is parallel to the magnetic field lines.

- If the coil cuts fewer magnetic field lines per second (or per unit time), this results in a smaller rate of change in magnetic flux, and therefore the current and emf induced in the circuit will decrease.
  This is when the plane of the coil is at an angle to the magnetic field lines.

- If the coil cuts no magnetic field lines, this results in no change in magnetic flux and no current or emf will be induced in the circuit.
  This is when the plane of the coil is perpendicular to the magnetic field lines.

The changes in the induced emf as the coil rotates can be represented in an induced **emf versus time graph**, as shown below. Below each point on the graph, the position of the plane of the coil is represented:

Induced emf vs time graph



NOTE: This graph represents one complete revolution

PRO-TIPS

Always note the position of the PLANE OF THE COIL to determine if the induced emf or current is at a maximum or minimum.

Explanation of the graph and the AC generator:

1 From **figure 3**, the coil is in a horizontal position initially (0°), where **the plane of the coil is parallel to the magnetic field lines** and the maximum number of magnetic field lines are being cut per second or per unit time, resulting in the greatest rate of change in magnetic flux, therefore the maximum induced emf and induced current.

2 As the coil rotates towards the vertical position (0° - 90°).
Fewer magnetic field lines are being cut per second or per unit time, therefore the induced emf and induced current decreases. **The plane of the coil is at an angle to the magnetic field lines.**

3 The coil reaches the vertical position (90°), where **the plane of the coil is perpendicular to the magnetic field lines.** The magnetic field lines pass through the surface of the coil. No magnetic field lines are cut, therefore the change in magnetic flux is zero and the induced emf and current is zero.

4 As the coil rotates towards the horizontal position (90° - 180°)
More magnetic field lines are being cut per second or per unit time, resulting in a greater rate of change in magnetic flux, therefore the induced emf and induced current increases. **The plane of the coil is at an angle to the magnetic field lines.**

5 The coil is in a horizontal position (180°) and the maximum number of magnetic field lines are being cut per second or per unit time, resulting in the greatest rate of change in magnetic flux, therefore the maximum induced emf and induced current. **The plane of the coil is parallel to the magnetic field lines.**

6 As the coil rotates towards a vertical position (180° - 270°), fewer magnetic field lines are being cut per second or per unit time, therefore the induced emf and induced current decreases. **The plane of the coil is at an angle to the magnetic field lines.**

7 The coil reaches the vertical position (270°), **where the plane of the coil is perpendicular to the magnetic field lines.** The magnetic field lines pass through the surface of the coil.
No magnetic field lines are cut, therefore the change in magnetic flux is zero and the induced emf and current is zero.

8 As the coil rotates towards the horizontal position (270°- 360°).
More magnetic field lines are being cut per second or per unit time, resulting in a greater rate of change in magnetic flux, therefore the induced emf and induced current increases.
The plane of the coil is at an angle to the magnetic field lines.

9 The coil is in a horizontal position (360°) and the maximum number of magnetic field lines are being cut per second or per unit time, resulting in the greatest rate of change in magnetic flux, therefore the maximum induced emf and induced current.

PRO-TIPS

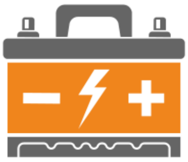
From figure 3:
If the coil started in a vertical position, such that no magnetic field lines were cut, the initial emf would be zero and the graph would form a sin wave.

The graph can also be an **induced current versus time graph**, which will represent how the induced current in the coil and the external circuit changes with time. The shape of the graph (cosine wave) remains the same.

Uses of AC generators

- AC generators are used **in electrical power plants to produce electricity** for mass distribution e.g. to homes, industries, offices, factories etc. This electricity is produced by massive power plants and usually has a low voltage which is converted to high voltage using a step- up transformer. **It is also used in diesel generators.**

- **In cars to recharge the battery while the car is being driven.** Cars have alternators. When the car's engine is running the alternator charges its battery and powers the car's electric system.



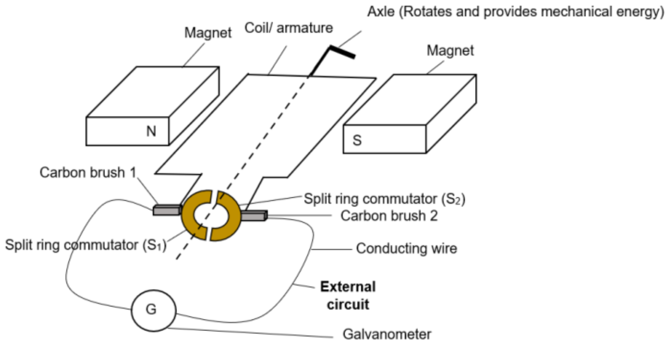
DC GENERATOR

2. Direct current (DC) generator (Dynamo)

What is direct current?
Direct current is current that flows in one direction only.
In a DC generator, the direction of the current in the **external circuit** flows in one direction only.

Below shows a simplified structure of a DC generator. A DC generator can also be called a dynamo.
Notice that the structure of a DC generator is similar to that of an AC generator, however there is ONE big difference. Can you notice ONE difference it has compared to an AC generator?

Simplified structure of a DC generator (Dynamo)



PRO-TIPS

Did you notice the ONE difference between an AC generator and a DC generator? An AC generator has slip rings, whereas a DC generator has split ring commutators!

Components and the functions of the components of a DC generator

- **Magnets:** Produce a magnetic field.
  Direction of the magnetic field: Magnetic field lines run away from north towards south (i.e., North to South).

- **Coil/ armature**: Made of conducting wire (e.g. copper wire) winded into a coil or loops.

- **Axle:** Ensures that the coil and the split ring commutators turn and function as a unit. When the axle is rotated it provides mechanical energy to the system.

- **Split ring commutator:** The DC generator consists of **split- ring commutators**.
  (notice that the ring is split in two!) Split ring commutators consist of a copper ring that is split in the middle resulting in two halves which can be referred to as S_1 and S_2, which are insulated from each other. The one half of the split ring commutator (S_1) is connected to the one side of the coil and the other half of the split ring commutator (S_2) is connected to the other side of the coil. Each half of the split – ring commutator is in contact with (but not connected to) ONE carbon brush, for example S_1 is in contact with carbon brush B_1 and S_2 is in contact with carbon brush B_2. When the coil rotates, the split ring commutator rotates with the coil.

Function of the split ring commutator:

The split ring commutator ensures that current in the external circuit flows in one direction only; It converts AC in the coil/armature to DC in the external circuit.

- **Carbon brushes:** The DC generator, like the AC generator consists of two carbon brushes, namely carbon brush **1** and carbon brush **2**. Each carbon brush makes contact with its own half of the split ring commutator; however, it is not connected to the split ring commutator.

  Functions:
  1. Conducts the current induced in the coil to the external circuit
  2. Provides electrical contact.

- **Galvanometer:** A device used for detecting and measuring small amounts of current and voltage (potential difference).

NOTE: The galvanometer can be replaced with a light bulb, ammeter or voltmeter.

Determining the direction of the induced current in the coil and in the external circuit of a generator or the direction the coil will rotate

When a coil is mechanically rotated through the magnetic field, it induces a current in the coil and in the external circuit. The galvanometer registers a reading.
Consider the DC generator shown below in **figure 4**, with the coil rotating clockwise.

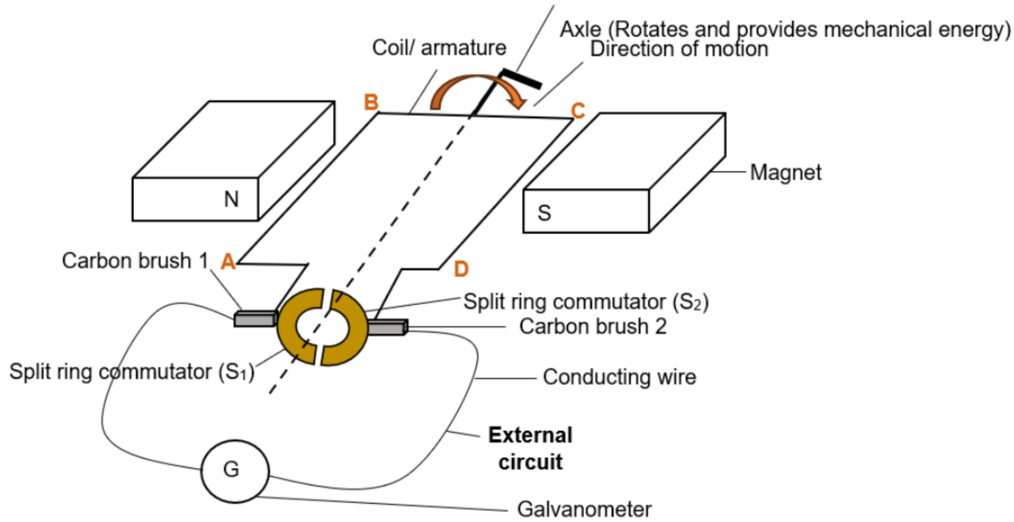


Figure 4

Fleming's Right Hand Rule for generators

Fleming's Right Hand Rule can be used to determine the:

- direction in which the induced current will flow in the coil and therefore the external circuit.

OR

- the direction that the coil will rotate (clockwise or anticlockwise)

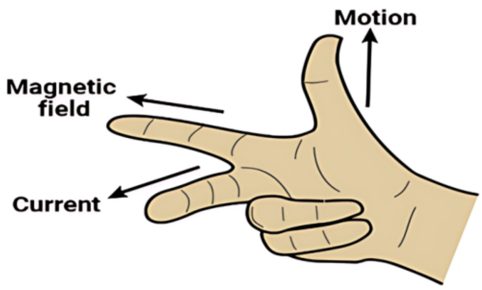
OR

- the direction of the magnetic field (and therefore the polarity of the magnets)

…in a **generator**.

How to use Fleming's Right Hand Rule (only applied to generators):

Hold your thumb, first finger (index finger) and second finger (middle finger) at **RIGHT ANGLES (90°)** to each other as seen below.



- **Thumb** points in the direction of **MOTION (Force).** **F**
- **First finger** points in the direction of the **magnetic FIELD, B,** from N to S. **B**
- **Second finger** points in the direction of the **CURRENT (induced current).** **I**

Apply Fleming's right hand rule to a DC generator, shown in **figure 4** below with the coil **rotating clockwise**.

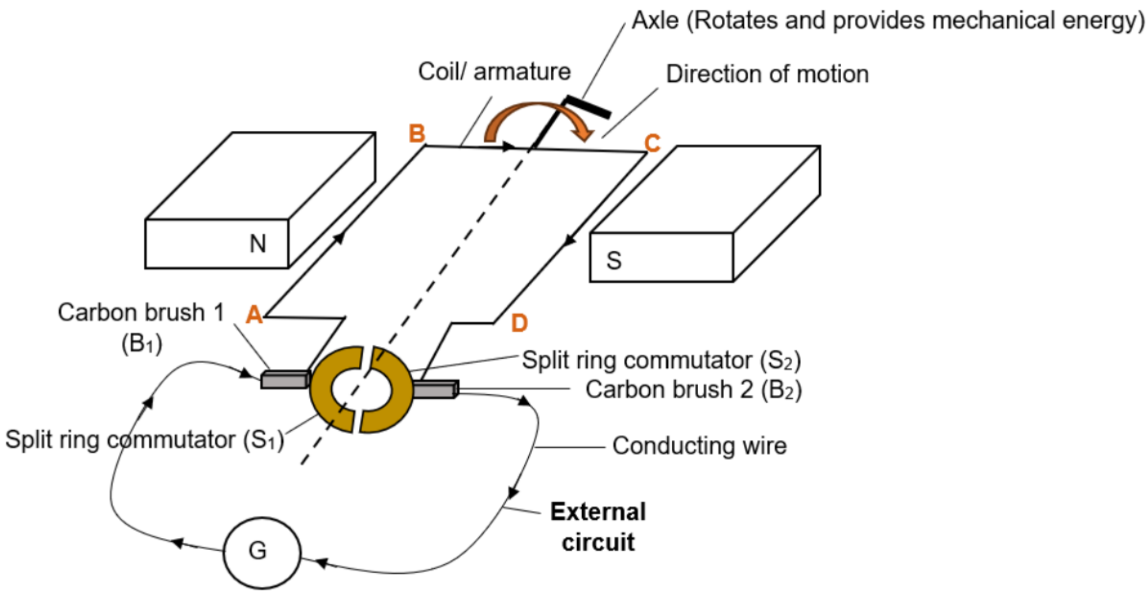


Figure 4

- In **figure 4** above, the coil is rotating in a clockwise direction, this means that side **AB** of the coil moves UP and side **CD** of the coil moves DOWN.
- Using Fleming's right – hand rule, the induced current will flow from **A** to **B** to **C** to **D** (**ABCD**) in the coil then to the external circuit, from brush **B**$_1$ to **B** $_2$.

- In **figure 5** below, the coil has completed half a revolution (180°).

When the coil reaches a vertical position, the brushes lose contact with the split ring commutators and no current flows, however, if we continue turning it clockwise, side **AB** is now moving DOWN and side **CD** is now moving UP, the brushes make contact with the split ring commutator and hence the current now flows from **D** to **C** to **B** to **A** (**DCBA**) in the coil and still from brush $\mathbf{B}_1$ to brush $\mathbf{B}_2$ in the external circuit. It is due to the loss of contact between the brushes and the split ring commutator that results in the current in the external circuit not changing direction.

NOTE: The current in the coil is alternating current and changes direction periodically. However, the current in the external circuit is direct current and does not change direction – it continues to move from brush 2 to brush 1 as the coil rotates.

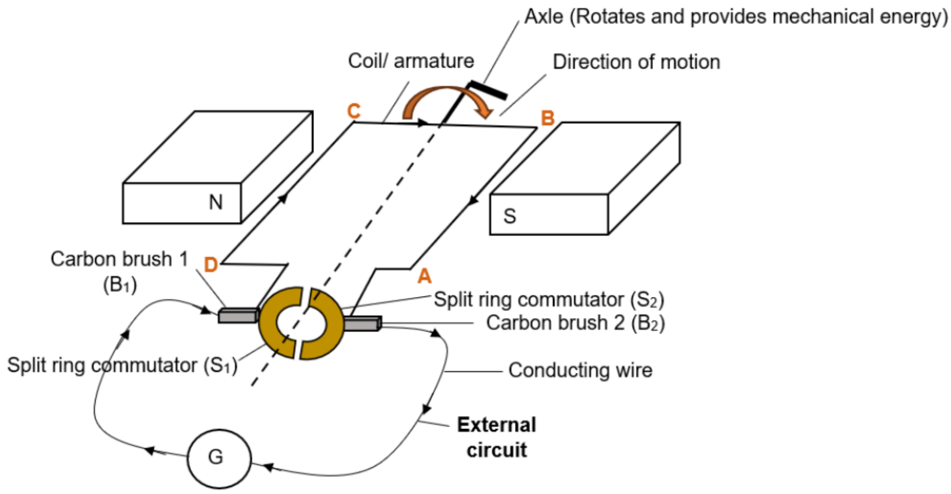
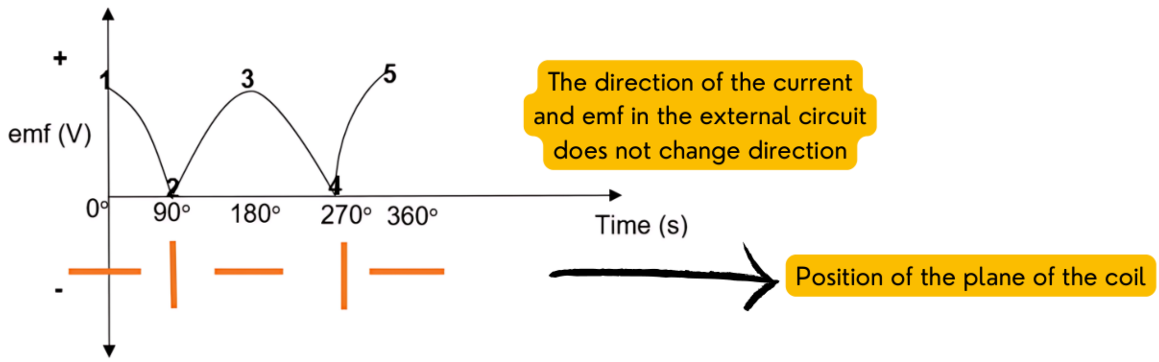


Figure 5

Emf vs time graph representing DC current in the external circuit

Direct current in the external circuit, is current that flows in one direction only, this can be depicted on an emf vs time graph as shown below.



The graph above shows how the **emf** in the external circuit of a DC generator varies with time and the position of the plane of the coil for one **full revolution**. The direction of the current in the external circuit **never** changes, but varies in **magnitude** as the coil rotates. This is because as the coil rotates, the rate of change of magnetic flux in the coil changes, inducing a changing emf in the coil.

In the DC generator, the graph for emf versus time is the same as for the AC generator inside the coil, the emf delivered to the external circuit, is also the same, **except** for the fact that it never becomes **negative**, because it is delivered in **one** direction **only**.

Explanation of the graph and the DC generator:

1. From **figure 5**, the coil is in a horizontal position initially (0°), where the plane of the coil is parallel to the magnetic field lines and the maximum number of magnetic field lines are being cut per second or per unit time, resulting in the greatest rate of change in magnetic flux, therefore the maximum induced emf and induced current.

2. The coil reaches the vertical position (90°), where the plane of the coil is perpendicular to the magnetic field lines. The magnetic field lines pass through the surface of the coil. No magnetic field lines are cut, therefore the change in magnetic flux is zero and the induced emf and current is zero.

3. The coil is in a horizontal position (180°), where the plane of the coil is parallel to the magnetic field lines and the maximum number of magnetic field lines are being cut per second or per unit time, resulting in the greatest rate of change in magnetic flux, therefore the maximum induced emf and induced current.

4. The coil reaches the vertical position (270°), where the plane of the coil is perpendicular to the magnetic field lines. The magnetic field lines pass through the surface of the coil. No magnetic field lines are cut, therefore the change in magnetic flux is zero and the induced emf and current is zero.

5. The coil is in a horizontal position (360°), where the plane of the coil is parallel to the magnetic field lines and the maximum number of magnetic field lines are being cut per second or per unit time, resulting in the greatest rate of change in magnetic flux, therefore the maximum induced emf and induced current.

Uses of DC generators

1. Used in bicycle dynamos to power bicycle lights.
2. Torches that use a DC generator to produce light by winding or shaking the torch.
3. Battery chargers.

AC and DC generators

Ways to increase the output/ induced emf/induced current of the generator:

1. **Increase** the number of **turns/windings** on the coil (i.e. increase the surface area of the loop).
2. **Increase** the **speed** at which the coil/ armature is rotated.
3. Use **stronger magnets** to increase the strength of the magnetic field (e.g. electromagnets).
4. Use curved magnets.
5. Use iron core.

Advantages of using AC (alternating current) over DC (direct current)

1. It is easier to **step** AC voltage up or down using a **transformer**.
2. It is easier and **cheaper** to convert AC to DC (than DC to AC).
3. AC voltages can be stepped up and electrical power can be transmitted over **long distances** at low current to **minimize** the **power loss** due to heating.

Less energy is converted into other forms of energy when transmitting AC current at high voltage compared to DC current over long distances.

Worked examples

Multiple choice question

C

1. The component of a DC generator that ensures that the current continuously moves in ONE DIRECTION in the external circuit is the …

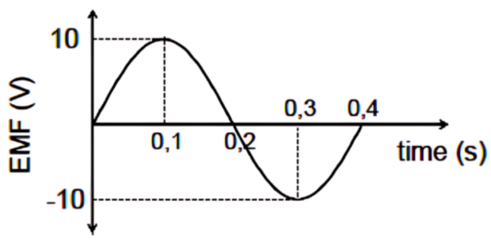
A Split ring commutator
B Slip rings
C Carbon brushes
D Battery

Answer: A
A DC generator has a split ring commutator and an AC generator has slip rings.
A generator does not have a battery as it converts mechanical energy into electrical energy.
The function of the split ring commutator is to ensure that current in the external circuit flows in one direction only; It converts AC in the coil/armature to DC in the external circuit.

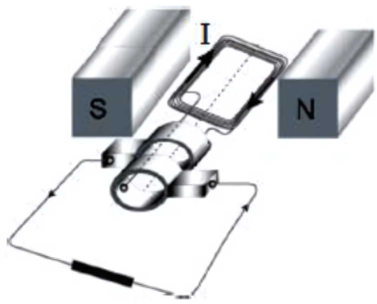
Multiple choice question

C + A

2. The graph below shows the graph of the induced emf versus time for a coil of a number turns rotating with a speed in a uniform magnetic field.



The electrical machine is represented below. The rotation speed of the coils is INCREASED.



Which ONE of the following represents the type of electrical machine shown above and how increasing the rotation speed will affect the induced emf represented by the graph?

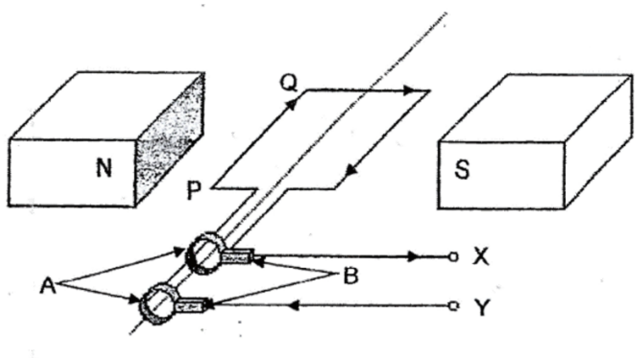
	TYPE OF ELECTRICAL MACHINE	INDUCED EMF
A	AC generator	10 V
B	AC generator	Greater than 10 V
C	DC generator	10 V
D	DC generator	Greater than 10 V

Answer: B

- The electrical machine is an AC generator because it has slip rings.
- If the rotational speed is increased, this will increase the induced emf because it will increase the number of magnetic field lines being cut per second or per unit time and therefore result in a a greater rate of change of magnetic flux. The induced emf will be GREATER than the original maximum emf (10 V) as depicted by the graph.

Worked example (C + A)

1. The simplified sketch below shows the principle of operation of the alternating current (AC) generator.



PRO-TIPS

Graph interpretation:
Period: Time taken to complete one full wave or oscillation. $T = \frac{1}{f}$

Frequency: Number of wave cycles or oscillations per second. $f = \frac{1}{T}$

Wave speed: $v = f\lambda$

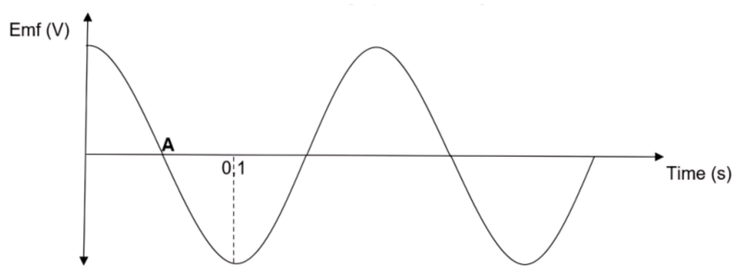
1.1 Name the parts labelled **A** and **B**. (2)

1.2 In which direction does segment **PQ** of coil have to be rotated to cause the current direction as shown in the diagram? Write down only CLOCKWISE or ANTI – CLOCKWISE. (1)

1.3 Write down TWO changes that can brought about improve the output of the generator. (2)

1.4 What change must be made to the AC generator to make it function as a direct – current (DC) generator? (1)

1.5 The induced emf versus time graph for the AC generator is shown below:



1.5.1 Write down the position of the coil at point **A**. Write down only VERTICAL or HORIZONTAL. (1)

1.5.2 Calculate the frequency of the wave. (3)

1.5.3 The coil is now turned at HALF the original speed. Write down the new period of the wave. (3)

1.5.4 Sketch a graph to show how the above waveform changes, after changing this generator into a DC generator. No values need to be indicated on the graph. (2)

1.1 **A** - slip rings
B - carbon brushes

1.2 Clockwise

This is an AC generator. Therefore, Faraday's Right Hand Rule can be used to determine the direction in which the coil is rotating. Remember to use your RIGHT HAND.

Consider side **PQ**, the left - hand side of the coil
The current is flowing **AWAY FROM YOU** or **INTO THE PAGE**. Using Fleming's Right
Hand Rule to determine the direction of motion:

1. Point your first finger in the direction of the magnetic field, namely **NORTH** to **SOUTH**
2. Point your second finger **AWAY FROM YOU**
3. Your thumb should point **UP** or **UPWARDS**, this is the direction of the force and therefore the motion of the left - hand side of the coil.

Repeat the above, looking at the right - hand side of the coil (closest to the South end of the magnet):
The current is flowing **TOWARDS YOU** or **OUT OF THE PAGE**. Use Fleming's Right Hand Rule to determine the direction of motion:

1. Point your first finger in the direction of the magnetic field, namely **NORTH** to **SOUTH**
2. Point your second finger **TOWARDS YOU.**
3. Your thumb should point **DOWN** or **DOWNWARDS**, this is the direction of the force and therefore the motion of the left - hand side of the coil.

Side **PQ** (and the coil) rotates **CLOCKWISE**.

1.3
1. Increase the number of turns/windings on the coil (i.e. increase the surface area of the loop)
2. Increase the speed at which the coil/ armature is rotated.
3. Use stronger magnets to increase the strength of the magnetic field (e.g. electromagnets).

Any two.
NOTE: The words in brackets do not need to be stated.

1.4 The slip rings must be replaced with a split ring commutator.

Remember that the split ring commutator ensures that current in the external circuit flowing in one direction only; It converts AC in the coil/armature to DC in the external circuit.

1.5 These questions involve graph interpretation questions.

1.5.1 Vertical position.

- From the graph, at position **A** the induced emf is 0V, therefore this means that the induced emf is 0V. This occurs when the plane of the coil is PERPENDICULAR to the magnetic field lines and therefore no magnetic field lines are being cut.
  The rate of change of magnetic flux is zero.
- From the diagram of an AC generator given, when the coil is in the VERTICAL position, the plane of the coil is perpendicular to the magnetic field lines, and the induced emf is zero.

1.5.2 The frequency is the number of oscillations or wave cycles per second, in the case of the generator, it is the number of complete revolutions per second (in one second).
The speed at which the coil moves is not given, but the period of the wave can be determined from the graph, which can be used to calculate the frequency: $f = \frac{1}{T}$

From the graph:

- The time given, 0,1 s, represents half a revolution (wave cycle). Therefore one full revolution or wave cycle will take DOUBLE the time. This is the period of the wave.

$T = 2(0{,}1)$

$T = 0{,}2$ s → It takes 0,2 s for the coil to make one full revolution or wave cycle

$f = \frac{1}{T}$

$f = \frac{1}{(0{,}2)}$

$f = 5$ Hz → The coil makes 5 revolutions or wave cycles in 1 second.

1.5.3 The speed of rotation is unknown, however:
From the formula $v = f\lambda$,$v \propto f$. From the formula $T = \frac{1}{f}$, frequency and period are inversely proportional.
The relationship between these variables can be used to determine how the period will be affected if the wave speed is halved.

$v = f\lambda$

$f \propto v$

$f \propto \frac{1}{2}$

$\therefore \frac{1}{2}f$ → If the rotation speed is halved, the frequency is halved

$T = \frac{1}{f}$

$T \propto \frac{1}{f}$

$T \propto \frac{1}{\left(\frac{1}{2}\right)}$

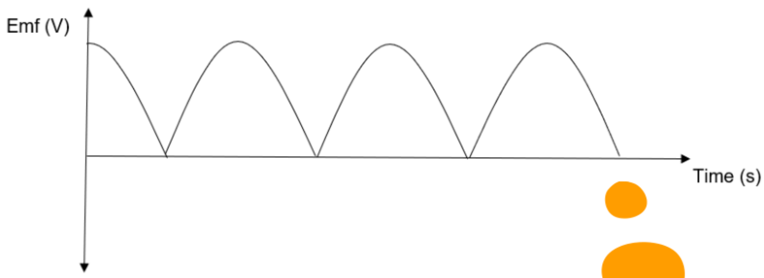
$\therefore 2T$ → If the frequency is halved, the period is doubled

$\therefore$ New period of wave = 2(0,2)

$\therefore$ New period of wave = 0,4 s → The new period is double the initial period of 0,2 s as calculated in question 1.5.2

1.5.4 A DC generator is a direct current generator, which means that the current in the external circuit flows in one direction and never changes direction. Therefore the emf versus time graph for a DC generator will only be drawn above the x - axis. Remember that as the coil is rotated, the rate of change of magnetic flux constantly changes, therefore the induced emf still varies:

<u>Emf versus time graph for a DC generator</u>



PRO-TIPS

Graph sketching tips:

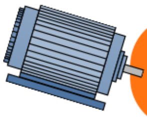
- All graphs must be drawn in pencil.
- Make sure your graph is large and easy to read.
- The graph must have a suitable heading that includes the independent and dependent variable. Underline the heading.
- Make sure that BOTH axes are labelled, with units.

REMINDER :QUESTION DIFFICULTY

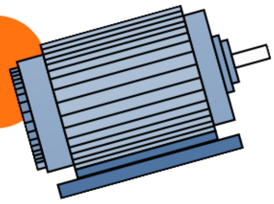
C — COMPREHENSION AND RECALL QUESTIONS
These are common questions which include definitions and calculation questions that look similar to the questions covered in class. Approximately **50%** of Paper 1 (Physics) will include questions on this level.

A — ANALYSIS AND APPLICATION QUESTIONS
These are more complex questions which involves applying the knowledge and skills learned in this chapter. Approximately **40%** of Paper 1 (Physics) will include questions on this level.

P — PROBLEM- SOLVING QUESTIONS
These are questions that require critical thinking and being able to make connections between different representations of information and integrating different topics. They are not familiar questions, but are able to be solved through critical analysis. Approximately **10%** of Paper 1 (Physics) will include questions on this level.



ELECTRIC MOTORS



What is an electric motor?

A motor is a device that converts electrical energy into mechanical energy.

Motor effect

Previously you learned that the mechanical rotation of a coil through a magnetic field induced an emf and a current in the coil. In motors, when a current flows through a coil and the current - carrying coil is placed in a magnetic field, **it produces a force (called a torque) that causes the coil to rotate**. This is called the **motor effect**. The motor effect is used to explain how electric motors work.

Motor effect: When a current carrying conductor is placed in a magnetic field it experiences a force.

A motor is connected to a a cell, battery power source, to supply the electrons in the conductor with energy, resulting in current flowing in the coil and in the external circuit.

Energy conversion that takes place in a motor: Motors convert electrical energy into mechanical energy.
A motor has the opposite function to a generator.

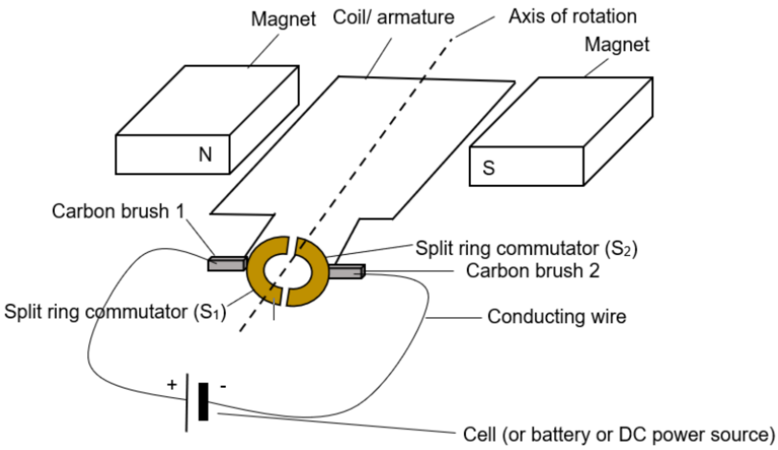
Uses of motors

Electric motors are used in common domestic appliances such as vacuum cleaners, tumble dryers, hair dryers, fans, in electric trains, in lifts, in industrial appliances such as drills etc.

The DC (direct current) motor

Direct current is current that flows in one direction. A cell or a battery is an example of a DC power source as it results in charge flowing in one direction only.
Conventional current is current that flows from the positive terminal of the cell or battery to the negative terminal of the cell or battery.
A DC motor, like a DC generator, has a split ring commutator.

The diagram below represents the simplified structure of a DC motor.



Components and the functions of the components of a DC motor

- **Magnets:** Produces a magnetic field.
  Direction of the magnetic field: Magnetic field lines run away from north towards south (i.e., North to South).

- **Coil/ armature**: Made of conducting wire (e.g. copper wire) winded into a coil or loops.

- **Split ring commutator:** The DC generator consists of **split- ring commutators**.
  (notice that the ring is split in two!)
  Split ring commutator consist of a copper ring that is split in the middle resulting in two halves which can be referred to as S_1 and S_2, which are insulated from each other. The one half of the split ring commutator (S_1) is connected to the one side of the coil and the other half of the split ring commutator (S_2) is connected to the other side of the coil.
  Each half of the split – ring commutator is in contact with (but not connected to) ONE carbon brush, for example S_1 is in contact with carbon brush B_1 and S_2 is in contact with carbon brush B_2. When the coil rotates, the split ring commutator rotates with the coil.

Function of the split ring commutator (in a motor):

The split ring commutator ensures that the current is reversed in the coil as the coil rotates through the vertical position.

- **Carbon brushes:** The DC motor, like the AC and DC generator consists of two carbon brushes, namely carbon brush **1** and carbon brush **2**. Each carbon brush makes contact with its own half of the split ring commutator; however, it is not connected to the split ring commutator.

Functions:

1. Provides electrical contact.
2. Conducts the current in the external circuit to the coil.

- **Cell/ battery/DC power source:** A device that provides a DC voltage to the external circuit, and therefore ensures that current flows.

Determining the direction of the current in the coil and in the external circuit of a motor or the direction the coil will rotate

Fleming's Left Hand Motor Rule is used to determine:

- in which direction (clockwise or anticlockwise) the coil will rotate.

OR

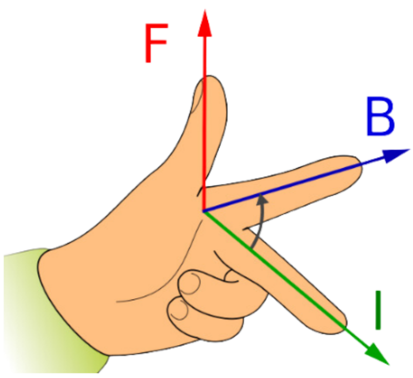
- the direction in which the current will flow in the coil.

OR

- the direction of the magnetic field (and therefore the polarity of the magnets) in a **motor**.

How to use Fleming's Left Hand Motor Rule (only applied to motors):

Hold your thumb, first finger (index finger) and second finger (middle finger) at **RIGHT ANGLES (90°)** to each other as seen below. Remember to use your **LEFT HAND** this time!



- **Thumb** points in the direction of **MOTION (Force).** F
- **First finger** points in the direction of the **magnetic FIELD, B,** from N to S. B
- **Second finger** points in the direction of the **CURRENT (conventional current).** I

Apply Fleming's Left Hand Motor rule to a DC motor, shown in **figure 6** below:

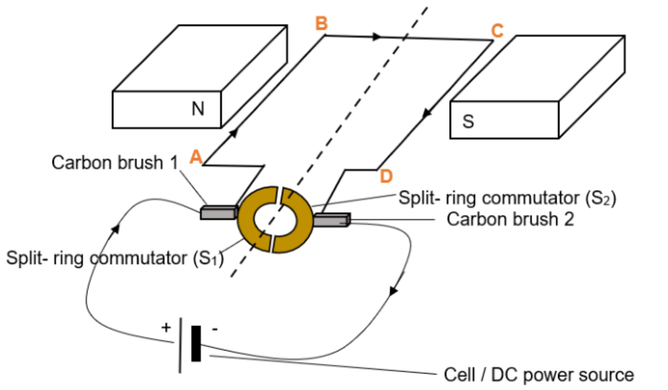


Figure 6

In **figure 6**, from the DC power source, i.e., the cell, the conventional current flows from positive to negative; from the cell via carbon brush 1, via split ring commutator $\mathbf{S_1}$, through **A** to **B** to **C** to **D** in the coil and back to the cell in the external circuit via split – ring commutator $\mathbf{S_2}$and carbon brush 2**.**

NOTE: When a current carrying conductor is placed in a magnetic field, the charges moving through the conductor experience a force. **The magnetic force is greatest when the conductor is placed at right angles to the magnetic field.**The direction of the force is at right angles to both the direction of the current and the direction of the magnetic field.

- Applying Fleming's Left Hand Motor rule to determine the direction in which the coil will rotate:

Side **AB** of the coil:

1. Make sure your thumb, first finger and second finger are at right angles to each other.
2. Point your first finger in the direction of the magnetic field (North to South) which from figure 6 is to the right.
3. Point your second finger in the direction of the current flowing in side **AB** of the coil. This is **AWAY FROM YOU** or **INTO THE PAGE.**
4. Your thumb should point **DOWN/ DOWNWARDS** in the direction of the force. Therefore, side **AB** of the coil experiences a downwards force (torque).

Repeat for side **CD** of the coil:

1. Make sure your thumb, first finger and second finger are at right angles to each other.
2. Point your first finger in the direction of the magnetic field (North to South) which from figure 6 is to the right.
3. Point your second finger in the direction of the current flowing in side **CD** of the coil. This is **TOWARDS YOU** or **OUT OF THE PAGE**.
4. Your thumb should point **UP/ UPWARDS** in the direction of the force. Therefore, side **CD** of the coil experiences an upwards force (torque).

Conclusion:

The downwards force on side AB of the coil and the upwards force on side CD of the coil, results in the coil rotating **ANTI – CLOCKWISE**.

Figure 7 is shown below where the coil has completed half a revolution (180°)

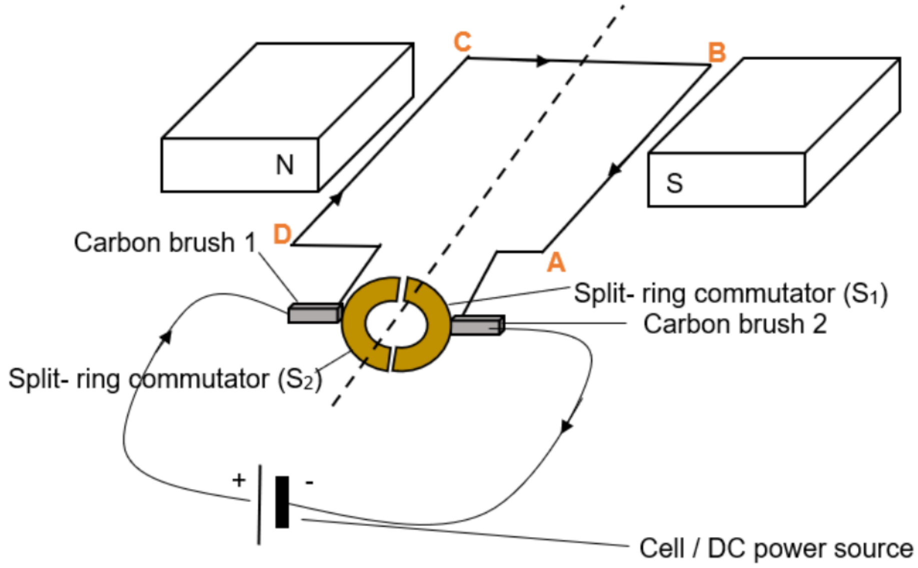


Figure 7

- As the coil moves towards the vertical position, **the force decreases**, and is zero when it is in the vertical position, this is because when the coil reaches the vertical position, the carbon brushes no longer make contact with the split ring commutator, due to the **split in the ring preventing contact**.

Remember that the carbon brushes remain stationary, but the split ring commutators rotate with the coil.

- The momentum of the coil makes it past this point, where the split ring commutator reverses the current in the coil because now the carbon brush 1 is in contact with split ring commutator 2 (S_2) hence the current now flows from the cell, via carbon brush 1, via split – ring commutator 2, from **D** to **C** to **B** to **A** (**DCBA**) and via split- ring commutator 1 (S_1), back to the cell. Side **CD** now experiences a downwards force and side **AB** an upwards force, hence, the coil **continues rotating ANTI - CLOCKWISE.**

NOTE: The current in the coil has reversed its direction.

Factors which affect the power of an electric motor

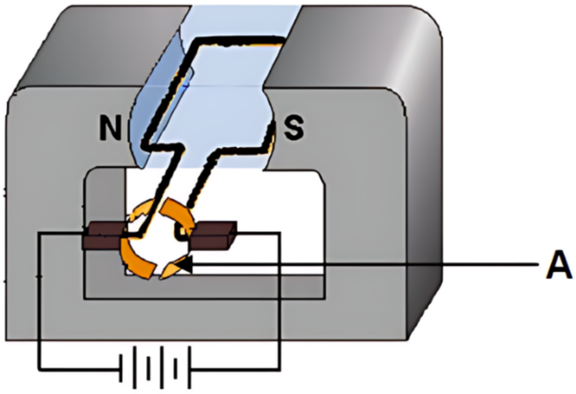
An electric motor can be made more powerful by increasing the magnetic force that turns the coil of the motor. The magnetic force that turns the coil of the motor can be **increased** by:

1. Increasing the strength of the magnetic field. Instead of using permanent magnets, use electromagnets or curved magnets.
2. Increasing the current in the coil, by increasing the emf of the DC power source, or decreasing the coils resistance (e.g., by using thicker wire).
3. Increasing the number of windings on the coil. Each coil will experience the same force, increasing the total force on each end of the coil.
4. Wrapping the coil around an iron core.
5. Using several coils wound at slightly different angles on the coil or armature.

Worked example

C + A

1. The diagram below shows a simple DC motor and its various parts.



1.1 Write down the type of energy conversion that takes place in the motor. (1)

1.2 Name the component marked **A** in the above diagram. (1)

1.3 Write down the direction (CLOCKWISE or ANTICLOCKWISE) the coil of the motor is rotating. (1)

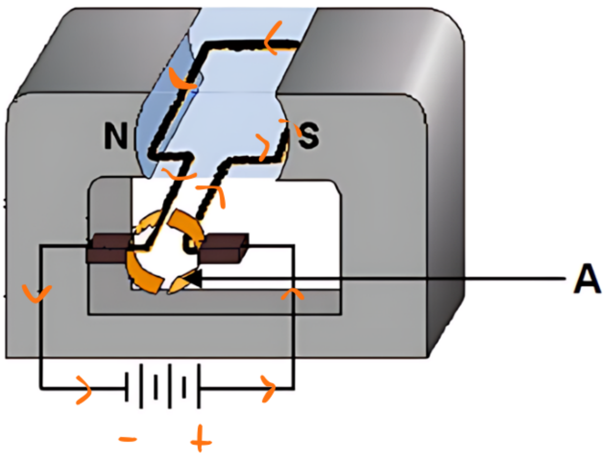
1.4 Write down ONE way in which a DC motor differs from a DC generator? (1)

1.1 Motors convert electrical energy into mechanical energy.

1.2 Split ring commutator

Remember that a DC motor (or a DC generator) has split ring commutators

1.3 Start by representing the direction of current flow.
Remember that **conventional current** flows from the POSITIVE terminal to the NEGATIVE terminal of the battery or cell, the direction of conventional current is always used.



- Applying Fleming's Left Hand Motor rule to determine the direction in which the coil will rotate:

<u>Left hand side of the coil (closest to the North end of the magnet):</u>

1. Make sure your thumb, first finger and second finger are at right angles to each other. Use your **LEFT HAND.**
2. Point your first finger in the direction of the magnetic field (North to South) which from the diagram is to the right.
3. Point your second finger in the direction of the current flowing in the left hand side of the coil. This is **TOWARDS YOU** or **OUT OF THE PAGE.**
4. Your thumb should point **UP/ UPWARD**S in the direction of the force. Therefore, the left hand of the coil experiences a upwards force (torque).

<u>Repeat for right hand side of the coil (closest to the South end of the magnet):</u>

1. Make sure your thumb, first finger and second finger are at right angles to each other.
2. Point your first finger in the direction of the magnetic field (North to South) which from the diagram is to the right.
3. Point your second finger in the direction of the current flowing in the right hand side of the coil. This is **AWAY FROM YOU** or **INTO THE PAGE.**
4. Your thumb should point **DOWN/ DOWNWARDS** in the direction of the force. Therefore, the right hand side of the coil experiences a downwards force (torque).

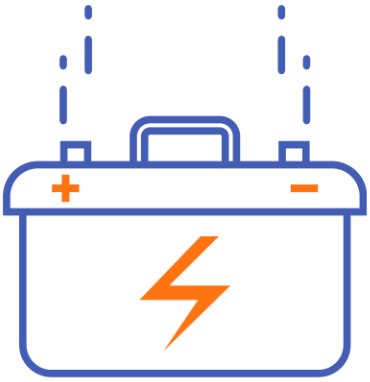
Conclusion:

The upwards force on the left side of the coil and the downwards force on right of the coil, results in the coil rotating **CLOCKWISE.**

1.4

A DC motor has a DC power source such as a cell or battery or a DC power supply, whereas a DC generator does not have a power source or power supply, it has a galvanometer/ light bulb/ ammeter or voltmeter.

NOTE: A DC generator also has an axle which is used to rotate the coil, this is not a common component in a DC motor, as a DC motor through a force, rotates the coil.



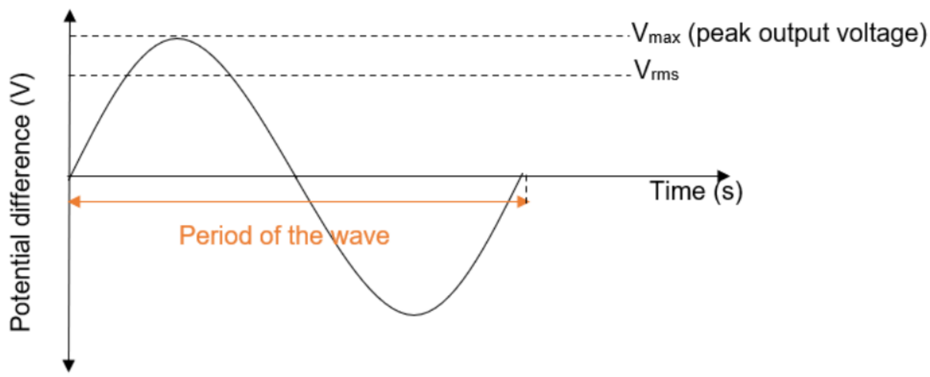
Voltage and current in an AC circuit: Alternating current theory

The voltage and current produced by a cell or battery is direct current (DC).
In DC, the electrons flow at a steadily rate in one direction in the circuit. The magnitude (size) of the direct current is unchanging (constant).
This is represented in the current - time graph shown below:

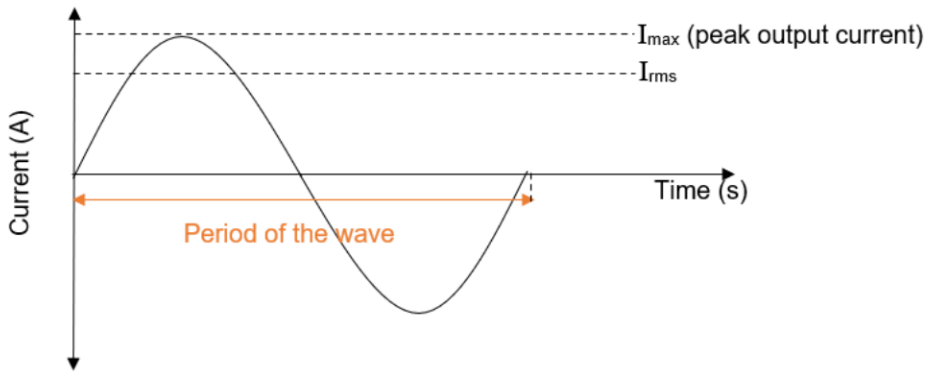
Current – time graph for a DC cell or battery:

Electric generators used at power plants produce alternating current (AC).
This alternating current is transported to our homes, shops, schools and businesses along the national power grid. The AC voltage is continually changing in magnitude and direction from zero up to a positive peak voltage (Vmax), through to zero to a negative peak voltage as shown in the diagram below:

The potential difference – time graph for an AC voltage can be represented by a sinusoidal (sine) waveform:



The current – time graph for an AC current can be represented by a sinusoidal (sine) waveform:



When you connect an appliance to the mains electricity supply at home, the appliance is supplied with **alternating current** via two wires at home, the live wire and the neutral wire. **The current in the appliance reverses direction periodically.** This means that the moving electrons keep changing direction periodically in the circuit.

Did you know?

The electricity supplied to homes in South Africa has a peak output voltage (Vmax) of approximately 311 V. This means that the output voltage alternates repeatedly between a positive maximum value (+311 V) and a negative maximum value (-311 V). This changing voltage produces an alternating current. The frequency of the alternating current is 50 Hz.

rms (root mean square) current and voltage

When calculating the **average** AC voltage and average alternating current, the sum of the positive and negative peak values is **zero** for both AC voltage and alternating current. Therefore, these values are useless to work with, especially when calculating the average power that an AC power source delivers. Instead, **the root mean square (rms) values of voltage and current is used in calculations**, for example, to calculate the average power delivered by an AC power source, **because it allows us to do calculations as if they were steady direct currents (DC).**

The effective value (rms value) of an alternating current is equal to the steady value of DC which develops the same quantity of heat energy in a resistor in a given time as AC does. Formulae to calculate V_{rms} and I_{rms}:

$$\boxed{\begin{array}{c} V_{rms} = \dfrac{V_{max}}{\sqrt{2}} \\[2ex] I_{rms} = \dfrac{I_{max}}{\sqrt{2}} \end{array}}$$

These formulae can be used to interchange between V_{rms} and V_{max}

What do these variables mean and what are the SI units?

V_{rms} = rms voltage measured in volts (V)
V_{max} = peak (output) voltage in volts (V)
I_{rms} = rms current measured in amperes (A)
I_{max} = peak (output) current measured in amperes (A)

Definition: rms voltage (V_{rms}): The rms potential difference is the AC potential difference which dissipates/produces the same amount of energy as an equivalent DC potential difference.

$$V_{rms} = \frac{V_{max}}{\sqrt{2}}$$

Definition: rms current (I_{rms}): The rms current is the alternating current which dissipates/produces the same amount of energy as an equivalent direct current (DC).

$$I_{rms} = \frac{I_{max}}{\sqrt{2}}$$

Average power (P_{ave}) in an AC circuit

Power is defined as the **rate at which work is done.** In electric circuits, it is the rate at which electrical energy is transferred.
Power is measured in Watts (W). 1 W = 1 $J.s^{-1}$.

Calculating the average power (P_{ave}) delivered by an AC source

In grade 11 you learned that power is calculated using the formula **P = VI**. In a DC circuit, the terminal voltage of the battery (V) and the current (I) is not changing. Therefore, the power (P) delivered by the DC source is not changing.
However, in an AC source, the magnitudes of both the voltage (V) and current (I) **vary** between zero and a maximum. It follows that the power (P = VI) delivered by a AC source also varies continually from zero to a maximum value.

The average power is therefore calculated as follows:

$$P_{ave} = \frac{1}{2}V_{max}I_{max} \quad \textbf{OR} \quad P_{ave} = \frac{V_{max}I_{max}}{2}$$

This can be written as:

$$P_{ave} = \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}}$$

Remember that $\sqrt{2} \times \sqrt{2} = \sqrt{4} = 2$

$$\therefore P_{ave} = V_{rms}I_{rms}$$

In summary:
The average power in an AC circuit can be calculated using the following formulae:

$$\boxed{\begin{aligned} P_{ave} &= V_{rms}I_{rms} \\ P_{ave} &= \frac{V^2_{rms}}{R} \\ P_{ave} &= I^2_{rms}R \end{aligned}}$$

PRO-TIPS

Ohm's law

$$V = IR$$
$$R = \frac{V}{I}$$

can be applied to circuits containing an AC power source.

Worked example

C

1. The AC voltage supplied to homes in South Africa is 220 V. Calculate the peak (maximum) voltage of the mains supply. (3)

Did you know? The rms voltage supplied to South African homes is between 220 V - 230 V. Therefore, the 220 V AC is the rms voltage.

---

1. $V_{rms} = \frac{V_{max}}{\sqrt{2}}$

$(220) = \frac{V_{max}}{\sqrt{2}}$

$V_{max} = (220)(\sqrt{2})$

$V_{max} = 311{,}13$ V

Worked example

C + A

2. Sarah's hairdryer is marked 1500 W and connected to a 230 V plug.

Calculate:

2.1 The maximum (peak) potential difference. (3)

2.2 The resistance of the hairdryer. (3)

2.3 The current flowing through the hairdryer. (3)

2. What information regarding Sarah's hairdryer do we have?

P_{ave} = 1500 W

V_{rms} = 230 V (This is the rms voltage supplied to the power points in homes)

2.1 The peak voltage (V_{max})can be calculated, because the rms voltage(230 V) is known:

$V_{rms} = \frac{V_{max}}{\sqrt{2}}$

$(230) = \frac{V_{max}}{\sqrt{2}}$

$V_{max} = (230)(\sqrt{2})$

$V_{max} = 325{,}27$ V

2.2 The power of the hairdryer (1500 W) and the rms voltage (230 V) across the circuit of hairdryer is known, and can be used to calculate the resistance of the hair dryer, using a power formula that has rms voltage and resistance in it:

$P_{ave} = \frac{V^2 rms}{R}$

$(1500) = \frac{(230)^2}{R}$

$1500R = 52900$

$R = \frac{52900}{1500}$

$R = 35{,}27\Omega$

2.3 There are many ways to calculate the current through the hairdryer (note this is the rms current)
The following is known:
The power of the hairdryer = 1500 W, rms voltage = 230 V (across the circuit of hairdryer) and the resistance of the hair dryer = 35,37Ω (calculated in question 2.2).

OPTION 1

Using a power formula containing the rms current:

$$P_{ave} = V_{rms}I_{rms}$$
$$(1500) = (230)I_{rms}$$
$$I_{rms} = \frac{(1500)}{(230)}$$
$$I_{rms} = 6{,}52\ A$$

OR

OPTION 2

Using a power formula containing the rms current:

$$P_{ave} = I^2{}_{rms}R$$
$$(1500) = I^2{}_{rms}(35{,}27)$$
$$I^2{}_{rms} = 42{,}529\ldots$$
$$\sqrt{I^2 rms} = \sqrt{45{,}529\ ...}$$
$$I_{rms} = 6{,}52\ A$$

OR

OPTION 3

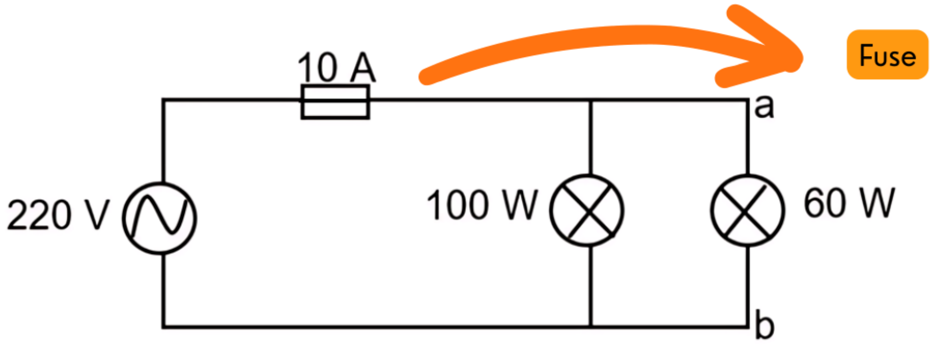
Ohm's law can be used to calculate the rms current, as long as the rms voltage is used.

$$R = \frac{V}{I}$$
$$R = \frac{V_{rms}}{I_{rms}}$$
$$(35{,}27) = \frac{(230)}{I_{rms}}$$
$$I_{rms} = \frac{(230)}{(35{,}27)}$$
$$I_{rms} = 6{,}52\ A$$

Worked example

A + P

3. Lights in most households are connected in parallel as shown in the simplified circuit below. Two light bulbs rated at 100 W; 220 V and 60 W; 220 V respectively are connected in an AC source of rms value 220 V. The fuse in the circuit can only allow 10 A.



3.1 Calculate the peak voltage of the source.
3.2 Calculate the resistance of the 100 W light bulb, when operating at optimal conditions.
3.3 An electric iron with a power rating of 2200 W, is now connected across points **a** and **b**. Explain, with the aid of a calculation, why this is not advisable.

NOTE: A fuse is an electrical safety device that provides protection to an electrical circuit. It contains a metal wire or strip that melts when too much current flows through it, thereby stopping or interrupting the flow of current.
For this example, assume the resistance of the fuse to be negligible.

3.1 The peak voltage (V_{max}) of the power source can be calculated, because the rms voltage(220 V) is known:

$$V_{rms} = \frac{V_{max}}{\sqrt{2}}$$

$$(220) = \frac{V_{max}}{\sqrt{2}}$$

$$V_{max} = (220)(\sqrt{2})$$

$$V_{max} = 311{,}13 \text{ V}$$

3.2 The power of the light bulb (100 W) and the rms voltage (220 V) across the light bulb is known, because the light bulb is in parallel to another 60 W light bulb, the potential difference across each of the light bulbs is the same as the rms voltage of the source (since there are no other resistors in the circuit - assume the resistance of the fuse is negligible). The resistance of the 100 W light bulb can be calculated using a power formula that has rms voltage and resistance in it:

$$P_{ave} = \frac{V^2 rms}{R}$$
$$(100) = \frac{(220)^2}{R}$$
$$100R = 48400$$
$$R = \frac{48400}{100}$$
$$R = 484\ \Omega$$

3.3

NOTE: When an iron is placed across points **a** and **b** it is being connected in parallel with the two light bulbs, therefore the potential difference across the iron is same as the potential difference across the two light bulbs, which is equal to the rms voltage across the AC power source, namely 220 V. The power rating of the iron is 2200 W.
The maximum current that the circuit (and fuse) can allow is 10 A. If the addition of the iron results in the TOTAL rms current exceeding 10 A, it will not be advised to connect the iron in parallel.

The rms current through the iron can be calculated, as the rms voltage (220 V) across the iron and the power of the iron (2200 W) is known:

rms current through the iron

$$P_{ave} = V_{rms} I_{rms}$$
$$(2200) = (220) I_{rms}$$
$$I_{rms} = \frac{(2200)}{(220)}$$
$$I_{rms} = 10\ A$$

The rms current through the iron, which is connected in parallel with the two other light bulbs is 10 A. Resistors in parallel divide the current in a fixed ratio, therefore the **sum of the currents** through the two light bulbs and the iron will **EXCEED 10 A**. The maximum current that can flow in the circuit (before the metal strip in the fuse melts) is 10 A, therefore, it is a safety hazard to connect the iron in parallel with the two light bulbs.

NOTE: The total rms current in the circuit does not have to calculated as it can already be concluded that the total rms current will be greater than 10 A.

OPTICAL PHENOMENA
THE PHOTOELECTRIC EFFECT
(WAVES, SOUND AND LIGHT)

The prefix "**photo**" in photoelectric effect is a Greek word, meaning **light.**
In grade 10 you learned that light has both wave-like and particle- like nature (referred to as wave - particle duality).
The photoelectric effect investigates the **particle nature of light.** Remember that light is a type of electromagnetic radiation.

Did you know?
The particles that make up light (electromagnetic waves) are called PHOTONS.

What is the photoelectric effect?

Photoelectric effect: is the process whereby electrons are ejected from a metal surface when light of a suitable frequency is incident on that surface.

The photoelectric effect was first observed by in 1887 by Heinrich Hertz, a German physicist, and explained by Albert Einstein in 1905. In order to understand the photoelectric effect, it is important to understand light and how atoms are arranged in a metal.

Revision of light and the electromagnetic spectrum

The Photoelectric effect deals with **light**. Light is a type of **electromagnetic radiation.** Examples of light include: visible light, infrared (light) and ultraviolet light. All these types of light have different frequencies and wavelengths, but travel at the speed of light, $c = 3 \times 10^{8} m.s^{-1}$, through a vacuum or air.

Figure 1 below represents the electromagnetic spectrum.

Visible light consists of a spectrum of colours:

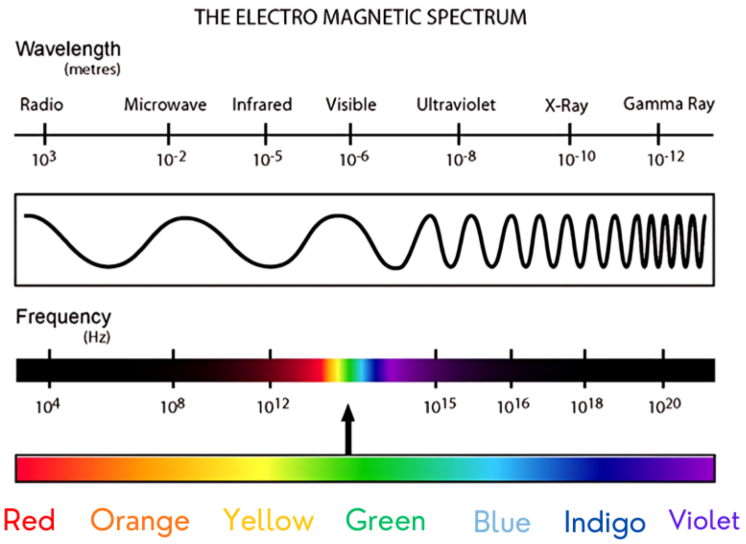


Figure 1

PRO-TIPS
The acronym '**ROYGBIV**' can be used to remember the visible light spectrum, in order of increasing frequency and decreasing wavelength.

Facts about electromagnetic waves

- Electromagnetic waves are transverse waves.
- Electromagnetic waves do not require a medium to pass through and can travel through a vacuum.
- All types of electromagnetic radiation travel at the speed of light, c, which is $3 \times 10^8 m.s^{-1}$ in a vacuum or air.
- The **universal wave equation** can be applied to all types of electromagnetic waves:

$$c = f\lambda$$

What do these variables mean and what are the SI units?

c = speed of light: $3 \times 10^8 m.s^{-1}$ in air or in a vacuum.
f = frequency in Hertz (Hz)
λ = wavelength in m

Photons - The particle nature of light (electromagnetic waves)

The photoelectric provides evidence that light has a particle - like nature. These 'particles' that make up light carry energy and are called photons.
Photon: Packet of energy found in light.

Note:
Photons have no mass and no charge.
The energy of a photon depends on the **frequency and wavelength** of the electromagnetic radiation.

Calculating the energy of a photon

Albert Einstein determined that when light is shone on a metal surface, all the energy of the photon is transferred to the electrons in the metal.

- The energy of a photon depends on the frequency of electromagnetic radiation.
- As the frequency of the electromagnetic radiation increases, the energy of the photon increases.
  For example: Blue light photons have more energy than a lower frequency red light photons.

The energy of a photon of a specific type of Electromagnetic radiation is directly proportional to the frequency of the radiation:

$$E \propto f$$

A proportionality constant, called Planck's constant, h, (in honour of Max Planck) with the constant value of $6{,}63 \times 10^{-34}$ J.s is introduced:

The equation for the energy of a photon:

$$E = hf$$

What do these variables mean and what are the SI units?

E = energy of **one** photon of the electromagnetic radiation in joules (J).
h = Planck's constant = $6{,}63 \times 10^{-34}$ J.s.
f = frequency of the electromagnetic radiation in Hertz (Hz).

The formula **E = hf** can be expanded. We know that all types of electromagnetic radiation travel at the speed of light, therefore, from the universal wave equation:

$$c = f\lambda$$

Make f the subject of the formula: $f = \frac{c}{\lambda}$ 1

$E = hf$ 2

Equation 1 into 2 $E = \frac{hc}{\lambda}$

PRO-TIPS

The energy of a photon is inversely proportional to the wavelength.
Therefore, the shorter the wavelength, the higher the energy of the photon and the longer the wavelength the lower the energy of the photon.

What do these variables mean and what are the SI units?

E = energy of the photon in J
c = speed of light in $m.s^{-1}$
h = Planck's constant ($6,63 \times 10^{-34}$ J.s)
λ = wavelength in m.

In summary:
There are two formulae to calculate the energy of a photon:

If the frequency (f) is known or asked to be calculated: **E = hf**

If the wavelength (λ) is known or asked to be calculated: $E = \frac{hc}{\lambda}$

Worked example

C + A

1. The wavelength of red light is 680 nm.
   1.1 Calculate the energy of a photon of red light. (3)
   1.2 Calculate the frequency of red light. (3)

1.1 Red light is part of the visible light spectrum and is a type of electromagnetic radiation. The wavelength of the red light is known, and must be converted to metres by $\times 10^{-9}$ this can be used to calculate the energy of ONE photon of red light, using the formula:

$$E = \frac{hc}{\lambda}$$

$$E = \frac{(6,63 \times 10^{-34})(3 \times 10^{8})}{(680 \times 10^{-9})}$$

$E = 2,925 \times 10^{-19}$ J

OR

$E = 2,93 \times 10^{-19}$ J

PRO-TIPS

Distance (wavelength) conversion chart

mm	→	m	$\times 10^{-3}$
µm	→	m	$\times 10^{-6}$
nm	→	m	$\times 10^{-9}$
pm	→	m	$\times 10^{-12}$

1.2 **OPTION 1**

Red light travels at the speed of light (c) in a vacuum. The wavelength of the red light is known, therefore the universal wave equation can be used to calculate the frequency of the red light:

$$c = f\lambda$$
$$(3 \times 10^{8}) = f(680 \times 10^{-9})$$
$$f = \frac{(3 \times 10^{8})}{(680 \times 10^{-9})}$$
$$f = 4{,}41 \times 10^{14}\ \text{Hz}$$

OR

OPTION 2

The energy of a photon of red light was calculated in question 1.1. Using the alternative formula to calculate the energy of the photon, E =hf, the frequency of red light can be determined.

$$E = hf$$
$$(2{,}925 \times 10^{-19}) = (6{,}63 \times 10^{-34})f$$
$$f = \frac{(2{,}925 \times 10^{-19})}{(6{,}63 \times 10^{-34})}$$
$$f = 4{,}41 \times 10^{14}\ \text{Hz}$$

THE PHOTOELECTRIC EFFECT EXPLAINED

Recall the phenomenon the **photoelectric effect** is the process whereby electrons are ejected from a metal surface when light of a suitable frequency is incident on that surface.

Metals

Some of the electrons in a metal are free to move in the metal. These are called delocalised valence electrons. These are electrons that are found in outermost energy level of an atom. It is these free electrons that move within the metal when a voltage is applied across the ends of a metal wire, resulting in an electric current flowing through the wire.

Figure 2 below represents how the free electrons move between the positive ions.

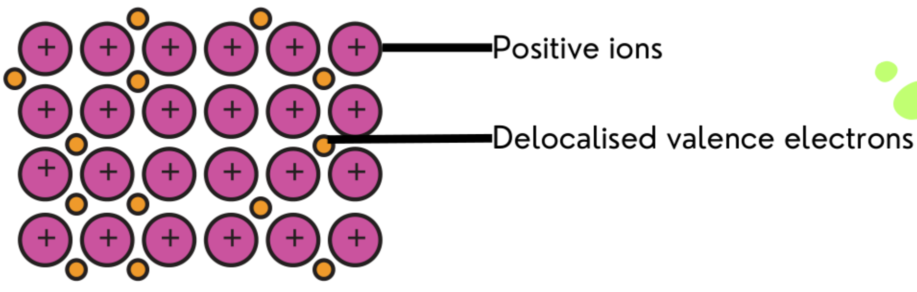


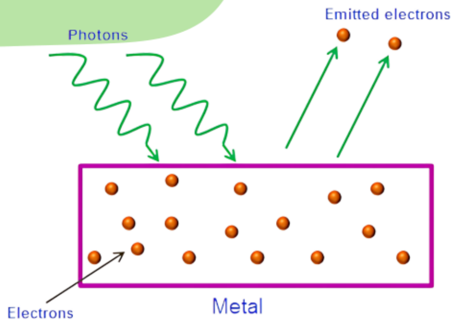
Figure 2

PRO-TIPS

The type of bonding in metal atoms is **metallic bonding.**

Photoelectric effect phenomena

- The negatively charged **free electrons** in a metal are held in the metal by the electrostatic force of attraction of the **positively** charged nuclei of the atoms.
- To remove an electron from the surface of the metal, the electron must be given enough energy to escape. When a **sufficient** amount of energy is provided in the form of light energy (as photons of light), the phenomenon is called the photoelectric effect.
- **EACH** photon of light transfers all its energy to ONE electron in the metal, if the energy of the photon is sufficient to remove an electron, the electron will be removed.
  The electrons emitted from the metal surface are called **PHOTOELECTRONS.**



NOTE:
- Not all frequencies of light cause electrons to be ejected from the surface of a metal.
- This is because each metal has its own work function (W_0).
- Different metals have different work functions because it depends on how tightly the positive ions in the metal hold onto the delocalised valence electrons.

Definition: Work function (W_0): The minimum energy that an electron in the metal needs to be emitted from the metal surface.

Examples of the work function, in Joules, of common metals are listed below:

Metal	Work function (J)
Sodium (Na)	$4,41 \times 10^{-19}$ J
Copper (Cu)	$7,53 \times 10^{-19}$ J
Zinc (Zn)	$6,63 \times 10^{-19}$ J

PRO-TIPS

The work function of a metal will either be given or you will have to calculate it.

Calculating the work function (W_0) of a metal

Depending on the metal, in order for an electron to be ejected from that metal surface, light of a certain **minimum frequency** is required, this is called the **threshold frequency** or the **cut- off frequency (f_0).**

Definition: Threshold frequency or cut - off frequency(f_0): is the minimum frequency of light needed to emit electrons from a certain metal surface.

The threshold frequency depends on the **type of metal**. Different metals have different threshold frequencies, that is minimum frequencies of light needed to remove an electron from that metal surface.

There is a **directly proportional** relationship between the threshold frequency and the work function of a metal:

$$f_0 \propto W_0$$

If a photon of light has a frequency EQUAL TO the threshold frequency (f_0), the energy of the photon will be equal to the energy of the work function (W_0):

From $E = hf$ (energy of a photon)

It follows that: $W_0 = hf_0$

Formula on the data sheet

What do these variables mean and what are the SI units?

W_0 = work function in J (joules)
h = Planck's constant $6{,}63 \times 10^{-34}$ J.s
f_0 = threshold or cut - off frequency in Hz.

Frequency of light incident on a metal surface and the kinetic energy (E_k)of the photoelectrons ejected

- **$f < f_0$** If the frequency of the light is **less than** the threshold frequency, **no** electrons will be ejected from the metal surface.

- **$f = f_0$** If the frequency of the light is **equal to** the threshold frequency, electrons will be brought to the metal surface. The kinetic energy (E_k)of the electrons = 0 J.

- **$f > f_0$** If the frequency of the light is **greater than** the threshold frequency, electrons will be ejected from the metal surface, with a **maximum kinetic energy** $E_{k(max)}$.

Explanation:

- If the photon of light has a frequency **GREATER THAN** the threshold frequency (f_0), then the energy of the photon will be greater than the work function (W_0) of the metal.
- **ALL** of the energy of the incident photon (E) is transferred to the electron. **SOME** of the energy of the photon (E) is used to remove the electron from the atom (work function), the remaining energy of the photon is transferred to the electron as **KINETIC ENERGY**. Electrons in the outermost energy level of the metal atom will be ejected with the **MAXIMUM KINETIC ENERGY**.

Energy of incident photon = Work function + maximum kinetic energy (of the photoelectron)

PRO-TIPS

This obeys the **Law of conservation of energy.**

Photoelectric effect equation

Formula on the data sheet

$E = W_0 + E_{k\,(max)}$

$E = W_0 + E_{k\,(max)}$

Expanded to:

$\therefore hf = hf_0 + \frac{1}{2} m v_{max}^2$ -------------------------- 1

OR

$\frac{hc}{\lambda} = hf_0 + \frac{1}{2} m v_{max}^2$ -------------------------- 2

Formulae on data sheet to expand the equation:

Energy of a photon:

$E = hf$ or $E = \frac{hc}{\lambda}$

Work function:

$W_0 = hf_0$

Kinetic energy

$E_k = \frac{1}{2}mv^2$

PRO-TIPS

These formulae can be adjusted and expanded, depending on the information you have and are given. Remember to always write down the original formula.

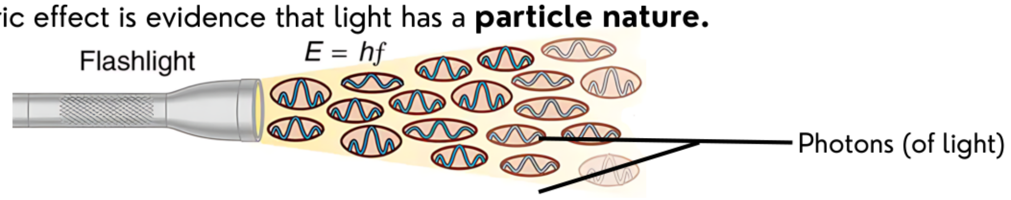
NOTE: it is an electron being ejected at the maximum speed, therefore the mass used is the mass of an electron ($9{,}11 \times 10^{-31}$ kg). This is m_e on the data sheet (under physical constants).

What do these new variables mean and what are the SI units?

E = energy of the photon in Joules (J)
m = mass of an electron = to $9{,}11 \times 10^{-31}$ kg (**given as m_e on data sheet**)
$\mathbf{v_{max}}$ = maximum speed at which the electron is ejected (photoelectron) in $m.s^{-1}$
$E_{k(max)}$ = maximum kinetic energy of the photoelectron in Joules (J).

Significance of the Photoelectric effect

The photoelectric effect is evidence that light has a **particle nature.**



Worked examples

Multiple choice question

C

1. The photoelectric effect is evidence that light has a… nature.

   A wave
   B particle
   C reflective
   D diffractive

Answer: B
The emission of free electrons from a metal surface when light of a suitable frequency is shined on it led to the conclusion that light is made up of packets of energy, called **photons**, which are **particles**.

Multiple choice question

C + A

2. Light of a frequency **f** is shined on a metal surface. Photoelectrons with a kinetic energy **K** are emitted. Light with a frequency 4**f** is now shined onto the surface of the same metal.

   Which ONE of the following represents the kinetic energy of the photoelectrons?
   A 2**K**
   B **K**
   C 4**K**
   D 16**K**

Answer: C

$E = W_0 + E_{k\ (max)}$
$hf = W_0 + E_{k\ (max)}$
$hf = W_0 + \mathbf{K}$

The work function (W_0) of the metal is constant because light is shined on the **SAME metal surface.**
Planck's constant is constant.

$\therefore f \propto \mathbf{K}$
$\therefore (4) \propto \mathbf{K}$
$\therefore 4\mathbf{K}$

Worked example

C + A

Platinum has a threshold frequency of $1,4 \times 10^{15}$ Hz.

Calculate:

1.1 The work function of platinum. (3)

1.2 The maximum wavelength of light needed to eject an electron from a platinum metal surface. (3)

1.1 The threshold frequency of platinum (f_0) is known, this can be used to determine the work function of platinum:

$W_0 = hf_0$

$W_0 = (6,63 \times 10^{-34})(1,4 \times 10^{15})$

$W_0 = 9,28 \times 10^{-19}$ J

This is the minimum energy a photon of light would need to emit an electron from a platinum surface.

1.2 The work function of platinum was calculated in question 1.1. Using the value, the formula $W_0 = hf_0$, can be adjusted by integrating the formula $c = f\lambda$, to determine the maximum wavelength of light needed to eject an electron from a platinum metal surface.

$W_0 = hf_0$

$W_0 = \frac{hc}{\lambda_0}$

$(9,28 \times 10^{-19}) = \frac{(6,63 \times 10^{-34})(3 \times 10^{8})}{\lambda_0}$

$\lambda_0 = \frac{(6,63 \times 10^{-34})(3 \times 10^{8})}{(9,28 \times 10^{-19})}$

$\lambda_0 = 2,14 \times 10^{-7}$ m

PRO-TIPS

λ_0 can be taken as the **threshold wavelength.** The maximum wavelength of light needed to eject an electron from a metal surface.

Worked example

C + A

2. The threshold frequency of caesium is $4,71 \times 10^{14}$ Hz.

2.1 Calculate the work function of caesium. (3)

2.2 Calculate the maximum speed of an ejected electron when blue light of wavelength 420 nm is shone on the caesium surface. (4)

2.1 The threshold frequency of caesium (f_0) is known, this can be used to determine the work function of caesium:

$W_0 = hf_0$
$W_0 = (6,63 \times 10^{-34})(4,71 \times 10^{14})$
$W_0 = 3,12 \times 10^{-19}$ J

This is the minimum energy a photon of light would need to emit an electron from a caesium surface.

2.2 All of the energy of the photon of blue light is transferred an electron in a caesium atom.The wavelength of blue light is known, this must be converted to metres, by x 10^{-9}
The work function of caesium was calculated in question 2.1.
Some of the energy of the photon of blue light is used to overcome the work function and the remaining energy is the kinetic energy with which the photoelectron is ejected. This can be determined and used to calculate the maximum speed with which the photoelectron is ejected:

$E = W_0 + E_{k\,(max)}$
$\frac{hc}{\lambda} = W_0 + \frac{1}{2}mv_{max}^2$
$\frac{(6,63 \times 10^{-34})(3 \times 10^{8})}{(420 \times 10^{-9})} = (3,12 \times 10^{-19}) + \frac{1}{2}(9,11 \times 10^{-31})v_{max}^2$
$v_{max}^2 = 3,54\ldots \times 10^{11}$
$\sqrt{v_{max}^2} = \sqrt{3,54 \times 10^{11}}$
$v_{max} = 595577,24 \text{ m.s}^{-1}$

Remember to expand the formula depending on the information available.

THE INTENSITY (BRIGHTNESS) OF LIGHT

The intensity (brightness) of the light determines the number of photons incident on the metal per second or per unit time and hence the number of electrons ejected per second or per unit time.

NOTE:

- A change in the intensity of the light does **NOT** change the frequency or the wavelength of the light. Therefore, a change in the intensity of the light **WILL NOT CHANGE** the **ENERGY** of the incoming photons.

Figure 3 below shows that if photons of light do not have enough energy to emit electrons from a metal surface ($E < W_0$). Increasing the intensity of the light will not result in photoelectrons being emitted because the energy of the photons remains the same.

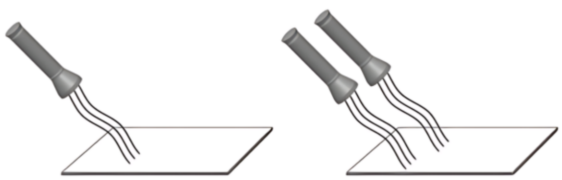


Figure 3

- Brighter light or high intensity light contains more photons than dimmer or low intensity light. Provided $E \geq W_0$, shining brighter light on a metal surface results in more photoelectrons being ejected per second or per unit time.

- The brightness or intensity of the light does **not** affect the kinetic energy (or speed) of the photoelectrons. This is because photons of the same light with the same frequency and wavelength, have the same energy, therefore each photoelectron is emitted with the same kinetic energy regardless of how bright or dim the light is.

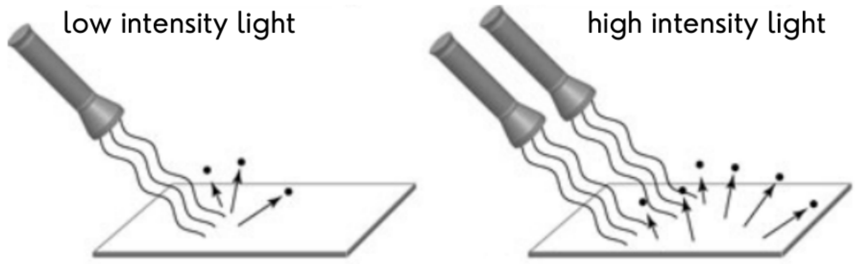


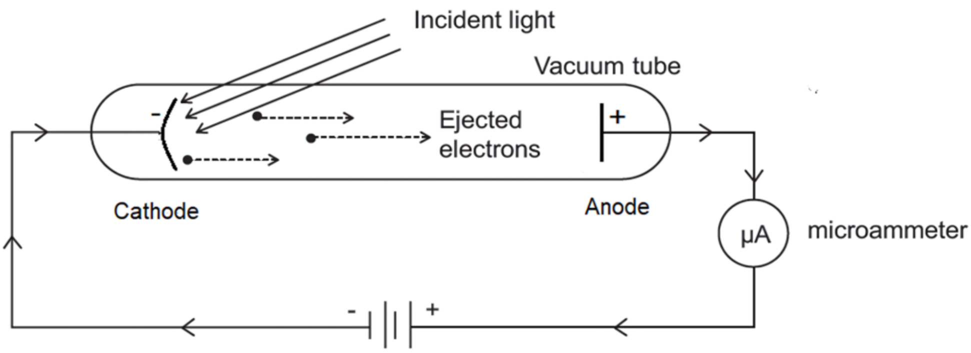
Figure 4: Low intensity light means fewer electrons are ejected per second or per unit time.

Figure 5: High intensity light means that more electrons are ejected per second or per unit time.

DEMONSTRATING THE PHOTOELECTRIC EFFECT

The photoelectric effect can be demonstrated using a **photosensitive vacuum tube** (**phototube**) as shown in **figure 6** below:

Figure 6



• A photocell has two electrodes, a cathode and an anode, enclosed in a glass vacuum tube containing an inert (unreactive) gas at low pressure.
• The cathode (-) is a curved metal plate having an emissive negatively charged surface that faces the anode (+) which is usually a single metal rod.
• When electromagnetic radiation (light) of a suitable frequency shines on the cathode, photoelectrons are emitted and attracted to the positive anode (+).
A current of a few microamperes flows through the circuit and increases with the intensity of the incident radiation.

Worked examples

Multiple choice question

C + A

1. Blue light with a low intensity is shined on the cathode of a phototube and the ammeter registers a current.
   Which ONE of the following represents the energy of a photon of blue light and reading on the ammeter if the intensity of the blue light is increased?

	ENERGY OF A PHOTON	READING ON THE AMMETER
A	Remains the same	Remains the same
B	Increases	Increases
C	Increases	Decreases
D	Remains the same	Increases

Answer: D
Increasing the intensity of the blue light does not change the frequency of the blue light, therefore it does not change the energy of a photon of blue light.

However, if the intensity of the blue light increases, this increases the number of photons that are incident onto the cathode surface per second or per unit time therefore increasing the number of electrons will be ejected from the cathode surface per second or per unit time. As a result, the ammeter reading will **INCREASE**, because more charge is flowing past a point per second.

Worked example

C + A

1. Dim green light of wavelength 570 nm is shone onto sodium metal which has a work function of 3,7 $\times 10^{-19}$ J.

   1.1 Prove, using a calculation, that no electrons will be ejected from the sodium metal's surface. (4)

1.2 The intensity of the green light is now increased and shone onto the sodium metal.
   1.2.1 Will electrons now be ejected from the sodium metal? Write down only YES or NO. (1)
   1.2.2 Explain the answer in QUESTION 1.2.1. (2)

1.1 To prove that no electrons will be ejected from sodium's metal surface, the energy of a photon of green light must be calculated and compared to the work function of sodium.The wavelength of green light is given. This must be converted to metres by x10 $^{-9}$.Since no photoelectrons are ejected from sodium's surface, it can be concluded that the energy of the photon (E) is less than sodium's work function (W_0).

$$E = \frac{hc}{\lambda}$$

$$E = \frac{(6{,}63 \times 10^{-34})(3 \times 10^{8})}{(570 \times 10^{-9})}$$

$$E = 3{,}49 \times 10^{-19}\ J$$

$3{,}49 \times 10^{-19}$ J < $3{,}7 \times 10^{-19}$ J

∴ $E < W_0$ (or the energy of the photon is less than the work function)

∴ no photoelectrons will be ejected when green light is shined onto a sodium metal surface.

1.2.1 No.

1.2.2
- Increasing the intensity of the green light does not change the frequency or wavelength of the green light. Therefore, the energy of a photon of green light remains the same.
- The energy of a photon of green light is still less than the work function of sodium.
- Therefore, no photoelectrons will be emitted.

PRO-TIPS

- Write explanations in point form.
- In **Question 1.2.2**, remember to reference:
  1. The frequency or wavelength of the light does not change when changing the intensity of light.
  2. That the energy of a photon of light therefore remains constant.

Worked example

A + P

2. Monochromatic light which has a period of 2×10^{-16} s is shined onto a metal which has a work function 2×10^{-18} J.

Monochromatic light is light of **ONE** colour.

2.1 Prove that this photon of light will be able to eject an electron from the metal. (4)

2.2 Calculate the maximum kinetic energy of the photoelectrons. (4)

2.3 The intensity of the light is decreased. How will this affect each of the following? Write down only **INCREASES**, **DECREASES** or **REMAINS THE SAME**.

2.3.1 The kinetic energy of the ejected electrons. (1)

2.3.2 The number of ejected electrons. (1)

2.1 OPTION 1

The period of the monochromatic light is given. This can used to determine the frequency of the monochromatic light since T = $T = \frac{1}{f}$
The work function of the metal is known. This can be used to determine the threshold frequency, which is the minimum frequency of light needed to eject an electron from a metal surface. The frequency of the monochromatic light and the threshold frequency can be compared. Since it is being proven that photoelectrons will be ejected, the frequency of the monochromatic light should be greater than the threshold frequency.

PRO-TIPS

Do you remember your waves, sound and light formulae? These can be applied to the Photoelectric effect:

$T = \frac{1}{f}$ $f = \frac{1}{T}$

Frequency of the monochromatic light

$f = \frac{1}{T}$
$f = \frac{1}{(2 \times 10^{-16})}$
$f = 5 \times 10^{15}$ Hz

Threshold frequency (or cut – off frequency) of the metal

$W_0 = hf_0$
$(2 \times 10^{-18}) = (6{,}63 \times 10^{-34})f_0$
$f_0 = \frac{(2 \times 10^{-18})}{(6{,}63 \times 10^{-34})}$
$f_0 = 3{,}02 \times 10^{15}$ Hz

$\therefore$ 5×10^{15} Hz > $3{,}02 \times 10^{15}$ Hz
$\therefore$ $f > f_0$ (the frequency of the light is greater than the threshold frequency)
$\therefore$ photoelectrons will be ejected.

OR

OPTION 2

The period of the monochromatic light is given. This can used to determine the frequency of the monochromatic light since $T = \frac{1}{f}$
The work function of the metal is known. The frequency of the monochromatic light can be used to determine the energy of a photon of the monochromatic light.
The energy of a photon of the monochromatic light and the work function of the metal can be compared. Since it is being proven that photoelectrons will be ejected, the energy of a photon of the monochromatic light should be greater than the work function of the metal.

PRO-TIPS

Do you remember your waves, sound and light formulae? These can be applied to the Photoelectric effect:

$T = \frac{1}{f}$ $f = \frac{1}{T}$

Frequency of the monochromatic light

$f = \frac{1}{T}$
$f = \frac{1}{(2 \times 10^{-16})}$
$f = 5 \times 10^{15}$ Hz

Energy of photon of this monochromatic light

$E = hf$
$E = (6{,}63 \times 10^{-34})(5 \times 10^{15})$
$E = 3{,}315 \times 10^{-18}$ J

$\therefore$ $3{,}315 \times 10^{-18}$ J > 2×10^{-18} J
$\therefore$ $E > W_0$ (the energy of the photon is greater than the work function of the metal)
$\therefore$ photoelectrons will be ejected.

2.2 **OPTION 1 (from option 1 in question 2.1)**

The frequency of the monochromatic light is known. This can be used to calculate the energy of a photon of this monochromatic light using the formula E = hf. The maximum kinetic energy of the photoelectrons can be calculated as the work function of the metal is known.

$E = W_0 + E_{k\,(max)}$
$hf = W_0 + E_{k\,(max)}$
$(6,63 \times 10^{-34})(5 \times 10^{15}) = (2 \times 10^{-18}) + E_{k(max)}$
$E_{k(max)} = 3,315 \times 10^{-18} - 2 \times 10^{-18}$
$E_{k(max)} = 1,315 \times 10^{18}$ J
OR
$E_{k(max)} = 1,32 \times 10^{18}$ J

OR

2.2 **OPTION 2 (from option 2 in question 2.1)**

The energy of a photon of this monochromatic light is known. The maximum kinetic energy of the photoelectrons can be calculated as the work function of the metal is known.

$E = W_0 + E_{k\,(max)}$
$(3,315 \times 10^{-18}) = (2 \times 10^{-18}) + E_{k(max)}$
$E_{k(max)} = 3,315 \times 10^{-18} - 2 \times 10^{-18}$
$E_{k(max)} = 1,315 \times 10^{18}$ J
OR
$E_{k(max)} = 1,32 \times 10^{18}$ J

2.3.1 Remains the same.

Remember that changing the intensity of the light does not change the kinetic energy with which the electrons are ejected, because it does not change the frequency or the wavelength of the light and therefore it does not change the energy of the photon of light.

2.3.2 Decreases.

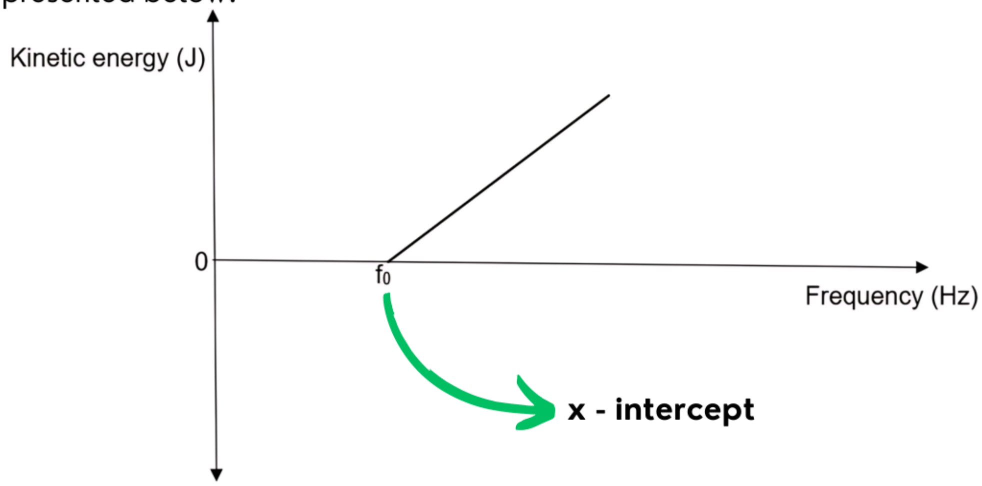
Remember that when the intensity of light is decreased, fewer photons of light are incident on the metal surface per second, or per unit time. Therefore, there will also be fewer electrons being ejected per second.

PHOTOELECTRIC EFFECT: COMMON GRAPHS

Kinetic energy versus frequency graph

Different light colours have different frequencies. As light of colours of increasing frequency are incident on a metal surface, the kinetic energy with which the electrons are ejected increases.

A kinetic energy (of the ejected electrons) versus frequency (of the incident light colours) graph is represented below:



Conclusions from the graph:

- The independent variable (x - axis) is the frequency of light.
- The dependent variable (y - axis) is the kinetic energy of the photoelectrons.
- As the frequency of the light colours increase, the kinetic energy with which the electrons are ejected, increases. The graph is a straight - line graph.
- Where the graph cuts the x - axis, the kinetic energy of the photoelectrons being ejected is zero. This frequency value is the threshold frequency (f_0). The x - intercept is therefore the threshold frequency (f_0).
- From the photoelectric effect equation:

$$E = W_0 + E_{k\,(max)}$$
$$hf = W_0 + E_{k(max)}$$
$$E_{k(max)} = hf - W_0$$

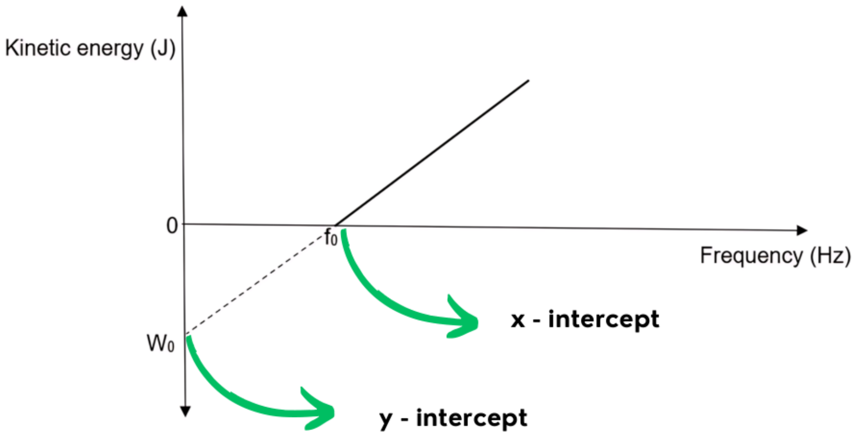
Rearrange the formula to match the variables in the equation y = mx + c

The equation of the straight - line graph: $y \quad = mx \; + \; c$

From the equation **y = mx + c:**

- The gradient (m) of the graph is Planck's constant, h. Remember that the graph is a straight - line graph with a constant gradient.
- The x– intercept of the graph is the threshold frequency (f_0)
- The y – intercept of the graph is the work function (W_0)

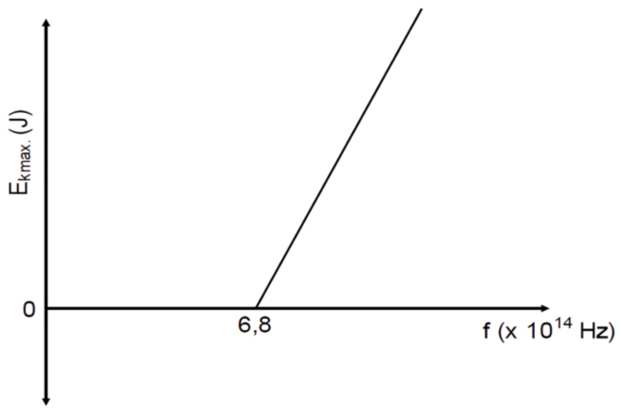
The **kinetic energy versus frequency graph** including the y - intercept, is extrapolated below:



(determined by extending the trend of the graph or extrapolating the graph)

Worked example

The graph below is obtained for an experiment on the photoelectric effect using different frequencies of light and a metal plate.



PRO-TIPS

Always take note of the variables on the x - axis and y - axis of the graph and the units.

1.1 Define the term threshold frequency (or cut- off frequency). (2)

In the experiment, the brightness of the light incident on the metal surface is increased.

1.2 State how this change will influence the speed of the photoelectrons emitted. Choose from INCREASES, DECREASES or REMAINS THE SAME. (1)

1.3 Show by means of a calculation whether the photoelectric effect will be OBSERVED or NOT OBSERVED if monochromatic light with a wavelength of 6×10^{-7} m is used in this experiment. (5)

One of the radiations used in this experiment has a frequency of $7,8 \times 10^{14}$ Hz.

1.4 Calculate the maximum speed of an ejected photoelectron. (5)

1.1 **Threshold frequency (or cut- off frequency):** The minimum frequency of light needed to emit electrons from a certain metal surface.

1.2 Remains the same.

- Increasing the intensity of the light does not change the frequency or the wavelength of the light, therefore, it does not change the energy of the photon of light.
- Therefore, the kinetic energy of the photoelectron (or electron that is ejected) remains the same, and so does the speed with which the photoelectron is emitted.

1.3 'Will the photoelectric effect be observed' is indirectly asking whether electrons will be ejected or emitted from the metal surface or not. In order for electrons to be emitted , the frequency of the light must be greater than or equal to the threshold frequency ($f \geq f_0$)

- From the graph, the threshold frequency is the x - intercept (as this is where the kinetic energy of the photoelectrons is zero).
  $f_0 = 6{,}8 \times 10^{14}$ Hz **Note the $\times 10^{14}$ on the x - axis of the graph**
- The frequency of the light source can be determined as its wavelength is known. Since light is a type of electromagnetic radiation, it travels at the speed of light, c. The formula $c = f\lambda$ can be used to determine the frequency of the light.

<u>Frequency of the light</u>

$c = f\lambda$
$(3 \times 10^8) = f(6 \times 10^{-7})$
$f = \frac{(3\times10^8)}{(6\times10^{-7})}$
$f = 5 \times 10^{14}$ Hz

<u>From the graph</u>

$f_0 = 6{,}8 \times 10^{14}$ Hz

$\therefore f < f_0$ (or the frequency of the light is less than the threshold frequency)

$\therefore$ photoelectrons will not be ejected or emitted from the metal surface (because the photons of this light colour do not have enough energy to transfer to the electrons).

1.4 $7{,}8 \times 10^{14}$ Hz (the frequency of light) is greater than the threshold frequency of this metal ($6{,}8 \times 10^{14}$ Hz), therefore the maximum speed of the photoelectrons can be determined using the photoelectric effect equation, as the energy of the photon (using the frequency of the light) can be calculated, and the work function of the metal can be determined as the threshold frequency of the metal is known. Remember, it is an electron being ejected at the maximum speed, therefore the mass used is the mass of an electron
($9{,}11 \times 10^{-31}$ kg). This is m_e on the data sheet (under physical constants).

$E = W_0 + E_{k\,(max)}$
$hf = hf_0 + \frac{1}{2}mv_{max}^2$
$(6{,}63 \times 10^{-34})(7{,}8 \times 10^{14}) = (6{,}63 \times 10^{-34})(6{,}8 \times 10^{14}) + \frac{1}{2}(9{,}11 \times 10^{-31})v_{max}^2$

$v_{max}^2 = 1{,}455\ldots \times 10^{11}$
$\sqrt{v_{max}^2} = \sqrt{1{,}455 \ldots \times 10^{11}}$
$v_{max} = 381515{,}84$ m.s^{-1} (or $3{,}82 \times 10^5$ m.s^{-1})

PHOTOELECTRIC EFFECT: SPECTRA

Analysing various types of spectra is important in astronomical spectroscopy to detect the presence of element.

There are **two** main types of spectra that will be discussed in detail in this section:

1. Atomic emission spectra
2. Atomic absorption spectra.

1. Atomic emission spectra

The word 'emission' means **emitting of light photons**.This happens when electrons transition from a higher energy state to a lower energy state.

Definition: Atomic emission spectrum: An atomic emission spectrum is formed when certain frequencies of electromagnetic radiation are emitted due to an electron making a transition from a higher energy state to a lower energy state.

1.1 Continuous emission spectra

When white light from a hot, dense substance such as light bulb or the sun is passed through a prism, the light is dispersed (split) into its component colours as shown in **figure 7** below. The band of colours produced is called a **continuous emission spectrum,** containing all seven colours of the rainbow.

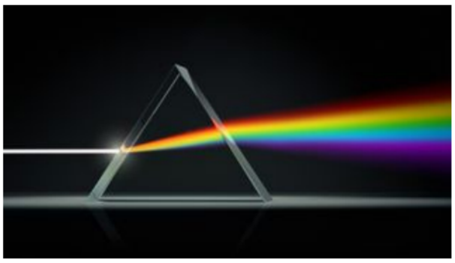
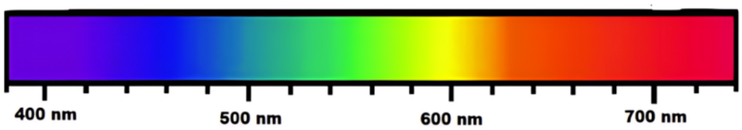


Figure 7



Note that this continuous emission spectra is arranged in increasing wavelength and decreasing frequency.

1.2 Atomic line emission spectra

Figure 8 below represents a gas discharge tube.
When a high voltage is applied across the two electrodes of a **gas discharge tube** containing a gas at low pressure. Light is emitted from the gas inside the tube. This is because some of the electrons are 'stripped' and therefore emitted from the atoms of the gas, and the gas ionises.

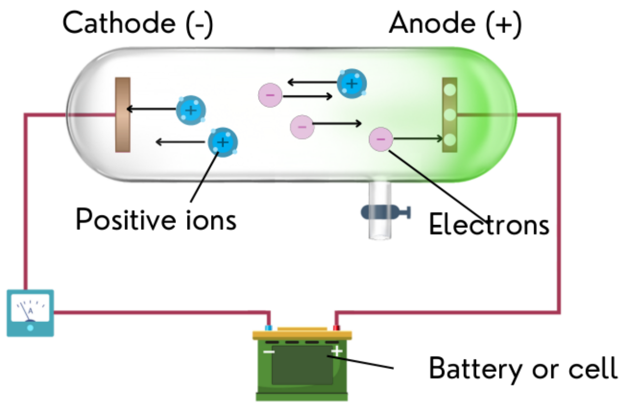
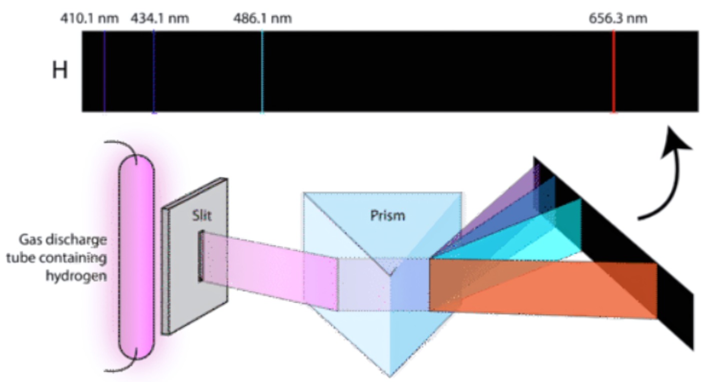


Figure 8

When the light emitted from a gas discharge tube containing a gas (hot gas) is passed through a prism (or a diffraction grating), the spectrum produced is no longer a continuous spectrum, it is an **atomic line emission spectrum.**
Each element has its own unique arrangement of electrons within the atom (electron configuration). Therefore, each gas inside a gas discharge tube with a voltage across it, is a hot gas that will glow and produce its own unique colour and its own unique atomic line emission spectrum.

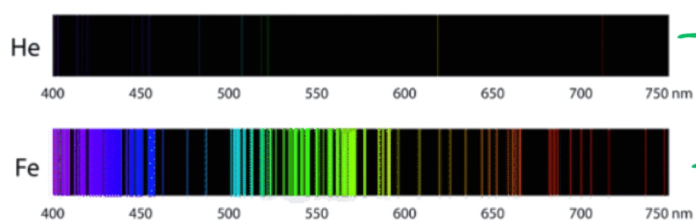
Figure 9 below represents light from a hydrogen gas discharge tube passing through a prism, producing a spectrum of lines.



Hydrogen's atomic line emission spectrum

Figure 9

Line emission spectra of other elements:



Helium's atomic line emission spectrum

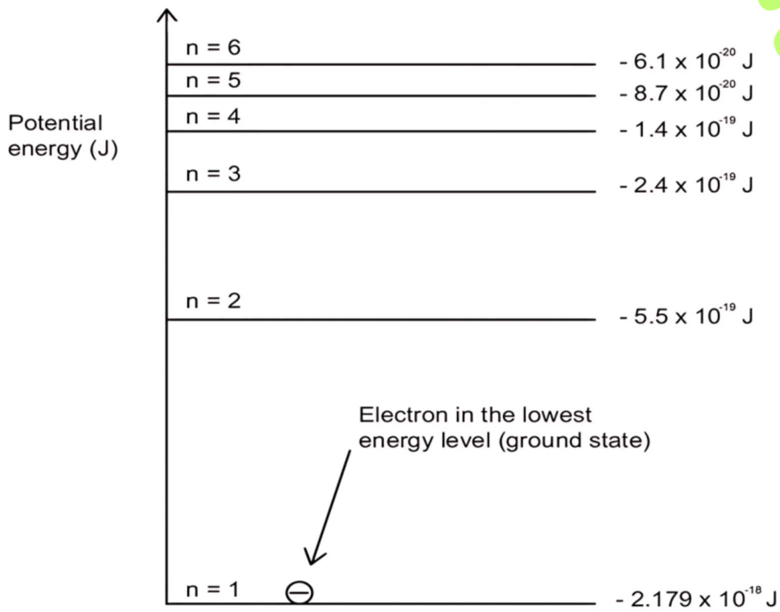
Iron's atomic line emission spectrum

When comparing the atomic line emission spectra of hydrogen, helium and iron, each element emits different frequencies and wavelengths of light. In fact, every single element emits its own unique atomic line emission spectrum.

Electron energy levels in atoms

To explain how an atomic line emission spectrum is produced, it is important to understand how the electrons in an atom behave.

Let's consider a hydrogen atom. The energy levels in a hydrogen atom can be represented as shown in **figure 10** below, with horizontal parallel lines against a vertical potential energy scale. This diagram is a similar representation to Bohr's model of the atom.



PRO-TIPS

According to Bohr's model of the atom, the electrons in different energy levels have different amounts of energy.

Figure 10

Electrons in energy level 1 (n = 1) are closest to the nucleus and in the lowest energy state. This is also called the **ground state**. The electrons are most stable in the ground state.

Why are the potential energy values in figure 10 above negative?

If an electron is free from an atom, it is considered to have zero potential energy. As the electron moves closer to the nucleus, its potential energy will decreases, hence the negative values.

Atomic line emission spectra explained

Usually, electrons occupy lower energy states, as the electrons are more stable in lower energy states.

In order for the electron to be in a higher energy level, it must **ABSORB ENERGY**. When a high voltage is applied to the discharge tube, the **ELECTRON GAINS ENERGY** and jumps to a **HIGHER ENERGY LEVEL** within the hydrogen atom. Another way an electron can absorb energy is from a photon of light.

The energy absorbed by the electron is exactly equal to the difference in energy between two energy levels. When the electrons are in a higher energy state, the hydrogen atom is said to be in the **excited state.** An excited atom is unstable. After a short while, the excited electron will fall to a lower energy level. In order to do this, it must **RELEASE ENERGY**. **It does this by emitting a photon of light.**

The movement of an electron between energy levels is known as an **electron transition.**

The atomic line emission spectra for hydrogen is represented in **figure 11** below:

Figure 11

From **figure 11** above, there are four possible transitions that can take place in a hydrogen atom.
The electron transitions that produce VIOLET, BLUE, LIGHT BLUE and RED LINES in the hydrogen line emission spectrum are shown in **figure 12** below: (see next page)

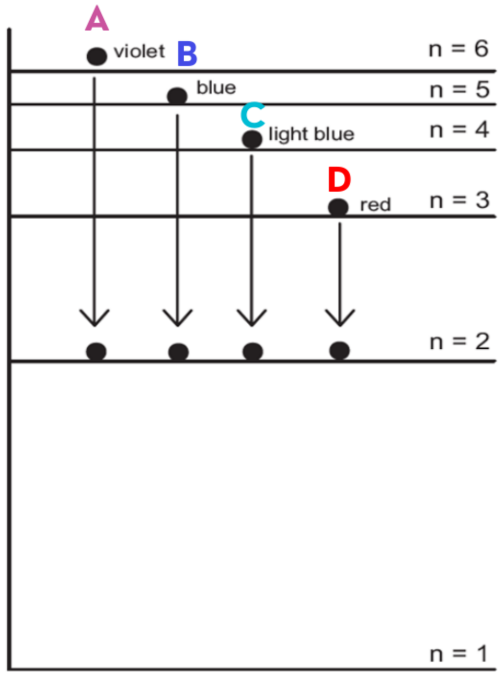


Figure 12

From **figure 12** the following transitions can be deduced:

Electron transition A : Represents the greatest change in energy of the electron. A photon of high energy will be emitted. This corresponds to the violet line in the line emission spectrum for hydrogen in **figure 12**.

Electron transition B : This transition results in a blue photon of light being emitted. The energy difference between the two levels is less than the transition that took place in A.
A photon of blue light will be emitted. This corresponds to the blue line in the line emission spectrum for hydrogen in **figure 12**.

Electron transition C : This transition results in a blue photon of light being emitted. The energy difference between the two levels is less than the transition that took place in A.
A photon of blue light will be emitted. This corresponds to the blue line in the line emission spectrum for hydrogen in **figure 12**.

Electron transition D : Represents the smallest change in energy of the electron. A photon of the lowest energy will be emitted. This corresponds to the red line in the line emission spectrum for hydrogen in **figure 12**.

Energy of the emitted photons

When an excited electron falls to a lower energy it emits a photon. The energy of the emitted photon (E = hf or $E = \frac{hc}{\lambda}$) depends on the difference in energy between the two levels:

$$E = E_{higher} - E_{lower}$$

OR

$$E = E_2 - E_1$$

Higher energy level

Lower energy level

PRO-TIPS

The energy of the photon is always positive.

Worked example

C +A

1. Refer to **figure 10** above. Consider an electron transitioning from energy level 6 to energy level 2.

1.1 Calculate the energy of the photon emitted. (3)

1.2 Calculate the wavelength of light emitted (in nm). (3)

1.3 Which colour of light is emitted with electrons making this transition? Use **figure 11** (on page 242) as a reference. (2)

1.1 The energy of the emitted photon (E) is equal to the **difference** in energy between the two energy levels ($E_6 - E_2$):

$E = E_6 - E_2$
$E = -6{,}1 \times 10^{-20} - (-5{,}5 \times 10^{-19})$
$E = 4{,}89 \times 10^{-19}$ J

1.2 The energy of the photon is known, this can be used to calculate the wavelength of a the light emitted:

$E = \frac{hc}{\lambda}$
$4{,}86 \times 10^{-19} = \frac{(6{,}63 \times 10^{-34})(3 \times 10^{8})}{\lambda}$
$4{,}86 \times 10^{-19}\lambda = (6{,}63 \times 10^{-34})(3 \times 10^{8})$
$\lambda = 4{,}09 \times 10^{-7}$ m
$\lambda \approx 410$ nm

1.3 A wavelength of 410 nm corresponds to the violet line in the line emission spectrum of hydrogen as shown in **figure 11**.

Significance of the atomic line emission spectra

Each element has its own unique atomic line emission spectrum; the line emission spectrum is like a finger print for that element. Astronomers are able to analyse the light coming from distant stars to identify the elements that are found in those starts.

2. Atomic absorption spectra

An atomic absorption is produced when WHITE LIGHT is passed through a cold gas at a low pressure e.g., hydrogen gas and is then passed through a prism or a diffraction grating. The atoms in the gas ABSORB the energy of a photon of light of certain light colours (from the white light). The photons absorbed raise electrons from lower energy levels to higher energy levels, as energy is absorbed.

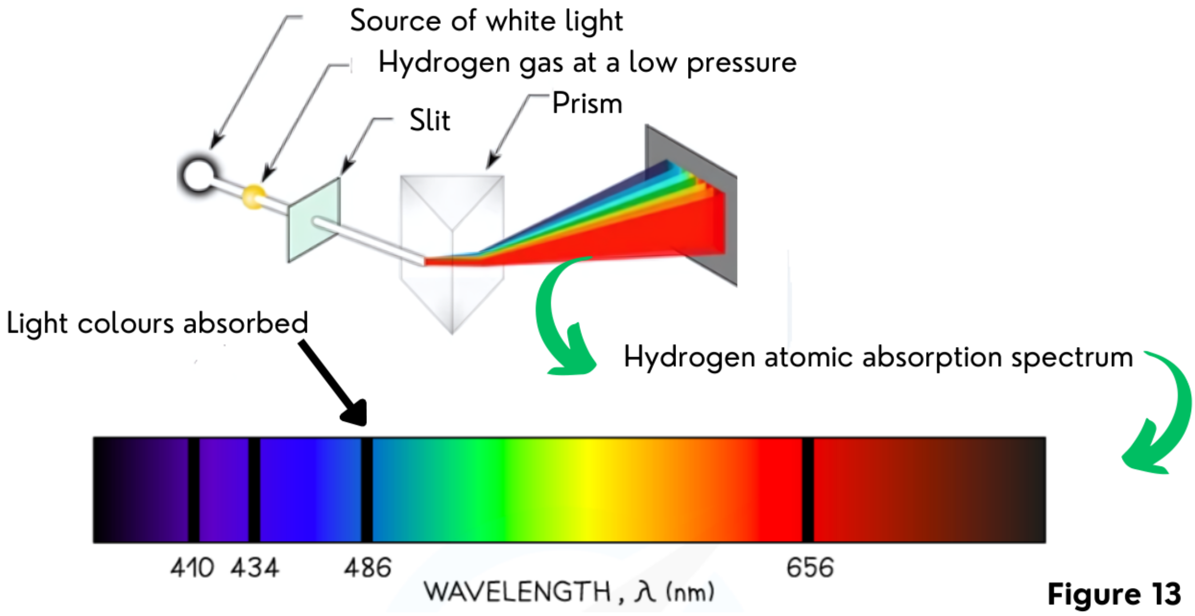


Figure 13

Distinct dark lines appear in the continuous spectrum. These are the light colours that have been **absorbed** and therefore their wavelengths and frequencies are missing from the continuous spectrum. **Figure 13** above represents the atomic absorption spectra for Hydrogen.

Definition: **Atomic absorption spectrum**: An atomic absorption spectrum is formed when certain frequencies of electromagnetic radiation passing through a substance is absorbed.

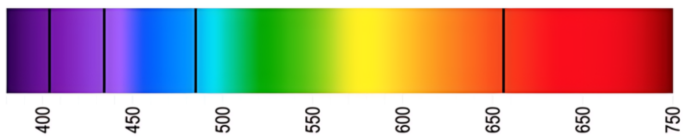
For example, when light passes through a cold gas, atoms in the gas absorb characteristic frequencies of the light and the spectrum observed is a continuous spectrum with dark lines where characteristic frequencies of light were removed. The frequencies of the absorption lines are unique to the type of atoms in the gas.

Comparing the atomic line emission and atomic absorption spectra

Comparing the atomic line emission spectrum to the atomic absorption spectrum of hydrogen as shown in **figure 14** below:

Atomic line emission spectrum for hydrogen:

Atomic line absorption spectrum for hydrogen:



The following can be concluded:

- The wavelengths and frequencies of light that is **emitted by the hot hydrogen gas** corresponds to the wavelengths and frequencies **absorbed by the cold hydrogen gas.**
- The atomic line absorption spectrum is produced when specific photons of light are absorbed by electrons.
- The atomic line emission spectrum is produced when specific photons of the same light colours as the atomic line emission spectrum are **emitted.**

Worked example

C + A

1. An electron makes a transition from an energy level of $-2{,}56 \times 10^{-19}$ J to $-2{,}24 \times 10^{-19}$ J in an atom.

1.1 Which ONE of the following will be observed due to the transition? Write down ATOMIC LINE EMISSION SPECTRA or ATOMIC ABSORPTION SPECTRA. Give a reason for the answer. (2)

1.2 Calculate the frequency of the radiation. (3)

1.1 Atomic absorption spectra. The electrons move from a lower energy level to higher energy level, therefore photons of light are **absorbed,** producing an atomic absorption spectrum.

1.2 The energy of the photon must first be determined before the frequency of the absorbed photon can be calculated:

The energy of the emitted photon (E) is equal to the difference in energy between the two energy levels:

$E = E_{higher} - E_{lower}$
$E = (-2{,}24 \times 10^{-19}) - (-2{,}56 \times 10^{-19})$
$E = 3{,}20 \times 10^{-20}$ J

This frequency of the emitted photon can be calculated, since the energy of the photon is known:

$E = hf$
$(3{,}20 \times 10^{-20}) = (6{,}63 \times 10^{-34})f$
$f = \frac{(3{,}20 \times 10^{-20})}{(6{,}63 \times 10^{-34})}$
$f = 4{,}83 \times 10^{13}$ Hz